Chapter 4

RATIO AND PROPORTION

We have already studied ratio and proportion in previous classes. In this chapter, we shall refresh that knowledge and solve a few tougher problems.

RATIO

A **ratio** is a comparison of the sizes of two or more quantities of the same kind by division.

If a and b are two quantities of the same kind (in same units), then the fraction $\frac{a}{b}$ is called the ratio of a to b. Thus, the ratio of a to $b = \frac{a}{b}$. It is written as $a : b$.

The quantities a and b are called the **terms** of the ratio, a is called the **first term** (or **antecedent**) and b is called the **second term** (or **consequent**).

‼ Remarks

- Both terms of a ratio can be multiplied or divided by the same (non-zero) number. Usually, a ratio is expressed in *lowest terms* (or *simplest form*).
- The order of the terms in a ratio is important.
- Ratio exists only between quantities of the same kind.
- Quantities to be compared (by division) must be in the same units.
- A ratio is a number, so it has no units.
- If the terms of a ratio are in fractions, convert them into natural numbers by multiplying each term by the L.C.M. of their denominators.
- To compare two ratios, convert them into equivalent like fractions.
- If a quantity increases or decreases in the ratio $a : b$, then

  new quantity $= \dfrac{b}{a}$ of the original quantity.

Ratio $a : b : c$ Defn

Three quantities of the same kind (in same units) are said to be in the ratio $a : b : c$ if the quantities are ak, bk and ck respectively, where k is any positive real number.

Similarly, four quantities of the same kind (in same units) are said to be in the ratio $a : b : c : d$ if the quantities are ak, bk, ck and dk respectively, where k is any positive real number.

Example 1. Find the ratio of each of the following in simplest form :

$\quad$ (i) 3 score to 4 dozen $\qquad$ (ii) 350 m to $1\frac{2}{5}$ km

Solution. (i) 3 score = 3 × 20 = 60,

$\qquad$ 4 dozen = 4 × 12 = 48

$\qquad \therefore$ Ratio of 3 score to 4 dozen $= \dfrac{60}{48} = \dfrac{5}{4} = 5 : 4.$

$\quad$ (ii) $1\frac{2}{5}$ km $= \dfrac{7}{5}$ km $= \left(\dfrac{7}{5} \times 1000\right)$ m = 1400 m

$\qquad \therefore$ Ratio of 350 m to $1\frac{2}{5}$ km $= \dfrac{350}{1400} = \dfrac{1}{4} = 1 : 4.$

Example 2. Simplify the following ratios :

$\quad$ (i) $2 \cdot 4 : 2\frac{2}{3}$ $\qquad\qquad$ (ii) $1\frac{1}{2} : \frac{1}{3} : \frac{1}{8}.$

Solution. (i) Given ratio $= 2 \cdot 4 : 2\frac{2}{3} = \dfrac{24}{10} : \dfrac{8}{3} = \dfrac{\frac{24}{10}}{\frac{8}{3}}$

$\qquad\qquad = \dfrac{24}{10} \times \dfrac{3}{8} = \dfrac{9}{10} = 9 : 10.$

$\quad$ (ii) Given ratio $= \dfrac{3}{2} : \dfrac{1}{3} : \dfrac{1}{8}$

> Multiply each term by the L.C.M. of their denominators

$\qquad\qquad = \dfrac{3}{2} \times 24 : \dfrac{1}{3} \times 24 : \dfrac{1}{8} \times 24$

$\qquad\qquad = 36 : 8 : 3.$

Example 3. Which ratio is greater $2\frac{1}{3} : 3\frac{1}{3}$ or $3 \cdot 6 : 4 \cdot 8$?

Solution. $2\frac{1}{3} : 3\frac{1}{3} = \dfrac{7}{3} : \dfrac{10}{3} = 7 : 10 = \dfrac{7}{10}$

and $3 \cdot 6 : 4 \cdot 8 = \dfrac{3 \cdot 6}{4 \cdot 8} = \dfrac{36}{48} = \dfrac{3}{4}$

> Convert into equivalent like fractions

L.C.M. of 10 and 4 is 20

$\dfrac{7}{10} = \dfrac{7 \times 2}{10 \times 2} = \dfrac{14}{20}$ and $\dfrac{3}{4} = \dfrac{3 \times 5}{4 \times 5} = \dfrac{15}{20}$

As $15 > 14, \dfrac{15}{20} > \dfrac{14}{20} \Rightarrow \dfrac{3}{4} > \dfrac{7}{10}.$

Hence, $3 \cdot 6 : 4 \cdot 8$ is the greater ratio.

Example 4. Arrange the following ratios in ascending order of magnitude :

$\quad$ 2 : 3, 8 : 15, 11 : 12 and 7 : 16

Solution. Given ratios are $\dfrac{2}{3}, \dfrac{8}{15}, \dfrac{11}{12}$ and $\dfrac{7}{16}$

> Convert into equivalent like fractions

L.C.M. of 3, 15, 12 and 16

$\qquad = 3 \times 4 \times 5 \times 4 = 240$

$\dfrac{2}{3} = \dfrac{2 \times 80}{3 \times 80} = \dfrac{160}{240}, \dfrac{8}{15} = \dfrac{8 \times 16}{15 \times 16} = \dfrac{128}{240}$

$\dfrac{11}{12} = \dfrac{11 \times 20}{12 \times 20} = \dfrac{220}{240}, \dfrac{7}{16} = \dfrac{7 \times 15}{16 \times 15} = \dfrac{105}{240}$

3	3,	15,	12,	16
4	1,	5,	4,	16
	1,	5,	1,	4

As $105 < 128 < 160 < 180$, $\dfrac{105}{240} < \dfrac{128}{240} < \dfrac{160}{240} < \dfrac{220}{240}$

$\Rightarrow \quad \dfrac{7}{16} < \dfrac{8}{15} < \dfrac{2}{3} < \dfrac{11}{12}$.

Hence, the given ratios in ascending order of magnitude are

$7 : 16$, $8 : 15$, $2 : 3$ and $11 : 12$.

Example 5. Divide ₹ 975 among Parul and Payush in the ratio $2\dfrac{1}{3} : 1\dfrac{5}{6}$.

Solution. Given ratio $= 2\dfrac{1}{3} : 1\dfrac{5}{6} = \dfrac{7}{3} : \dfrac{11}{6}$ L.C.M. of 3, 6 = 6

$= \dfrac{7}{3} \times 6 : \dfrac{11}{6} \times 6$

$= 14 : 11$

Sum of the terms of the ratio $= 14 + 11 = 25$

$\therefore$ Share of Parul $= \dfrac{14}{25}$ of ₹ 975 $= ₹ \left(\dfrac{14}{25} \times 975 \right) = ₹ 546$

Share of Payush $= \dfrac{11}{25}$ of ₹ 975 $= ₹ \left(\dfrac{11}{25} \times 975 \right) = ₹ 429$.

Example 6. Divide ₹ 2324 among three children in the ratio $1\dfrac{1}{4} : 1\dfrac{1}{3} : \dfrac{7}{8}$.

Solution. Given ratio $= 1\dfrac{1}{4} : 1\dfrac{1}{3} : \dfrac{7}{8} = \dfrac{5}{4} : \dfrac{4}{3} : \dfrac{7}{8}$ L.C.M. of 4, 3, 8 = 24

$= \dfrac{5}{4} \times 24 : \dfrac{4}{3} \times 24 : \dfrac{7}{8} \times 24$

$= 30 : 32 : 21$

Sum of the terms of the ratio $= 30 + 32 + 21 = 83$

Share of first child $= \dfrac{30}{83}$ of ₹ 2324 $= ₹ \left(\dfrac{30}{83} \times 2324 \right) = ₹ 840$

Share of second child $= \dfrac{32}{83}$ of ₹ 2324 $= ₹ \left(\dfrac{32}{83} \times 2324 \right) = ₹ 896$

Share of third child $= \dfrac{21}{83}$ of ₹ 2324 $= ₹ \left(\dfrac{21}{83} \times 2324 \right) = ₹ 588$.

Example 7. If $p : q = 1\dfrac{1}{2} : 3\dfrac{1}{2}$ and $q : r = 3 : 4\dfrac{2}{3}$, express $p : q : r$ in simplest terms. Also find $r : p$.

Solution. $p : q = 1\dfrac{1}{2} : 3\dfrac{1}{2} = \dfrac{3}{2} : \dfrac{7}{2} = 3 : 7$

$q : r = 3 : 4\dfrac{2}{3} = 3 : \dfrac{14}{3} = 9 : 14$

L.C.M. of two values of q *i.e.* 7 and 9 is 63. To find $p : q : r$, make q same in both cases

Thus, $p : q = 3 : 7 = 27 : 63$

$q : r = 9 : 14 = 63 : 98$

$\therefore \quad p : q : r = 27 : 63 : 98$, which is in simplest form.

Also $\quad p : r = 27 : 98 \Rightarrow r : p = 98 : 27$, which is also in simplest form.

Example 8.   If $3A = 5B = 6C$, find $A : B : C$.

Solution.   Let $3A = 5B = 6C = x$, then

$$A = \frac{x}{3}, \; B = \frac{x}{5}, \; C = \frac{x}{6}$$

$$\therefore \; A : B : C = \frac{x}{6} : \frac{x}{3} : \frac{x}{6} = \frac{1}{3} : \frac{1}{5} : \frac{1}{6} \qquad \boxed{\textbf{L.C.M. of 3, 5, 6 = 30}}$$

$$= \frac{1}{3} \times 30 : \frac{1}{5} \times 30 : \frac{1}{6} \times 30 = 10 : 6 : 5.$$

Example 9.   A certain sum of money is divided into three parts in the ratio $4 : 7 : 10$. If the second part is ₹350, find the total amount and the other two parts.

Solution.   Since the money has been divided into three parts in the ratio $4 : 7 : 10$,

Let the three parts be ₹$4x$, ₹$7x$ and ₹$10x$ respectively.

Then total amount = ₹$(4x + 7x + 10x) = $ ₹$21x$

According to the given condition, $7x = 350 \Rightarrow x = 50$

$$\therefore \quad \text{Total amount} = ₹(21 \times 50) = ₹1050$$
$$\text{1st part} = ₹(4 \times 50) = ₹200 \text{ and}$$
$$\text{3rd part} = ₹(10 \times 50) = ₹500.$$

Example 10.   Divide ₹645 into three parts such that the first part is $\frac{2}{5}$ of the second part and the ratio between second and third parts is $4 : 3$.

Solution.   Since the ratio between second and third parts is $4 : 3$, let second and third parts be ₹$4x$ and ₹$3x$ respectively.

Then first part = $\frac{2}{5}$ of ₹$4x$ = ₹$\frac{8x}{5}$.

According to the given condition, $\frac{8}{5}x + 4x + 3x = 645$

$$\Rightarrow \frac{8x + 20x + 15x}{5} = 645 \quad \Rightarrow \quad \frac{43}{5}x = 645 \quad \Rightarrow \quad x = \frac{645 \times 5}{43} = 75.$$

$\therefore$ First part = ₹$\left(\frac{8}{5} \times 75\right)$ = ₹120, second part = ₹$(4 \times 75) = ₹300$ and

third part = ₹$(3 \times 75) = ₹225.$

Example 11.   A bag contains ₹59·50 in the form of 50 paise, 25 paise and 20 paise coins in the ratio $3 : 7 : 5$. Find the number of each type of coins.

Solution.   Since 50 paise, 25 paise and 20 paise coins are in the ratio $3 : 7 : 5$, let the number of these coins be $3x$, $7x$ and $5x$ respectively.

The value of 50 paise coins = ₹$\left(\frac{50}{100} \times 3x\right)$ = ₹$\frac{3x}{2}$,

the value of 25 paise coins = ₹$\left(\frac{25}{100} \times 7x\right)$ = ₹$\frac{7x}{4}$ and

the value of 20 paise coins = ₹$\left(\frac{20}{100} \times 5x\right)$ = ₹x

According to the given condition, $\frac{3x}{2} + \frac{7x}{4} + x = 59·50$

$$\Rightarrow \quad \frac{6x + 7x + 4x}{4} = \frac{119}{2} \Rightarrow \frac{17}{4}x = \frac{119}{2} \Rightarrow x = \frac{119}{2} \times \frac{4}{17} = 14$$

∴ The number of 50 paise coins = 3 × 14 = 42,

the number of 25 paise coins = 7 × 14 = 98 and

the number of 20 paise coins = 5 × 14 = 70.

Example 12. Payal weighs 56·7 kg. If she reduces her weight in the ratio 7 : 6, find her new weight.

Solution. Weight of Payal = 56·7 kg.

Since she reduces her weight in the ratio 7 : 6,

her new weight = $\frac{6}{7}$ of 56·7 kg = $\left(\frac{6}{7} \times 56\cdot7\right)$ kg

$$= (6 \times 8\cdot1) \text{ kg} = 48\cdot6 \text{ kg.}$$

Example 13. Two numbers are in the ratio 5 : 6. When 2 is added to first and 3 is added to second, they are in the ratio 4 : 5. Find the numbers.

Solution. As the numbers are in the ratio 5 : 6, let the numbers be $5x$ and $6x$.

When 2 is added to the first number and 3 is added to the second number, then the numbers become $5x + 2$ and $6x + 3$.

Since these numbers are in the ratio 4 : 5, therefore,

$$\frac{5x + 2}{6x + 3} = \frac{4}{5} \qquad \Rightarrow \quad 5(5x + 2) = 4(6x + 3)$$

$$\Rightarrow \quad 25x + 10 = 24x + 12 \Rightarrow 25x - 24x = 12 - 10 \Rightarrow x = 2.$$

∴ The numbers are 5 × 2 and 6 × 2 *i.e.* 10 and 12.

Example 14. The ratio of the number of girls to the number of boys in a school of 720 students is 3 : 5. If 18 new boys are admitted in the school, find how many new girls may be admitted so that the ratio of number of girls to the number of boys may change to 2 : 3.

Solution. The ratio of the number of girls to the number of boys is 3 : 5.

Sum of the terms of the ratio = 3 + 5 = 8

∴ The number of girls in the school = $\frac{3}{8} \times 720 = 270$ and

the number of boys in the school = $\frac{5}{8} \times 720 = 450$.

Let the number of new girls admitted be x, then the number of girls become $(270 + x)$.

After admitting 18 new boys, the number of boys become 450 + 18 *i.e.* 468.

According to the given condition, $\frac{270 + x}{468} = \frac{2}{3}$

$$\Rightarrow \quad 3(270 + x) = 2 \times 468$$

$$\Rightarrow \quad 810 + 3x = 936 \quad \Rightarrow 3x = 936 - 810$$

$$\Rightarrow \quad 3x = 126 \quad \Rightarrow x = 42$$

Hence, the number of new girls admitted = 42.

Exercise 4.1

1. Find the ratio of each of the following in simplest form :

   (i) 7 days to 1 week

   (ii) $1\frac{1}{2}$ hours to 45 seconds

   (iii) 30 cm to 2·4 m

   (iv) 1·2 kg to 90 gm

2. Simplify the following ratios :

   (i) $2\frac{1}{3} : 4\frac{1}{5}$

   (ii) $3·5 : 5\frac{1}{2}$

   (iii) $\frac{1}{6} : \frac{1}{8} : \frac{5}{12}$

   (iv) $2\frac{1}{2} : 1\frac{1}{6} : 1\frac{3}{4}$

3. An alloy consists of $27\frac{1}{2}$ kg of copper and $2\frac{1}{2}$ kg of tin. Find the ratio by weight of tin to alloy.

4. Which ratio is greater?

   (i) 7 : 12 or 3 : 5

   (ii) $3\frac{1}{3} : 4\frac{1}{6}$ or 0·9 : 1

   (iii) $\frac{1}{2} : \frac{1}{3}$ or $\frac{1}{4} : \frac{1}{5}$

   (iv) $\frac{1}{8} : \frac{1}{6}$ or 3·5 : 4·5

5. Arrange the following ratios in ascending order of magnitude:

   (i) 3 : 4, 5 : 12 and 9 : 16

   (ii) 5 : 7, 3 : 8, 9 : 14 and 20 : 21

6. Karim secured 35 marks out of 50 in English and 29 marks out of 40 in Science. In which subject did he perform better?

7. Divide ₹ 1050 among Champa and Tara in the ratio $2\frac{2}{3} : 6\frac{2}{3}$.

8. Three children are aged 2 years, 3 years and 4 years respectively. Divide ₹ 90 among them in the ratio of their ages.

9. Divide ₹ 810 into three parts in the ratio $\frac{1}{4} : \frac{2}{5} : 1\frac{3}{8}$.

10. If A : B = 4 : 5 and B : C = 9 : 13, find A : C.

11. If A : C = 2 : 3 and B : C = 5 : 6, find A : B.

12. Find A : B : C, when

   (i) A : B = 6 : 5 and B : C = 7 : 8

   (ii) A : B = 2 : 3 and B : C = 4 : 11

   (iii) A : B = $\frac{1}{4} : \frac{1}{5}$ and B : C = $\frac{1}{7} : \frac{1}{6}$.

13. If A : B : C = 2 : 3 : 5 and B = 21, find A and C.

14. If 3A = 4B = 6C, find A : B : C.

15. A sum of money is divided in the ratio 3 : 5. If the larger part is ₹ 315, find the smaller part.

16. A sum of money is divided between Marry and Jean in the ratio 4 : 7. If Jean's share is ₹ 595, find the total amount.

17. Two numbers are in the ratio 5 : 8. If their difference is 48, find the numbers.

18. A certain sum of money is divided into three parts in the ratio 5 : 7 : 8. If the first part is ₹ 225, find the total amount and the other two parts.

19. Divide ₹1312 into three parts such that first part is $\frac{2}{3}$ of the second and the ratio between second and third parts is 4 : 7.

20. Divide ₹7095 into three parts such that the second part is 90% of the first part and the ratio between first and third parts is 5 : 7.

21. A bag contains ₹135 in the form of 1 rupee, 50 paise and 25 paise coins in the ratio 3 : 5 : 8. Find the number of each type of coins.

22. Increase the number 215 in the ratio 5 : 7.

23. A woman reduces her weight in the ratio 7 : 5. What does her weight become if originally it was 84 kg?

24. Two numbers are in the ratio 1 : 2. When 10 is added to each, they are in the ratio 2 : 3. Find the numbers.

25. Two numbers are in the ratio 9 : 10. When 3 is subtracted from the first and 10 is subtracted from the second, they are in the ratio 3 : 2. Find the numbers.

26. The ratio of number of boys to the number of girls in a school of 1430 students is 7 : 6. If 26 new girls are admitted in the school, find how many new boys may be admitted so that the ratio of number of boys to the number of girls may change to 8 : 7.

PROPORTION

*An equality of two ratios is called a **proportion**.*

Four quantities a, b, c and d are said to be in proportion if a : b = c : d i.e.

$$\text{if } \frac{a}{b} = \frac{c}{d} \text{ i.e. if } ad = bc.$$

The quantities a, b, c and d are called **terms** of the proportion; a, b, c and d are called its first, second, third and fourth terms respectively. First and fourth terms are called **extremes** (or **extreme terms**). Second and third terms are called **means** (or **middle terms**).

If $a : b = c : d$, then d is called the **fourth proportional**.

If a, b, c and d are in proportion then $\frac{a}{b} = \frac{c}{d}$ i.e. $ad = bc$ i.e.

$$\boxed{\textit{product of extremes} = \textit{product of means}.}$$

This is called **cross product rule**.

*Three quantities a, b and c of the same kind (in same units) are said to be in **continuous***

proportion *if* $a : b = b : c$ *i.e. if* $\frac{a}{b} = \frac{b}{c}$ *i.e. if* $b^2 = ac$.

If a, b and c are in continuous proportion, then the middle term b is called the **mean proportional** between a and c, a is the **first proportional** and c is the **third proportional**.

Thus, if b is mean proportional between a and c, then

$$b^2 = ac \text{ i.e. } b = \sqrt{ac}.$$

Hence, the mean proportional between a and c = $\sqrt{ac}$.

‼ Remark

☑ In a ratio $a : b$, both quantities must be of the same kind while in a proportion $a : b = c : d$, all the four quantities need not be of the same type. The first two quantities should be of the same kind and last two quantities should be of the same kind.

Example 1. Check whether the following numbers form a proportion or not :

(i) 2·4, 3·2, 1·5, 2   (ii) $1\frac{1}{5}$, $1\frac{3}{5}$, 2·1, 3·5

Solution. (i) Ratio $\quad 2\cdot4 : 3\cdot2 = \dfrac{2\cdot4}{3\cdot2} = \dfrac{24}{32} = \dfrac{3}{4}$ and

Ratio $\quad 1\cdot5 : 2 = \dfrac{1\cdot5}{2} = \dfrac{15}{20} = \dfrac{3}{4}$.

As the two ratios are equal, the numbers 2·4, 3·2, 1·5 and 2 form a proportion.

(ii) Ratio $\quad 1\frac{1}{5} : 1\frac{3}{5} = \dfrac{6}{5} : \dfrac{8}{5} = 6 : 8 = 3 : 4 = \dfrac{3}{4}$ and

ratio $\quad 2\cdot1 : 3\cdot5 = \dfrac{2\cdot1}{3\cdot5} = \dfrac{21}{35} = \dfrac{3}{5}$.

As the two ratios are not equal, the numbers $1\frac{1}{5}$, $1\frac{3}{5}$, 2·1 and 3·5 do not form a proportion.

Example 2. Find the value of x if :

(i) 21 : 27 : : 3·5 : x (ii) $3\frac{1}{3} : x : : 2\frac{1}{2} : 1\frac{1}{4}$

Solution. (i) Using cross product rule, we get

$$21 \times x = 27 \times 3\cdot5 \qquad \boxed{\text{product of extremes = product of means}}$$

$$\Rightarrow \qquad x = \frac{27 \times 3\cdot5}{21} = \frac{9 \times 3\cdot5}{7} = 9 \times 0\cdot5 = 4\cdot5.$$

(ii) Given $3\frac{1}{3} : x = 2\frac{1}{2} : 1\frac{1}{4} \qquad \Rightarrow \qquad \dfrac{10}{3} : x = \dfrac{5}{2} : \dfrac{5}{4}$

Using cross product rule, we get

$$x \times \frac{5}{2} = \frac{10}{3} \times \frac{5}{4} \qquad \boxed{\text{product of means = product of extremes}}$$

$$\Rightarrow \qquad x = \frac{10}{3} \times \frac{5}{4} \times \frac{2}{5} = \frac{5}{3} = 1\frac{2}{3}.$$

Example 3. Find the fourth proportional to $\dfrac{2}{3}, \dfrac{3}{5}, 2$.

Solution. Let the fourth proportional be x, then $\dfrac{2}{3}, \dfrac{3}{5}, 2$ and x are in proportion.

Using cross product rule, we get

$$\frac{2}{3} \times x = \frac{3}{5} \times 2 \Rightarrow x = \frac{3}{5} \times 2 \times \frac{3}{2} = \frac{9}{5} = 1\cdot8.$$

Example 4. Find the third proportional to 2·4 kg, 9·6 kg.

Solution. Let the third proportional to 2·4 kg, 9·6 kg be x kg.

Then 2·4 kg, 9·6 kg and x kg are in continued proportion

$$\Rightarrow \quad \frac{2 \cdot 4}{9 \cdot 6} = \frac{9 \cdot 6}{x} \Rightarrow \frac{24}{96} = \frac{9 \cdot 6}{x} \Rightarrow \frac{1}{4} = \frac{9 \cdot 6}{x}$$

$$\Rightarrow \quad x = 4 \times 9 \cdot 6 = 38 \cdot 4$$

Hence, the third proportional is 38·4 kg.

Example 5. Find the mean proportional between

(i) 1·25 and 1·8   (ii) $8\frac{1}{3}$ and 27.

Solution. (i) Mean proportional between 1·25 and 1·8

$$= \sqrt{1 \cdot 25 \times 1 \cdot 8} = \sqrt{2 \cdot 25} = 1 \cdot 5.$$

(ii) Mean proportional between $8\frac{1}{3}$ and 27

$$= \sqrt{\frac{25}{3} \times 27} = \sqrt{25 \times 9} = \sqrt{225} = 15.$$

Example 6. In a school, the ratio between the number of girls and the number of boys is 6 : 7. If there are 504 girls in the school, find

(i) the number of boys in the school.

(ii) the total number of students in the school.

Solution. (i) Let there be x boys in the school, then the ratio of number of girls in the school to that of boys is 504 : x.

According to the given condition, 504 : x = 6 : 7

$$\Rightarrow \quad \frac{504}{x} = \frac{6}{7} \Rightarrow 6x = 7 \times 504$$

$$\Rightarrow \quad x = \frac{7 \times 504}{6} \Rightarrow x = 7 \times 84 \Rightarrow x = 588.$$

Hence, there are 588 boys in the school.

(ii) The total number of students in the school = 504 + 588 = 1092.

Exercise 4.2

1. Which of the following statements are true?

(i) 18 : 24 = 7·5 : 10   (ii) $\frac{4}{5} : \frac{21}{20} : : \frac{2}{3} : \frac{7}{8}$

(iii) ₹12 : ₹18 = 24 toffees : 36 toffees   (iv) 5·4 kg : 2·7 kg = 3 puppies : 2 puppies

2. Check whether the following numbers are in proportion or not :

(i) 6, 9, 3, 4   (ii) 4·8, 2·7, 80, 45   (iii) 0·24, 1·2, 7·5, 37·5

3. Find x in the following proportions :

(i) 2·5 : 1·5 = x : 24   (ii) $\frac{1}{5} : \frac{2}{3} = \frac{3}{7} : x$ (iii) 0·16 : x = 0·4 : 7

4. Find the missing term in each of the following proportions :

   (i) 11 : 55 = 9 : ...   (ii) ₹ 15 : ₹ 21 = ... : 7 pencils

5. Find the fourth proportional to

   (i) 23, 115, 2·7   (ii) $\dfrac{2}{3}$, $\dfrac{4}{7}$, $1\dfrac{5}{6}$

   (iii) ₹ 8, ₹ 25, ₹ 44   (iv) ₹ 72, ₹ 168, 3 m

6. Check whether 2·5, 12·5, 62·5 are in continued proportion or not.

7. Find the third proportional to

   (i) 16, 12   (ii) 1·8, 2·7

   (iii) $3\dfrac{1}{2}$, $8\dfrac{3}{4}$ (iv) 5 rupee 40 paise, 90 paise

8. Find the mean proportional between

   (i) 3 and 48   (ii) $\dfrac{1}{7}$ and $\dfrac{4}{63}$

   (iii) 0·2 and 0·8   (iv) 0·027 and 0·003

9. If 50 is the third proportional to 8 and x, find the value of x.

10.   (i) Find the fourth proportional to 6, 7 and 8, correct to one decimal place.

      (ii) Find the third proportional to 7 and 8, correct to two decimal places.

      (iii) Find the mean proportional between 2·7 and 3·8, correct to two significant figures.

11. The heights of Gopal and Harish are in the ratio 7 : 9. If the height of Gopal is 1 m 5 cm, find the height of Harish.

Summary

- Ratio is a comparison of the sizes of two or more quantities of the same kind by division.
- If a and b are two quantities of the same kind (in same units), then the fraction $\dfrac{a}{b}$ is called the ratio of a to b and is written as $a : b$. a is called the first term (or antecedent) and b is called the second term (or consequent).
- The terms of a ratio can be multiplied or divided by the same (non-zero) number. Usually, the ratio is written in simplest form.
- Ratio is a number, so it has no units.
- If the terms of a ratio are in fractions, convert them into natural numbers by multiplying each term by the L.C.M. of their denominators.
- To compare two ratios, convert them into equivalent like fractions.
- If $a : b$ and $b : c$ are given, then to find $a : b : c$, make b same in both cases.
- If a quantity increases or decreases in the ratio $a : b$, then new quantity = $\dfrac{b}{a}$ of the original quantity.
- An equality of two ratios is called a proportion.
- Four quantities a, b, c and d are in proportion if $a : b = c : d$ i.e. if $\dfrac{a}{b} = \dfrac{c}{d}$.
- If $a : b = c : d$, then a and d are called extremes (or extreme terms) and b and c are called means (or mean terms).

➡ If $a : b = c : d$, then d is called the fourth proportional to a, b and c.

➡ a, b, c and d are in proportion if $ad = bc$ i.e. if product of extremes = product of means. This is called cross product rule.

➡ Three quantities a, b and c of the same kind (in same units) are said to be in continued proportion if $\dfrac{a}{b} = \dfrac{b}{c}$ i.e. if $b^2 = ac$.

➡ If a, b and c are in continued proportion, then b is called the mean proportional between a and c, a is called the first proportional and c is called the third proportional.

➡ The mean proportional between a and $c = \sqrt{ac}$.

Check Your Progress

1. Find the ratio of 10 minutes 15 seconds to 1 hour 13 minutes 30 seconds.

2. 5 grams of an alloy contains $3\dfrac{3}{4}$ grams copper and the rest is nickel. Find the ratio of weights of nickel and copper.

3. The ages of two sisters are in the ratio 7 : 5. If the age of elder sister is 15 years 9 months, what is the age of the younger sister?

4. The ratio between length and breadth of a rectangular plot is 3 : 2. If the breadth of the plot is 18 m, find :

   (i) the length of the plot   (ii) the perimeter of the plot

   (iii) the ratio between length and perimeter of the plot.

5. The marks of three students in a Mathematics test are in the ratio 3 : 4 : 5. If the second student obtained 76 marks, what are the marks of the other two?

6. If A : B = $\dfrac{1}{3} : \dfrac{1}{4}$ and B : C = $\dfrac{1}{6} : \dfrac{1}{5}$, find A : C and A : B : C.

7. Divide ₹ 1645 among A, B and C such that A's share : B's share = 2 : 3 and B's share : C's share = 4 : 5.

   [Hint. A : B = 2 : 3, B : C = 4 : 5 $\Rightarrow$ A : B : C = 8 : 12 : 15.]

8. Divide ₹ 1870 into three parts in such a way that half of the first part, one-third of the second part and one-sixth of the third part all are equal.

   [Hint. Let $\dfrac{1}{2}$ (1st part) = $\dfrac{1}{3}$ (2nd part) = $\dfrac{1}{6}$ (3rd part) = x

   $\Rightarrow$ 1st part = $2x$, 2nd part = $3x$, 3rd part = $6x$

   According to the given condition, $2x + 3x + 6x = 1870 \Rightarrow x = 170$.]

9. A brother and a sister share their collection of 5000 stamps in the ratio 5 : 3. The brother then shares his stamps with his two friends in the ratio 3 : 1 : 1, keeping most for himself. How many stamps does each of his friends receive?

10. The income of a man is increased in the ratio 7 : 8. If the increase in his income is ₹ 4500 per month, find his new income.

11. When the fare of a certain journey by an airlines was increased in the ratio 5 : 7, the cost of a ticket for the journey became ₹ 14210. Find the increase in the fare.

12. Find the fourth proportional to $\dfrac{1}{3}, \dfrac{1}{4}, \dfrac{1}{5}$.

13. Find the third proportional to ₹ 4 and ₹ 5.

14. Find the mean proportional between 0·5 kg and 4·5 kg.

15. Find the mean proportional of 0·1 and 0·2 correct to 2 decimal places.