Chapter 2 OPERATIONS ON SETS; VENN DIAGRAMS

In previous classes, you studied the union and intersection of two sets, and also the complement of a set. You also learnt how to draw simple Venn diagrams to represent sets and the various relationships between them. In this chapter, we shall strengthen these ideas. We shall also introduce another operation on sets—difference of two sets.

OPERATIONS ON SETS

Union of sets

*The **union** of two sets A and B, written as $A \cup B$ (read as 'A union B'), is the set consisting of all those elements which belong to either A or B or both.*

Thus, $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$.

For example :

(*i*) If $A = \{a, b, c, d, e, f\}$ and $B = \{a, e, i, o, u\}$, then

$A \cup B = \{a, b, c, d, e, f, i, o, u\}$.

(*ii*) If $A = \{1, 5, 9\}$ and $B = \{0, 2, 4, 6, 8, 10\}$, then

$A \cup B = \{1, 5, 9, 0, 2, 4, 6, 8, 10\}$.

(*iii*) If $A = \{0, 3, 6, 9, 12, 15, 18, 21, 24\}$ and $B = \{6, 12, 18, 24\}$, then

$A \cup B = \{0, 3, 6, 9, 12, 15, 18, 21, 24\} = A$.

Intersection of sets

*The **intersection** of two sets A and B, written as $A \cap B$ (read as 'A intersection B'), is the set consisting of all those elements which belong to both A and B.*

Thus, $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$.

For example :

(*i*) If $A = \{a, b, c, d, e, f\}$ and $B = \{a, e, i, o, u\}$, then

$A \cap B = \{a, e\}$.

(*ii*) If $A = \{1, 5, 9\}$ and $B = \{0, 2, 4, 6, 8, 10\}$, then

$A \cap B = \phi$.

(*iii*) If $A = \{0, 3, 6, 9, 12, 15, 18, 21, 24\}$ and $B = \{6, 12, 18, 24\}$, then

$A \cap B = \{6, 12, 18, 24\} = B$.

Difference of two sets

If A and B be two sets, then $A - B$ is the set consisting of all those elements which belong to A but do not belong to B.

Thus, $A - B = \{x \mid x \in A \text{ and } x \notin B\}$.

Similarly, $B - A = \{x \mid x \in B \text{ and } x \notin A\}$.

Note that A – B is the set consisting of elements of A **only** and B – A is the set consisting of elements of B **only**.

For example :

(*i*) If A = {*a*, *b*, *c*, *d*, *e*, *f*} and B = {*a*, *e*, *i*, *o*, *u*}, then
A – B = {*b*, *c*, *d*, *f*} and B – A = {*i*, *o*, *u*}.
Note that A – B ≠ B – A.

(*ii*) If A = {1, 5, 9} and B = {0, 2, 4, 6, 8, 10}, then
A – B = {1, 5, 9} and B – A = {0, 2, 4, 6, 8, 10}.

(*iii*) If A = {0, 3, 6, 9, 12, 15, 18, 21, 24} and B = {6, 12, 18, 24}, then
A – B = {0, 3, 9, 15, 21} and B – A = ϕ.

Complement of a set

If ξ is the universal set and A is any set, then the ***complement*** *of A, denoted by A′ or $\overline{A}$ or A^c (read as 'complement of A'), is the set consisting of all those elements of ξ which do not belong to A.*

Thus, A′ = {x | x ∈ ξ and x ∉ A}.

Note that A′ = ξ – A.

For example :

(*i*) If A = {1, 3, 5, 7, 9} and ξ = {1, 2, 3, ..., 10}, then
A′ = {2, 4, 6, 8, 10}.

(*ii*) If A = {*a*, *b*, *c*, *d*, *e*, *f*} and ξ = {letters of English alphabet}, then
A′ = {last twenty letters of English alphabet}.

(*iii*) If A = {January, June, July} and ξ = {months of a year}, then
A′ = {months of a year which do not begin with letter 'J'}.

Remarks

- If A is any set, then
  (*i*) A ∪ ϕ = A, A ∪ ξ = ξ, A ∪ A = A
  (*ii*) A ∩ ϕ = ϕ, A ∩ ξ = A, A ∩ A = A.
- ξ′ = ϕ, ϕ′ = ξ.
- If A is any set, then
  A ∪ A′ = ξ, A ∩ A′ = ϕ.
- If A and B are any sets, then
  (*i*) A ∪ B = B ∪ A, A ∩ B = B ∩ A (Commutative laws)
  (*ii*) A ⊆ A ∪ B, B ⊆ A ∪ B, A ∪ B ⊆ ξ
  (*iii*) A ∩ B ⊆ A, A ∩ B ⊆ B
  (*iv*) A – B = A ∩ B′, B – A = B ∩ A′
  (*v*) (A ∪ B)′ = A′ ∩ B′, (A ∩ B)′ = A′ ∪ B′. (De Morgan's laws)
- If A and B are two sets, then
  (*i*) A and B are disjoint sets if and only if A ∩ B = ϕ
  (*ii*) A and B are overlapping sets if and only if A ∩ B ≠ ϕ.
- If A ⊆ B, then
  (*i*) A ∪ B = B, A ∩ B = A (*ii*) A – B = ϕ.

Example 1. If A = {factors of 24} and B = {factors of 36}, then find

(*i*) A ∪ B (*ii*) A ∩ B

(*iii*) A – B (*iv*) B – A.

Solution. The given sets in the roster form are :

A = {1, 2, 3, 4, 6, 8, 12, 24} and B = {1, 2, 3, 4, 6, 9,12, 18, 36}

(*i*) A ∪ B = {1, 2, 3, 4, 6, 8, 12, 24, 9, 18, 36}

(*ii*) A ∩ B = {1, 2, 3, 4, 6, 12}

(*iii*) A – B = {8, 24}

(*iv*) B – A = {9, 18, 36}.

Example 2. If A = {letters of SECUNDRABAD} and B = {letters of BENGALURU}, then find

(*i*) A ∪ B (*ii*) A ∩ B

(*iii*) A – B (*iv*) B – A.

Also verify that :

(*a*) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(*b*) $n(A - B) = n(A \cup B) - n(B)$

(*c*) $n(B - A) = n(B) - n(A \cap B)$

(*d*) $n(A - B) + n(B - A) + n(A \cap B) = n(A \cup B)$.

Solution. The given sets in the roster form are :

A = {S, E, C, U, N, D, R, A, B} and

B = {B, E, N, G, A, L, U, R}

(*i*) A ∪ B = {S, E, C, U, N, D, R, A, B, G, L}.

(*ii*) A ∩ B = {E, U, N, R, A, B}.

(*iii*) A – B = {S, C, D}.

(*iv*) B – A = {G, L].

Write each element of the set once and only once

Verification

Here $n(A) = 9$, $n(B) = 8$, $n(A \cup B) = 11$, $n(A \cap B) = 6$,

$n(A - B) = 3$ and $n(B - A) = 2$. Therefore,

(*a*) $n(A) + n(B) - n(A \cap B) = 9 + 8 - 6 = 11 = n(A \cup B)$

(*b*) $n(A \cup B) - n(B) = 11 - 8 = 3 = n(A - B)$

(*c*) $n(B) - n(A \cap B) = 8 - 6 = 2 = n(B - A)$

(*d*) $n(A - B) + n(B - A) + n(A \cap B) = 3 + 2 + 6 = 11 = n(A \cup B)$.

Example 3. If ξ = {all digits in our number system}, A = {x : x is prime}, B = {x : x is a factor of 18} and C = {multiples of 3}, then verify the following :

(*i*) (A ∪ B)′ = A′ ∩ B′ (*ii*) (A ∩ B)′ = A′ ∪ B′

(*iii*) A – C ≠ C – A (*iv*) A – B = A ∩ B′.

Solution. Here ξ = {0, 1, 2, ..., 9}

A = {2, 3, 5, 7},

B = {1, 2, 3, 6, 9} and

C = {0, 3, 6, 9}.

A, B, C are subsets of ξ

(*i*) A ∪ B = {1, 2, 3, 5, 6, 7, 9}

∴ (A ∪ B)′ = {0, 4, 8}

Also, $A' = \{0, 1, 4, 6, 8, 9\}$ and $B' = \{0, 4, 5, 7, 8\}$

$\therefore \quad A' \cap B' = \{0, 4, 8\}$

Hence, $(A \cup B)' = A' \cap B'$.

(*ii*) $A \cap B = \{2, 3\}$

$\therefore \quad (A \cap B)' = \{0, 1, 4, 5, 6, 7, 8, 9\}$

Also $A' \cup B' = \{0, 1, 4, 6, 8, 9\} \cup \{0, 4, 5, 7, 8\}$

$= \{0, 1, 4, 6, 8, 9, 5, 7\}$

Hence, $(A \cap B)' = A' \cup B'$.

(*iii*) $A - C = \{2, 5, 7\}$ and $C - A = \{0, 3, 9\}$

Hence, $A - C \neq C - A$.

(*iv*) $A - B = \{5, 7\}$,

$A \cap B' = \{2, 3, 5, 7\} \cap \{0, 4, 5, 7, 8\} = \{5, 7\}$

Hence, $A - B = A \cap B'$.

Some basic results about cardinal number :

- If A and B are finites sets, then
  - (*i*) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
  - (*ii*) $n(A - B) = n(A \cup B) - n(B) = n(A) - n(A \cap B)$
  - (*iii*) $n(B - A) = n(A \cup B) - n(A) = n(B) - n(A \cap B)$
  - (*iv*) $n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$
- *If ξ (universal set) is finite set and A is any set, then* $n(A) + n(A') = n(\xi)$.

These results are very useful in solving problems.

Example 4. If $n(\xi) = 40$, $n(A) = 25$, $n(B) = 12$ and $n((A \cup B)') = 8$, find

(*i*) $n(A \cup B)$ (*ii*) $n(A \cap B)$ (*iii*) $n(A - B)$.

Solution. (*i*) We know that $n(A \cup B) + n((A \cup B)') = n(\xi)$

$[\because n(A) + n(A') = n(\xi)]$

$\Rightarrow \quad n(A \cup B) + 8 = 40$

$\Rightarrow \quad n(A \cup B) = 40 - 8 = 32.$

(*ii*) We know that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\Rightarrow \quad 32 = 25 + 12 - n(A \cap B)$

$\Rightarrow \quad n(A \cap B) = 25 + 12 - 32 = 5.$

(*iii*) We know that $n(A - B) = n(A) - n(A \cap B)$

$\Rightarrow \quad n(A - B) = 25 - 5 = 20.$

Example 5. If $n(\xi) = 50$, $n(A) = 35$, $n(B) = 20$ and $n((A \cap B)') = 40$, find

(*i*) $n(A')$ (*ii*) $n(B')$ (*iii*) $n(A \cap B)$

(*iv*) $n(A \cup B)$ (*v*) $n((A \cup B)')$ (*vi*) $n(B - A)$.

Solution. (*i*) $n(A') = n(\xi) - n(A) = 50 - 35 = 15$

(*ii*) $n(B') = n(\xi) - n(B) = 50 - 20 = 30$

(*iii*) $n(A \cap B) = n(\xi) - n((A \cap B)') = 50 - 40 = 10$

(*iv*) $n(A \cup B) = n(A) + n(B) - n(A \cap B) = 35 + 20 - 10 = 45$

(*v*) $n((A \cup B)') = n(\xi) - n(A \cup B) = 50 - 45 = 5$

(*vi*) We know that $n(B - A) = n(A \cup B) - n(A)$

$\Rightarrow \quad n(B - A) = 45 - 35 = 10.$

Example 6. If $n(A - B) = 15$, $n(B - A) = 10$ and $n(A \cap B) = 5$, find

(i) $n(A \cup B)$ (ii) $n(A)$ (iii) $n(B)$.

Solution. (i) We know that $n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$

$\Rightarrow n(A \cup B) = 15 + 10 + 5 = 30$

(ii) We know that $n(A - B) = n(A) - n(A \cap B)$

$\Rightarrow 15 = n(A) - 5 \Rightarrow n(A) = 15 + 5 = 20$

(iii) We know that $n(B - A) = n(B) - n(A \cap B)$

$\Rightarrow 10 = n(B) - 5 \Rightarrow n(B) = 10 + 5 = 15$.

Exercise 2.1

1. If A $\{x : x \in \mathbf{N}$ and $3 < x < 7\}$ and B = $\{x : x \in \mathbf{W}$ and $x \le 4\}$, find
   (i) $A \cup B$ (ii) $A \cap B$ (iii) $A - B$ (iv) $B - A$.
2. If P = $\{x : x \in \mathbf{W}$ and $x < 6\}$ and Q = $\{x : x \in \mathbf{N}$ and $4 \le x < 9\}$, find
   (i) $P \cup Q$ (ii) $P \cap Q$ (iii) $P - Q$ (iv) $Q - P$.
   Is $P \cup Q \supset P \cap Q$?
3. If A = {letters of word INTEGRITY} and B = {letters of word RECKONING}, find
   (i) $A \cup B$ (ii) $A \cap B$ (iii) $A - B$ (iv) $B - A$.
   Also verify that :
   (a) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
   (b) $n(A - B) = n(A \cup B) - n(B) = n(A) - n(A \cap B)$
   (c) $n(B - A) = n(A \cup B) - n(A) = n(B) - n(A \cap B)$
   (d) $n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$.
4. If A = {factors of 24} and B = {factors of 30}, then find
   (i) $A \cup B$ (ii) $A \cap B$ (iii) $A - B$ (iv) $B - A$.
   Also verify that :
   (a) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
   (b) $n(A - B) = n(A) - n(A \cap B) = n(A \cup B) - n(B)$
   (c) $n(B - A) = n(B) - n(A \cap B) = n(A \cup B) - n(A)$
   (d) $n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$.
5. If $\xi = \{1, 2, 3, ..., 9\}$, A = {1, 2, 3, 4, 6, 7, 8} and B = {4, 6, 8}, then find
   (i) A' (ii) B' (iii) $A \cup B$ (iv) $A \cap B$
   (v) $A - B$ (vi) $B - A$ (vii) $(A \cap B)'$ (viii) $A' \cup B'$.
   Also verify that :
   (a) $(A \cap B)' = A' \cup B'$ (b) $n(A) + n(A') = n(\xi)$
   (c) $n(A \cap B) + n((A \cap B)') = n(\xi)$ (d) $n(A - B) + n(B - A) + n(A \cap B) = n(A \cup B)$.
6. If $\xi = \{x : x \in \mathbf{W}, x \le 10\}$, A = $\{x : x \ge 5\}$ and B = $\{x : 3 \le x < 8\}$, then verify that:
   (i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$
   (iii) $A - B = A \cap B'$ (iv) $B - A = B \cap A'$.
7. If $n(A) = 20$, $n(B) = 16$ and $n(A \cup B) = 30$, find $n(A \cap B)$.

8. If $n(\xi) = 32$, $n(A) = 20$, $n(B) = 16$ and $n((A \cup B)') = 4$, find :
   (i) $n(A \cup B)$ (ii) $n(A \cap B)$ (iii) $n(A - B)$.

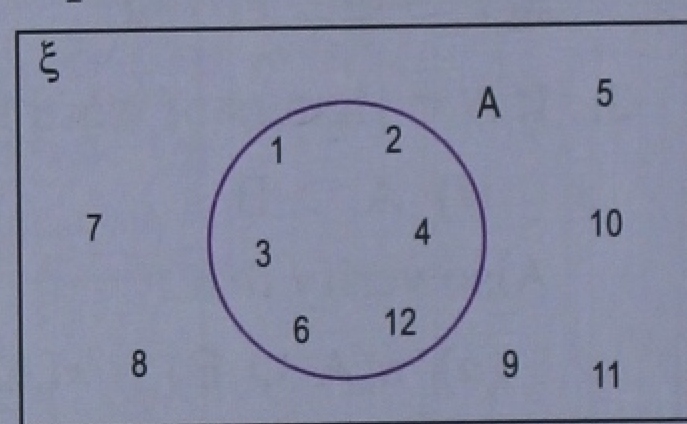
9. If $n(\xi) = 40$, $n(A') = 15$, $n(B) = 12$ and $n((A \cap B)') = 32$, find :
   (i) $n(A)$ (ii) $n(B')$ (iii) $n(A \cap B)$
   (iv) $n(A \cup B)$ (v) $n(A - B)$ (vi) $n(B - A)$.

10. If $n(A - B) = 12$, $n(B - A) = 16$ and $n(A \cap B) = 5$, find :
   (i) $n(A)$ (ii) $n(B)$ (iii) $n(A \cup B)$.

VENN DIAGRAMS

Often diagrams help us to understand and solve problems. Many ideas about sets and various relationships between them can be visualized by means of geometric figures known as Venn diagrams. Usually the universal set ξ is represented by a rectangle and its subsets by closed figures within the rectangle, such as circles, ovals etc. If necessary, we mark individual elements as points inside the diagram. Sometimes points are not marked, only the elements are written inside the diagram.

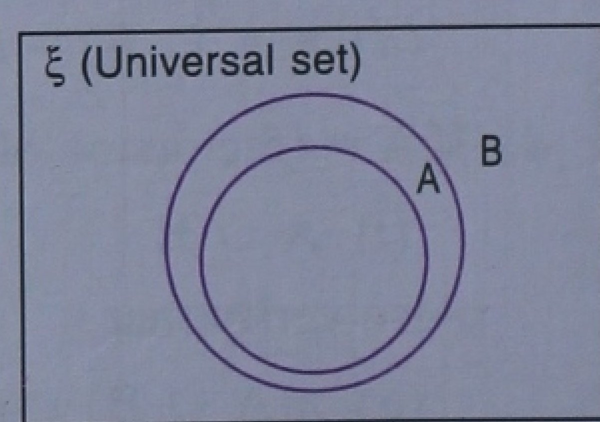
Thus, the set A of factors of 12
i.e. {1, 2, 3, 4, 6, 12} can be represented by the adjoining Venn diagram.
Here, ξ = {1, 2, 3, ..., 12}.



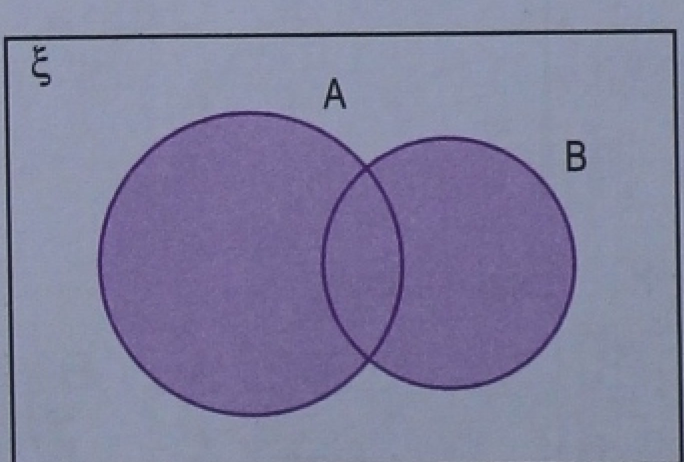
Venn diagrams showing relationships between given sets

If A, B and C are sets whose members are represented by points within circles, then some relationships between them are shown in the following Venn diagrams :

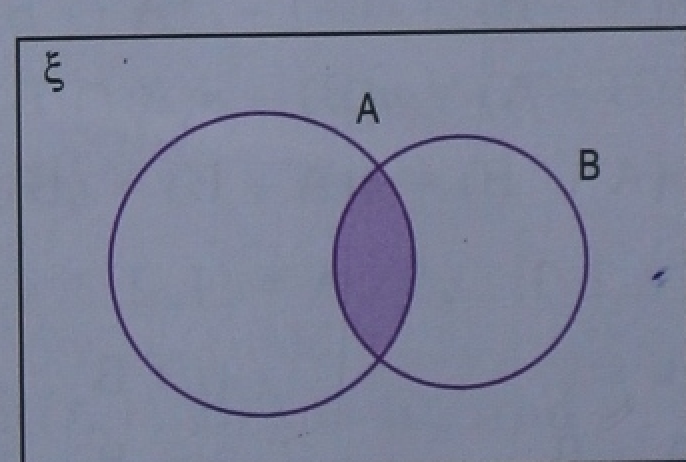
(i) The adjoining diagram represents that $A \subseteq B$.



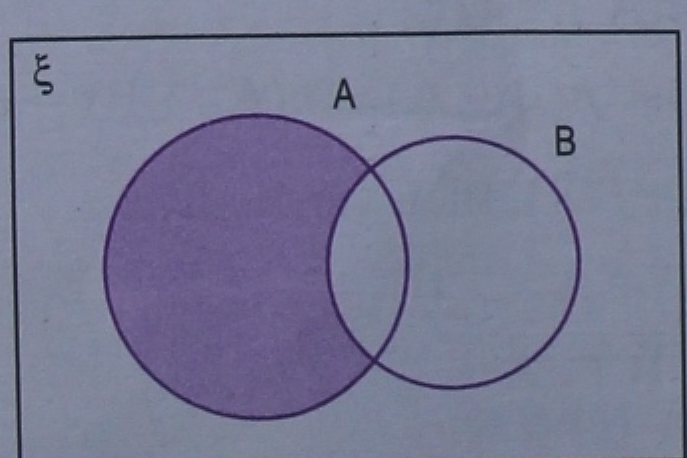
(ii) If the sets A and B are overlapping sets, then $A \cup B$, $A \cap B$, $A - B$ and $B - A$ may be shown as



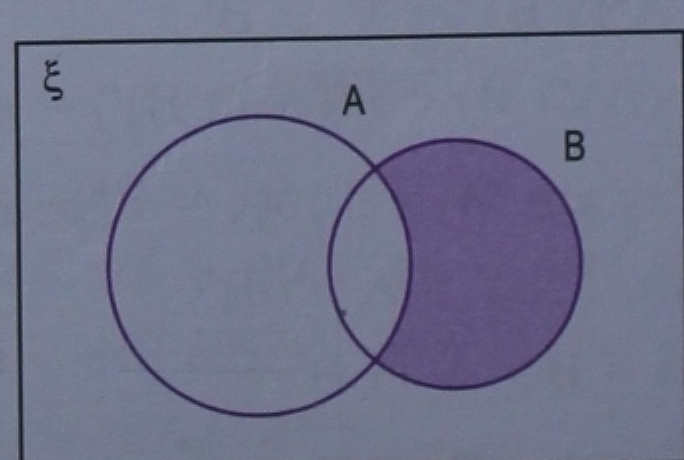
$A \cup B$ (shaded)



$A \cap B$ (shaded)

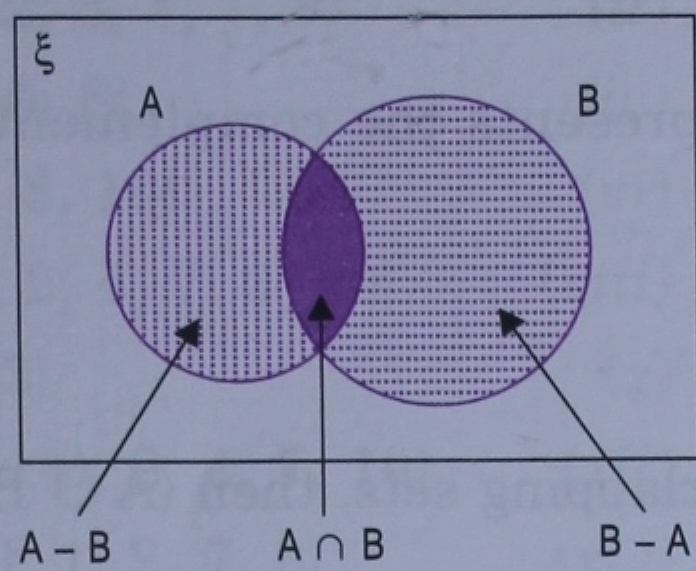


$A - B$ (shaded)



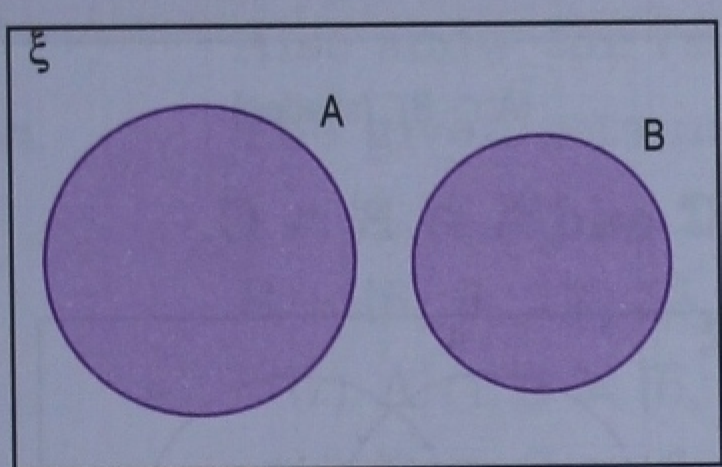
$B - A$ (shaded)

The sets A ∪ B, A ∩ B, A – B and B – A can be shown by single diagram as

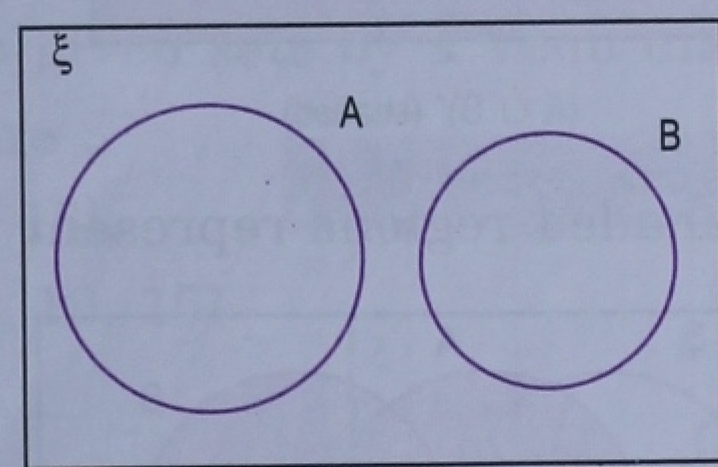


A ∪ B is represented by the whole shaded region.

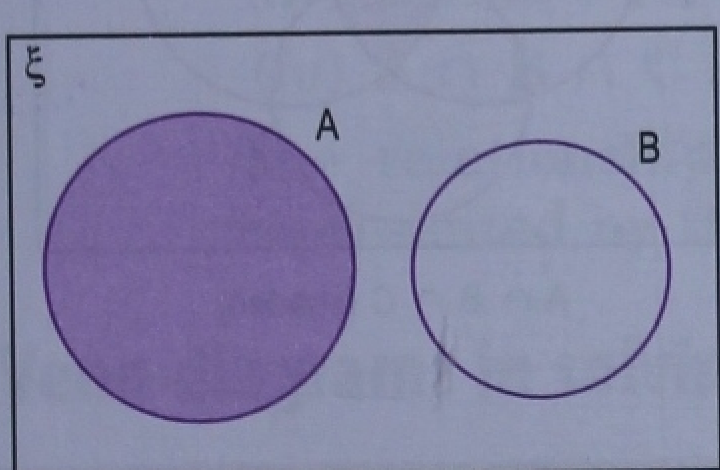
(*iii*) If the sets A and B are disjoint sets, then A ∪ B, A ∩ B, A – B and B – A may be shown as



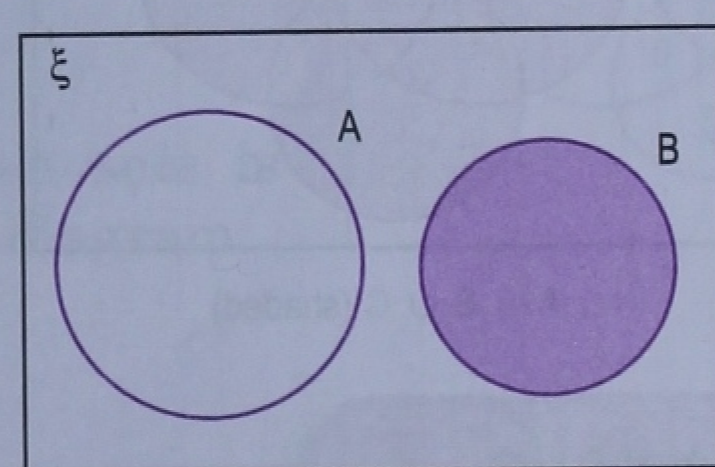
A ∪ B (shaded)



A ∩ B = ϕ

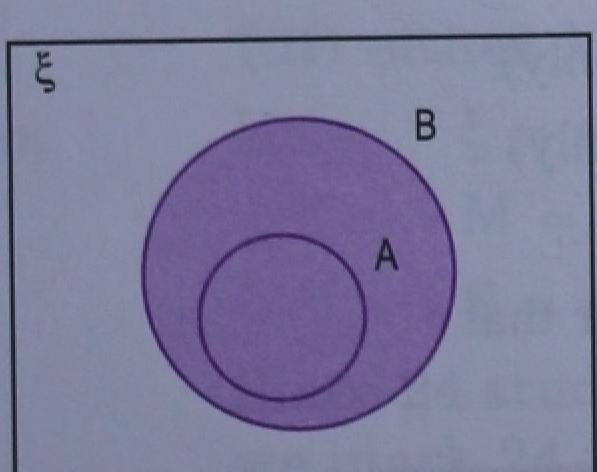


A – B (shaded)

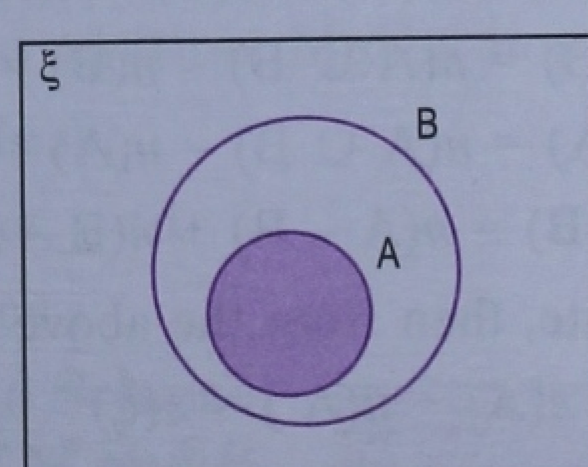


B – A (shaded)

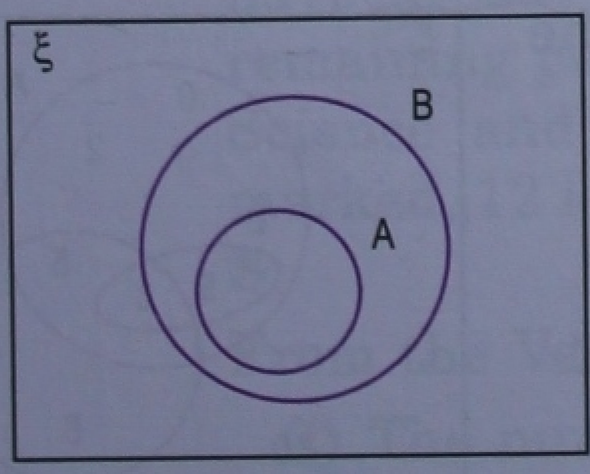
(*iv*) If A ⊆ B, then A ∪ B, A ∩ B, A – B and B – A may be shown as



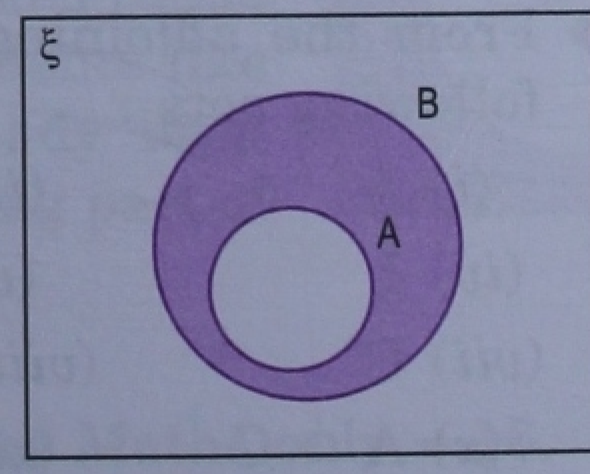
A ∪ B (shaded)



A ∩ B (shaded)

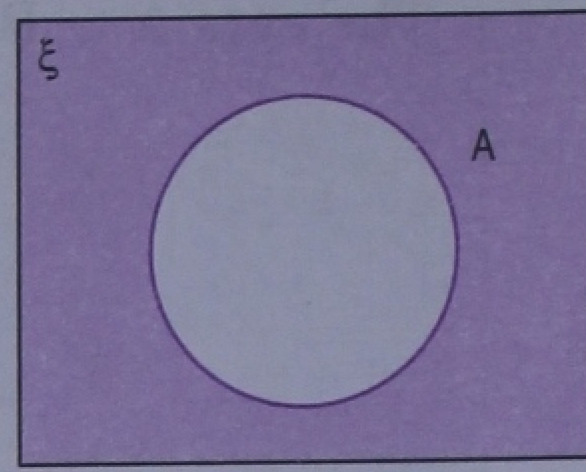


A – B = ϕ



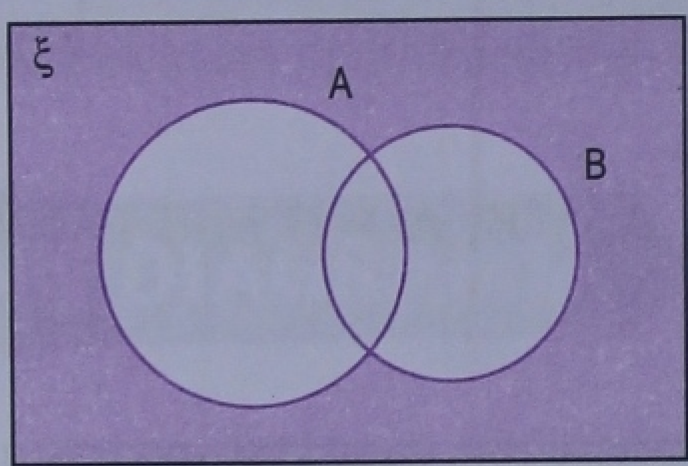
B – A (shaded)

(*v*) The adjoining diagram represents the complement of set A *i.e.* the set A′.

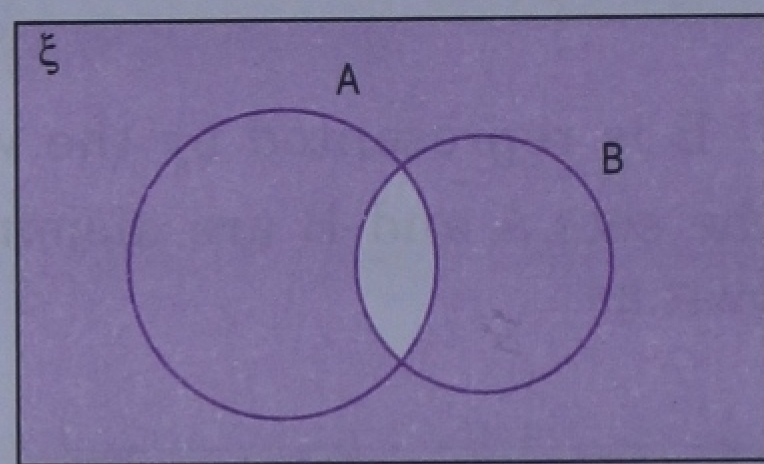


A′ (shaded)

(*vi*) If the sets A and B are overlapping sets, then (A ∪ B)′ and (A ∩ B)′ may be shown as

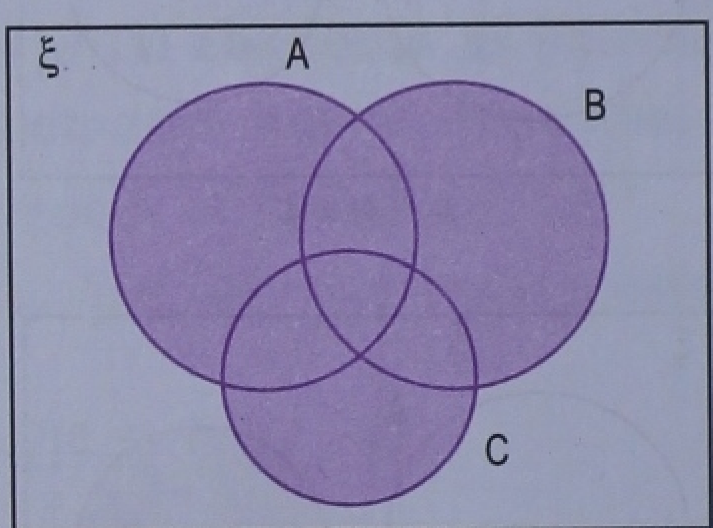


(A ∪ B)′ (shaded)

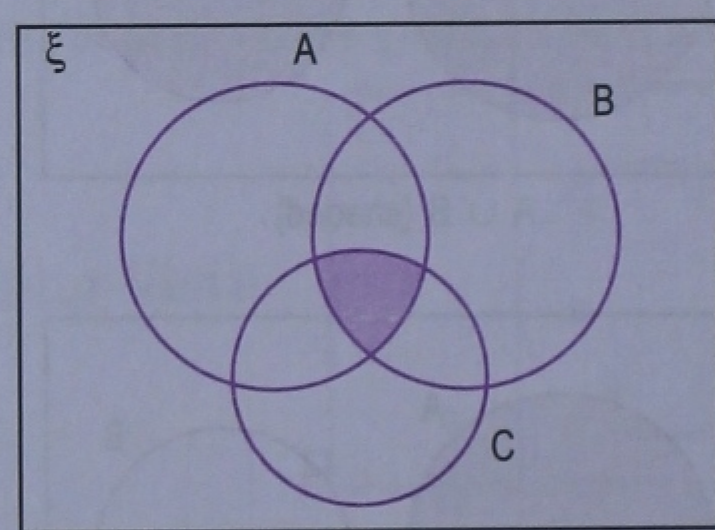


(A ∩ B)′ (shaded)

(*vii*) The shaded regions represent the sets A ∪ B ∪ C and A ∩ B ∩ C.



A ∪ B ∪ C (shaded)



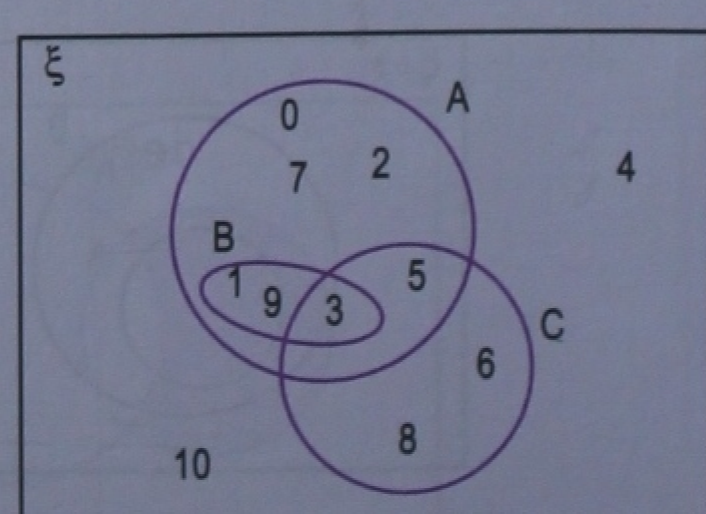
A ∩ B ∩ C (shaded)

Remarks

- If A and B are finite sets, then from the above Venn diagrams (*ii*), (*iii*) and (*iv*) it is clear that
  1. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
  2. $n(A - B) = n(A \cup B) - n(B) = n(A) - n(A \cap B) = n(A \text{ only})$
  3. $n(B - A) = n(A \cup B) - n(A) = n(B) - n(A \cap B) = n(B \text{ only})$
  4. $n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$
- If ξ is finite, then from the above Venn diagram (*v*) it is clear that

$$n(A) + n(A') = n(\xi)$$

Example 1. From the adjoining Venn diagram, find the following sets :



(*i*) A	(*ii*) B	(*iii*) C
(*iv*) ξ	(*v*) A′	(*vi*) B′
(*vii*) C′	(*viii*) B ∪ C	(*ix*) B ∩ C
(*x*) A ∩ C	(*xi*) (B ∪ C)′	(*xii*) A – B
(*xiii*) A – C	(*xiv*) B – C	(*xv*) C – B.

Solution. From the given Venn diagram, we find that

(*i*) A = {0, 1, 2, 3, 5, 7, 9} (*ii*) B = {1, 3, 9}

(*iii*) C = {3, 5, 6, 8} (*iv*) ξ = {0, 1, 2, ..., 10}

(*v*) A′ = {4, 6, 8, 10} (*vi*) B′ = {0, 2, 4, 5, 6, 7, 8, 10}

(*vii*) C′ = {0, 1, 2, 4, 7, 9, 10} (*viii*) B ∪ C = {1, 3, 5, 6, 8, 9}

(*ix*) B ∩ C = {3} (*x*) A ∩ C = {3, 5}

(*xi*) (B ∪ C)′ = {0, 2, 4, 7, 10} (*xii*) A – B = {0, 2, 5, 7}

(*xiii*) A – C = {0, 1, 2, 7, 9} (*xiv*) B – C = {1, 9}

(*xv*) C – B = {5, 6, 8}.

Example 2. Given ξ = {x : x ∈ **N**, 4 ≤ x < 20}, A = {even numbers}, B = {multiples of 3} and C = {factors of 30}.

Find : (*i*) A ∩ B (*ii*) B ∩ C (*iii*) C ∩ A (*iv*) A ∩ B ∩ C.

Also show the relationship between given sets by a Venn diagram.

Solution. The given sets in the roster form are :

ξ = {4, 5, 6, ..., 19}, A = {4, 6, 8, ..., 18},

B = {6, 9, 12, 15, 18} and C = {5, 6, 10, 15}

(*i*) A ∩ B = {6, 12, 18}

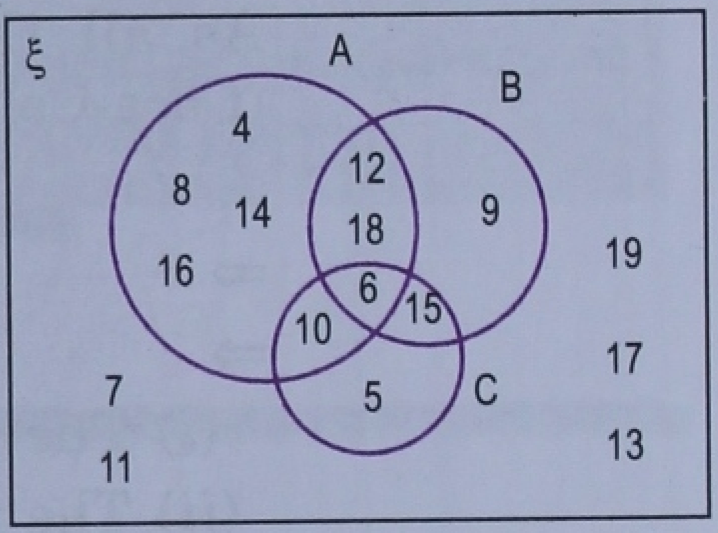
(*ii*) B ∩ C = {6, 15}

(*iii*) C ∩ A = {6, 10}

(*iv*) A ∩ B ∩ C = {6}

The relationship between the given sets is represented by the adjoining Venn diagram.

A, B and C are subsets of ξ



Use of Venn diagrams in solving problems

Example 3. In a class of 60 students, 40 students like Maths, 36 like Science, and 24 like both the subjects. Draw a Venn diagram and find the number of students who like

(*i*) Maths only (*ii*) Science only

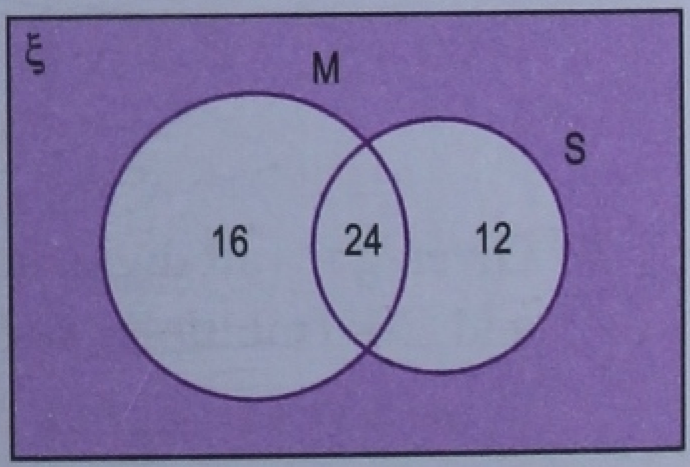
(*iii*) either Maths or Science (*iv*) neither Maths nor Science.

Solution. Here ξ = all the students of the class,

M = students who like Maths, and

S = students who like Science.

Since 24 students like both Maths and Science, we mark 24 in the common region of M and S. Since 40 students like Maths and 24 of these have already been marked, 16 is marked in the remaining part of M. Also, as 36 students like Science and 24 of these have already been marked, 12 is marked in the remaining part of S.



From the Venn diagram :

(*i*) The number of students who like Maths only = 16.

(*ii*) The number of students who like Science only = 12.

(*iii*) The number of students who like either Maths or Science

$= 16 + 24 + 12 = 52.$

(*iv*) The number of students who like neither Maths nor Science (shown shaded in the diagram) $= 60 - 52 = 8.$

Example 4. In a group of 50 students, 28 students had Coca-Cola and 32 had Pepsi. All the students had atleast one soft drink. Draw Venn diagram and find how many students had

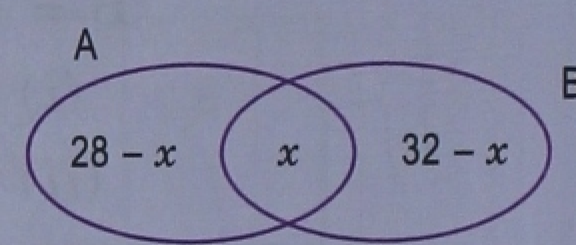
(*i*) both Coca-Cola and Pepsi

(*ii*) Coca-Cola only (*iii*) Pepsi only.

Solution. Let A be the set of students who had Coca-Cola and B be the set of students who had Pepsi. Then $A \cap B$ is the set of students who had both Coca-Cola and Pepsi.

Let $n(A \cap B) = x$. Mark x in the common region of A and B. Since 28 students had Coca-Cola and x students had both the drinks, so the number of students who had only Coca-Cola is $28 - x$. Similarly, $32 - x$ students had only Pepsi.

This information is shown in the adjoining Venn diagram.



As all the students had atleast one soft drink (Coca-Cola or Pepsi), from Venn diagram, we get

$$50 = (28 - x) + x + (32 - x)$$

$$\Rightarrow \quad 50 = 60 - x$$

$$\Rightarrow \quad x = 60 - 50 = 10$$

(*i*) The number of students who had both Coca-Cola and Pepsi = 10

(*ii*) The number of students who had Coca-Cola only

$= 28 - x = 28 - 10 = 18.$

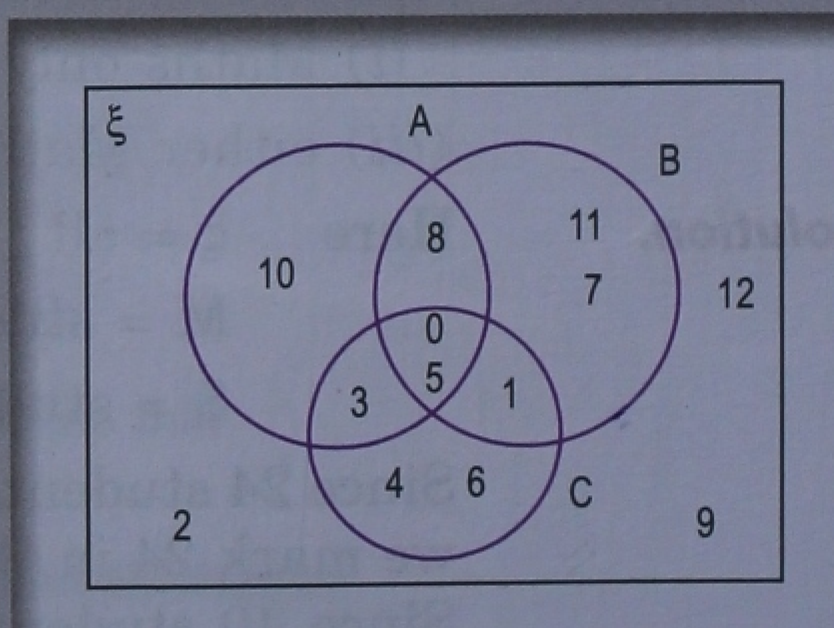
(*iii*) The number of students who had Pepsi only

$= 32 - x = 32 - 10 = 22.$

Home work

Exercise 2.2

1. From the adjoining Venn diagram, find the following sets :



(*i*) ξ (*ii*) $A \cap B$

(*iii*) $A \cap B \cap C$ (*iv*) C'

(*v*) $A - C$ (*vi*) $B - C$

(*vii*) $C - B$ (*viii*) $(A \cup B)'$

(*ix*) $(A \cup B \cup C)'$.

2. Given ξ = {all digits in our number system}, A = {multiples of 3}, B = {multiples of 4} and C = {multiples of 5}.
Draw a Venn diagram to show the relationship between the given sets.
Hence find : (*i*) $A \cap B \cap C$ (*ii*) $(A \cup B \cup C)'$.

3. Given $\xi = \{x : x \in \mathbf{N}, x \leq 21\}$, A = {multiples of 3}, B = {prime numbers} and C = {factor of 42}.
Draw a Venn diagram to show the relationship between the given sets.

4. Draw a Venn diagram to illustrate the following information :

$n(A) = 22$, $n(B) = 18$ and $n(A \cap B) = 5$.

Hence find : (*i*) $n(A \cup B)$ (*ii*) $n(A - B)$ (*iii*) $n(B - A)$.

5. Draw a Venn diagram to illustrate the following information :

$n(A) = 25$, $n(B) = 16$, $n(A \cap B) = 6$ and $n((A \cup B)') = 5$.

Hence find : (*i*) $n(A \cup B)$ (*ii*) $n(\xi)$ (*iii*) $n(A - B)$ (*iv*) $n(B - A)$.

6. Given $n(\xi) = 25$, $n(A') = 7$, $n(B) = 10$ and $B \subset A$. Draw a Venn diagram to illustrate this information. Hence find the cardinal number of the set $A - B$.

7. In a group of 50 boys, 20 play only cricket, 12 play only football and 5 boys play both the games. Draw a Venn diagram and find the number of boys who play

(*i*) atleast one of the two games cricket or football.

(*ii*) neither cricket nor football.

8. In a group of 40 students, 26 students like orange but not banana, while 32 students like orange. If all the students like atleast one of the two fruits, find the number of students who like

(*i*) both orange and banana (*ii*) only banana.

Draw a Venn diagram to represent the given data.

9. In a group of 60 persons, 45 speak Bengali, 28 speak English and all the persons speak atleast one language. Find how many people speak both Bengali and English. Draw a Venn diagram.

Summary

- The union of two sets A and B, written as $A \cup B$, is the set consisting of all those elements which belong to either A or B or both.

  Thus, $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$.

- The intersection of two sets A and B, written as $A \cap B$, is the set consisting of all those elements which belong to both A and B.

  Thus, $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$.

- Let A and B be two sets, then $A - B$ is the set consisting of all those elements which belong to A but do not belong to B.

  Thus, $A - B = \{x \mid x \in A \text{ and } x \notin B\}$.

  Similarly, $B - A = \{x \mid x \in B \text{ and } x \notin A\}$.

- Complement of a set A, written as A' (or A or A^c), is the set consisting of all those elements of ξ (universal set) which do not belong to A.

  Thus, $A' = \{x \mid x \in \xi \text{ and } x \notin A\}$.

- If A is any set, then

  (*i*) $A \cup \phi = A$, $A \cup \xi = \xi$, $A \cup A = A$ (*ii*) $A \cap \phi = \phi$, $A \cap \xi = A$, $A \cap A = A$.

- If A and B are two sets, then

  (*i*) $A \cup B = B \cup A$, $A \cap B = B \cap A$ (*ii*) $A \subseteq A \cup B$, $B \subseteq A \cup B$, $A \cup B \subseteq \xi$

  (*iii*) $A \cap B \subseteq A$, $A \cap B \subseteq B$ (*iv*) $A - B = A \cap B'$, $B - A = B \cap A'$

  (*v*) $(A \cup B)' = A', \cap B'$, $(A \cap B)' = A' \cup B'$ (De Morgan's laws)

  (*vi*) Sets A and B are disjoint sets if and only if $A \cap B = \phi$, and sets A and B are overlapping sets if and only if $A \cap B \neq \phi$.

- If A and B are finite sets, then

  (*i*) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(*ii*) $n(A - B) = n(A \cup B) - n(B) = n(A) - n(A \cap B)$
(*iii*) $n(B - A) = n(A \cup B) - n(A) = n(B) - n(A \cap B)$
(*iv*) $n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$.

- If ξ is a finite set and A is any set, then $n(A) + n(A') = n(\xi)$.
- In Venn diagrams, sets are represented by closed figures like rectangle, square, circle, oval etc. Elements of the set are written inside the figure. Usually, the universal set is represented by a rectangle and its subsets by circles (or ovals) within this rectangle.

Check Your Progress

1. If $\xi = \{x : x \in \mathbf{N}, x < 25\}$ and $A = \{x : x$ is a composite number$\}$, then find A′ in set builder form and also in roster form.

2. If $\xi = \{x : x \in \mathbf{N}, x \leq 12\}$, $A = \{x : x \geq 7\}$ and $B = \{x : 4 < x < 10\}$, then find :

(*i*) A' (*ii*) B' (*iii*) $A \cup B$ (*iv*) $A \cap B$
(*v*) $A - B$ (*vi*) $B - A$ (*vii*) $(A \cup B)'$ (*viii*) $A' \cap B'$

Also verify that :

(*i*) $(A \cup B)' = A' \cap B'$ (*ii*) $A - B = A \cap B'$
(*iii*) $n(A \cup B) + n((A \cup B)') = n(\xi)$ (*iv*) $n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$.

3. If $\xi = \{x : x \in \mathbf{N}, x \leq 21\}$, A = {even numbers}, B = {multiples of 3} and C = {multiples of 7}, then prove that :

(*i*) $(A \cap B)' = A' \cup B'$ (*ii*) $A - B = A \cap B'$
(*iii*) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (*iv*) $A - (B \cup C) = (A - B) \cap (A - C)$.

4. If $n(A \cup B) = 40$, $n(A \cap B) = 8$ and $n(A - B) = n(B - A)$, then find :

(*i*) $n(A)$ (*ii*) $n(B)$.

5. If $n(\xi) = 60$, $n(A) = 35$, $n(B') = 36$ and $n((A \cap B)') = 51$, then find :

(*i*) $n(B)$ (*ii*) $n(A \cap B)$ (*iii*) $n(A \cup B)$ (*iv*) $n(A - B)$.

6. Given ξ = {all triangles drawn in a plane}, I = {isosceles triangles} and R = {right angled triangles}.

Draw a Venn diagram to show these sets in their correct relationship. Shade the region representing $I \cap R$ and write the measures of the angles of the triangles of this region.

7. Out of 200 labourers employed to carry bricks and mounting them in the construction of a multistorey building, 120 only carried the bricks while 50 helped in carrying the bricks as well as mounting them. How many did the work of mounting the bricks only? Draw a Venn diagram to represent the data.

8. In a city, 50% people read newspaper A, 45% read newspaper B, and 25% read neither A nor B. What percentage of people read both the newspapers A as well as B?