Chapter I

SETS

In previous classes, you learnt the basic concepts of sets. The following topics were covered :

- Concept of a set
- Methods of describing a set, by listing its members (roster or tabular form) or by using rule method (set builder form)
- Finite sets, infinite sets, empty set, universal set
- Cardinal number of a finite set
- Equal and equivalent sets
- Overlapping and disjoint sets
- Subsets
- Union and intersection of sets; complement of a set
- Venn diagrams
- Solving problems by using Venn diagrams

In this chapter and the next chapter, we shall review and strengthen these topics and introduce following new ideas :

- Proper subsets
- Number of subsets of a finite set
- Difference of two sets
- Combination of operations like union, intersection, complement

SETS

Any well defined collection of objects is called a ***set****. The objects which belong to the set are called its* ***members*** *or* ***elements****.*

By a '*well defined collection of objects*', we mean that given a set and an object, it should be possible (beyond doubt) whether the object belongs to the set or not. For example, the collection of all brave students of your school is not well defined, since a student of your school considered brave by one person might be considered coward by another. Thus, such a collection is not a set.

The sets are usually denoted by capital letters A, B, C etc., and the members of a set are denoted by small letters x, y, z etc.

If x is a member of the set A, we write $x \in A$ (read as 'x belongs to A') and if x is not a member of the set A, we write $x \notin A$ (read as 'x does not belong to A'). If x and y both are members of the set A, we write $x, y \in A$.

Representation of a set

A set can be represented by the following methods :

(*i*) Description method

(*ii*) Roster method or tabular form

(*iii*) Rule method or set builder form.

Description method

In this method, we make a (well defined) description of the elements of the set and this description of elements is enclosed in curly brackets.

For example :

(*i*) The set of even whole numbers less than 20 is written as
{even whole numbers less than 20}.
Note that 0 ∈ {even whole numbers less than 20} while 20 ∉ {even whole numbers less than 20}.

(*ii*) The set of all months of a year is written as
{months of a year}.

Roster method or tabular form

In this method, we list all the members of the set within braces (curly brackets) and separate these by commas.

For example :

(*i*) The set A of all odd natural numbers less than 15 in the roster form is written as
A = {1, 3, 5, 7, 9, 11, 13}.

(*ii*) The set M of months of a year having less than 31 days in the roster form is written as
M = {February, April, June, September, November}.

(*iii*) The set L of letters in the word 'JODHPUR' in the tabular form can be written as
L = {J, O, D, H, P, U, R}.

Remarks

- *The order of listing the elements in a set can be changed.*
  Thus, the set {3, 5, 6, 9} may also be written as {3, 6, 9, 5} or {6, 3, 9, 5} etc.
- *If one or more elements of a set are repeated, the set remains the same.*
  Thus, the set $\{a, b, c, b, b, a\}$ is the same as $\{a, b, c\}$.
- *Each element of a set is listed once and only once, repetitions are removed.*
  Thus, the set A of letters in the word 'PROFESSOR' is written as
  A = {P, R, O, F, E, S}
- If the number of elements in a set is very large, then we can represent the set by writing a few members which clearly indicate the structure of the elements of the set followed (or preceded) by three dots '...' and then writing the last element (if it exists).
  Thus, the set A of even natural numbers between 50 and 500 in the tabular form is written as
  A = {52, 54, 56,... , 498}
  The set B of odd integers less than 9 in the roster form is written as
  B = {..., −5, −3, −1, 1, 3, 5, 7}

Rule method or set builder form

In this method, we write a variable (say x) representing any member of the set followed by a property satisfied by each element of the set, and enclose it in braces.

If A is the set consisting of elements x having property p, we write

$$A = \{x \mid x \text{ has property } p\}$$

which is read as 'the set of elements x such that x has property p'. The symbol '$\mid$' stands for the words 'such that'. Sometimes, we use the symbol ':' in place of the symbol '$\mid$'.

For example :

(*i*) The set A of all odd natural numbers less than 15 in the set builder form is written as

$A = \{x : x = 2n - 1, n \in \mathbf{N} \text{ and } n \leq 7\}$

or $A = \{x : x = 2n + 1, n \in \mathbf{W} \text{ and } n \leq 6\}$.

(*ii*) The set $P = \{0, 3, 6, 9, 12, 15, 18\}$ in the set builder form is written as

$P = \{x : x = 3n, n \in \mathbf{W} \text{ and } n < 7\}$.

(*iii*) The set $S = \{9, 16, 25, 36, 49, 64, 81, 100\}$ in the set builder form can be written as

$S = \{x \mid x = n^2, n \in \mathbf{N} \text{ and } 3 \leq n \leq 10\}$.

Example 1. Write the following sets in roster form and also in set builder form:

(*i*) The set of prime numbers between 10 and 50

(*ii*) The set of whole numbers which are divisible by 7 and less than 100

(*iii*) {factors of 48}.

Solution. The given sets can be written as

(*i*) $\{11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47\}$ (roster form)

or $\{x \mid x \text{ is a prime number}, 10 < x < 50\}$. (set builder form)

(*ii*) $\{0, 7, 14, 21, \ldots, 98\}$ (roster form)

or $\{x : x = 7n, n \in \mathbf{W} \text{ and } n \leq 14\}$. (set builder form)

(*iii*) $\{1, 2, 3, 4, 6, 8, 12, 16, 24, 48\}$ (roster form)

or $\{x \mid x \text{ is a factor of } 48\}$. (set builder form)

Example 2. Write the following sets in roster form :

(*i*) $\{x \mid x = 5p, p \in \mathbf{I} \text{ and } -3 \leq p < 2\}$

(*ii*) $\{x \mid x \in \mathbf{W}, 2x - 3 < 8\}$

(*iii*) $\{x : x$ is a two digit number such that the sum of its digits is $9\}$.

Solution. (*i*) Given $p \in \mathbf{I}$ and $-3 \leq p < 2$

$\Rightarrow \quad p = -3, -2, -1, 0, 1$

Also, $x = 5p$; putting $p = -3, -2, -1, 0, 1$, we get

$x = 5(-3), 5(-2), 5(-1), 5(0), 5(1)$

$= -15, -10, -5, 0, 5.$

$\therefore$ The given set in the roster form can be written as

$\{-15, -10, -5, 0, 5\}$.

(*ii*) Given $2x - 3 < 8, x \in \mathbf{W}$

$\Rightarrow \quad 2x < 11, x \in \mathbf{W}$

$\Rightarrow \quad x < \frac{11}{2}, x \in \mathbf{W}$

$\Rightarrow \quad x = 0, 1, 2, 3, 4, 5.$

∴ The given set in the roster form can be written as

{0, 1, 2, 3, 4, 5}

(iii) As x is a two digit number and the sum of whose digits is 9, such numbers are

18, 27, 36, 45, 54, 63, 72, 81, 90.

∴ The given set in the roster form can be written as

{18, 27, 36, 45, 54, 63, 72, 81, 90}.

Example 3. Write the following sets in set builder form :

(i) $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, ..., \frac{99}{100}\right\}$

(ii) {0, 1, 8, 27, 64, 125, 216}

(iii) {–36, –30, –24, ..., 42}

Solution. *(i)* Here, we observe that each member in the given set has denominator one more than the numerator. Also the numerator begins with 1 and ends with 99.

Hence, the given set in the builder form can be written as

$$\left\{x : x = \frac{n}{n+1}, n \in \mathbf{N} \text{ and } n < 100\right\}.$$

(ii) The given numbers are perfect cubes of the first seven whole numbers.

∴ Given set = $\{x : x = n^3, n \in \mathbf{W} \text{ and } n \le 6\}$. (set builder form)

(iii) The given numbers are multiples of 6 lying between –36 and 42 (both inclusive).

∴ Given set = $\{x : x = 6p, p \in \mathbf{I} \text{ and } -6 \le p \le 7\}$. (set builder form)

Exercise 1.1

1. State which of the following collections are sets :
   - *(i)* Collection of four rivers flowing in Asia
   - *(ii)* Collection of all rivers of India
   - *(iii)* Collection of all handsome boys of Lucknow
   - *(iv)* Collection of students of your class whose heights lie between 140 cm and 155 cm
   - *(v)* Collection of all good football players of your school
   - *(vi)* Collection of popular cinema actors of India
   - *(vii)* Collection of competent maths teachers of Mumbai
   - *(viii)* Collection of three Prime Ministers of India
   - *(ix)* Collection of five most talented students of your class.
2. Write the following sets in tabular form and also in set builder form :
   - *(i)* The set of odd natural numbers
   - *(ii)* The set of even whole numbers
   - *(iii)* The set of even integers
   - *(iv)* {factors of 72}
   - *(v)* {natural numbers which are perfect cubes and less than 100}.

3. Write the following sets in roster form :
   (i) $\{x \mid x = 3p, p \in \mathbf{I} \text{ and } -2 \le p < 10\}$
   (ii) $\{x \mid x$ is a composite number and $11 < x < 25\}$
   (iii) $\{x \mid x \in \mathbf{W}$, x is divisible by 4 and 6, $x \le 100\}$
   (iv) $\{x \mid x$ is a two digit number whose sum of digits is $13\}$
   (v) $\{x : x = 2n - 3, n \in \mathbf{W} \text{ and } n < 6\}$
   (vi) $\{x \mid x = \frac{2n+1}{2n+3}, n \in \mathbf{W} \text{ and } n \le 10\}$
   (vii) $\{x : x \in \mathbf{I}, x^2 < 20\}$
   (viii) $\{x : 2x - 3 \le 7, x \in \mathbf{W}\}$
   (ix) $\{x : 3x - 5 < 15, x \in \mathbf{W}\}$.
4. Write the following sets in the set builder form :
   (i) $\left\{1, \frac{1}{2}, \frac{1}{3}, \ldots, \frac{1}{9}\right\}$
   (ii) $\left\{\frac{1}{3}, \frac{3}{5}, \frac{5}{7}, \frac{7}{9}, \ldots\right\}$
   (iii) $\left\{1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \ldots, \frac{1}{100}\right\}$
   (iv) $\left\{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots, \frac{1}{256}\right\}$
   (v) $\{-10, -5, 0, 5, 10, 15, \ldots, 100\}$
   (vi) $\{1, 2, 3, 4, 6, 8, 12, 16, 24, 48\}$.

TYPES OF SETS

Finite set

*A set that contains limited (countable) number of different elements is called **finite set**. In other words, a set is called **finite set** if and only if the counting of its different elements comes to an end.*

For example :

(i) A = {Red, Blue, Green}
(ii) B = $\{x : x \in \mathbf{W}, x < 1000\}$
(iii) C = {planets of our solar system}

Each one of these is a finite set.

Infinite set

*A set that contains unlimited (uncountable) number of different elements is called **infinite set**. In other words, a set is called **infinite set** if and only if the counting of its different elements does not come to an end.*

For example :

(i) The set of natural numbers $\mathbf{N} = \{1, 2, 3, \ldots\}$
(ii) The set of whole numbers $\mathbf{W} = \{0, 1, 2, 3, \ldots\}$
(iii) The set of integers $\mathbf{I} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$
(iv) $\{x : x \in \mathbf{I}, x < 5\}$

Each one of these is an infinite set.

Singleton set

A set containing one element is called a ***singleton*** *(or* ***unit****)* ***set****.*

For example :

(*i*) {–9}
(*ii*) {months of a year having less than 30 days}
(*iii*) {x : x is President of India}
(*iv*) {x : $3x + 1 < 5, x \in \mathbf{N}$}

Each one of these is a singleton set.

Empty set

A set that contains no elements is called ***empty set****. It is also called* ***null*** *(or* ***void****)* ***set****.*
There is only one such set. It is denoted by ϕ or { }. As the empty set has no elements, it is a finite set.

For example :

(*i*) {x | $3x - 1 = 4, x \in \mathbf{N}$}
(*ii*) {prime numbers between 13 and 17}
(*iii*) {x | $x \in \mathbf{W}, x < 0$}

Each one of these is the empty set.
Note that {0} and {ϕ} are not empty sets because each of these sets has one element.

Universal set

A set that contains all the elements under consideration in a given problem is called ***universal set****.*
It is a kind of 'parent set'. It is denoted by U or ξ. Universal set may vary from problem to problem. Therefore, we shall always specify the universal set in a given problem.

For example :

(*i*) For A = {b, c, f, g}, the universal set may be {letters of English alphabet}.
(*ii*) For A = {x | $x \in \mathbf{N}, 3 \le x < 11$}, the universal set may be {1, 2, 3, ..., 20} or **N**.
(*iii*) For A = {Earth, Mars}, the universal set may be {planets of our solar system}.

Note that the choice of universal set is not unique.

Cardinal number of a finite set

The number of distinct elements in a finite set is called its ***cardinal number****.*
The cardinal number of a finite set A is denoted by n(A).

For example :

(*i*) A = {days of a week} has 7 elements, so $n(A) = 7$.
(*ii*) A = {x : $2x - 3 < 5, x \in \mathbf{N}$} = {1, 2, 3} has 3 elements, so $n(A) = 3$.

Remarks

- Cardinal number of an infinite set is not defined.
- Cardinal number of the empty set is zero *i.e.* $n(\phi) = 0$.
- The cardinal number of a singleton set is 1.

Equal sets

Two sets A and B are called ***equal sets****, written as A = B, if they have the same elements.*

If sets A and B are not equal, we write A ≠ B.

For example :

(*i*) If A = {5, 6, 9, 7} and B = {9, 6, 6, 5, 7, 6}, then A = B because the elements in a set can be repeated or rearranged.

(*ii*) If A = {–4, –3, –2, –1, 0, 1, 2, 3, 4} and B = {x : $x \in$ **I**, $x^2 < 20$}, then A = B.

(*iii*) If P = {vowels in EQUALITY} and Q = {vowels in QUANTITATIVE}, then P = Q, because each set = {E, U, A, I}.

(*iv*) If P = {x | $x \in$ **N**, $x^2 - 2 = 0$} and Q = {triangles having 5 sides}, then P = Q, because each set = ϕ.

Remarks

If A and B are finite sets and A = B then n(A) = n(B), but the converse may not be true *i.e.* if n(A) = n(B) then A and B may not be equal. For example, let A = {3, 5} and B = {5, a} then n(A) = 2 = n(B) but A ≠ B.

Equivalent sets

Two finite sets A and B are called ***equivalent sets*** *if they have the same number of elements.*

Thus, two finite sets A and B are equivalent sets, written as A ↔ B (read as A is equivalent to B), if n(A) = n(B).

For example :

(*i*) If A = {a, b, c, d, e} and B = {2, 5, 6, 9, 10}, then n(A) = 5 = n(B). So A ↔ B. Note that A ≠ B.

(*ii*) If A = {colours of rainbow} and B = {x | $x \in$ **W**, $x < 7$}, then n(A) = 7 = n(B). So A ↔ B. Note that A ≠ B.

(*iii*) If A = {letters of FLOWER} and B = {letters of FOLLOWER}, then A = B because each set = {F, L, O, W, E, R}.
Also n(A) = 6 = n(B), so A ↔ B.

Remarks

- Two (finite) sets A and B are equivalent sets if n(A) = n(B).
- If two sets are equal, then they are equivalent sets *i.e.* if A = B then A ↔ B. Converse is not necessarily true. In the above example, in parts (*i*) and (*ii*), we notice that A ↔ B but A ≠ B.
- Two infinite sets are always equivalent sets.

Overlapping (intersecting) sets

Two sets A and B are called ***overlapping (intersecting*** *or* ***joint) sets*** *if they have atleast one element in common.*

In other words, two sets A and B are overlapping sets if A ∩ B ≠ ϕ.

For example :

(*i*) The sets {5, 7, 9, 11} and {1, 2, 3, ..., 10} are overlapping sets because these sets have elements 5, 7 and 9 in common.
Here, $A \cap B = \{5, 7, 9\} \neq \phi$.

(*ii*) The sets A = {letters of GEORGE} and B = {letters of FORGET} are joint sets because these sets have all the elements of set A in common.
Here, $A \cap B = \{G, E, O, R\} = A \neq \phi$.

Disjoint sets

Two sets A and B are called ***disjoint*** *(or* ***non-overlapping****)* ***sets*** *if they have no element in common.*

In other words, two sets A and B are disjoint set if $A \cap B = \phi$.

For example :

(*i*) The set {0, 2, 4, 6, 8, 10} and {1, 5, 9} are disjoint sets because these sets have no element in common.

(*ii*) The sets {vowels} and {consonants} are disjoint sets because these sets have no element in common.

(*iii*) The sets {days of a week} and {colours of a rainbow} are disjoint sets because these sets have no element in common.

Subsets

Let A and B be any two sets, then A is called a ***subset*** *of B if every member of A is also a member of B.*

We write it as $A \subseteq B$ (read as 'A is a subset of B' or 'A is contained in B').

If $A \subseteq B$ *i.e.* A is contained in B, we may also say that B contains A, or B is a **superset** of A. We write it as $B \supseteq A$ (read as 'B contains A' or 'B is a superset of A').

If there exists atleast one element in A which is not a member of B, then A is not a subset of B and we write it as $A \not\subseteq B$ (read as 'A is not a subset of B').

For example :

(*i*) If A = {1, 3, 5, 7} and B = {1, 2, 3, ..., 10}, then every member of A is a member of B, so A is a subset of B *i.e.* $A \subseteq B$. Note that $B \not\subseteq A$.

(*ii*) If A = {Red, Green, Blue} and B = {colours of a rainbow}, then every member of A is a member of B, so $A \subseteq B$. Note that $B \not\subseteq A$.

(*iii*) Let P = {letters of CHARM} and Q = {letters of MARCH}. Here, every member of P is a member of Q and every member of Q is a member of P, so $P \subseteq Q$ and $Q \subseteq P$. In fact, P = Q.

Proper subsets

Let A be any set and B be a non-empty set, then A is called a ***proper subset*** *of B if every member of A is also a member of B and there exists atleast one element in B which is not a member of A. We write it as $A \subset B$ (read as 'A is a proper subset of B').*

If A is a proper subset of B *i.e.* $A \subset B$, then B is called a **proper superset** of A and we write it as $B \supset A$ (read as 'B is a proper superset of A').

In the above example, in (*i*) A is a proper subset of B; in (*ii*) A is a proper subset of B; in (*iii*) P is not a proper subset of Q.

Note. We have used the symbol '$\subseteq$' to represent a subset and the symbol '$\subset$' to represent a proper subset. However, NCERT and many authors use the symbol '$\subset$' to represent a subset.

Remarks

- Let A be any set, then
  - (*i*) $A \subseteq A$ *i.e.* every set is a subset of itself, but not a proper subset. A subset which is not a proper subset is called an **improper subset**.
  - (*ii*) Every set has only one *improper subset*.
  - (*iii*) Since the empty set has no elements, $\phi \subseteq A$ *i.e.* empty set is a subset of every set.
  - (*iv*) Empty set is a proper subset of every set except itself.
  - (*v*) Every set is a subset of universal set *i.e.* $A \subseteq \xi$.
- If A and B are two sets, then
  - (*i*) $A \subseteq B$ if and only if $x \in A$ implies $x \in B$.
  - (*ii*) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

NUMBER OF SUBSETS OF A FINITE SET

If A is a finite set with $n(A) = m$, then

(*i*) *the number of subsets of A* $= 2^m$

(*ii*) *the number of proper subsets of A* $= 2^m - 1$.

For example :

(*i*) Let $A = \{a\}$, then the subsets of A are ϕ, A.
Note that $n(A) = 1$, number of subsets of $A = 2 = 2^1$ and number of proper subsets of $A = 1 = 2^1 - 1$.

(*ii*) Let $A = \{a, b\}$, then the subsets A are ϕ, $\{a\}$, $\{b\}$, A.
Note that $n(A) = 2$, number of subsets of $A = 4 = 2^2$ and number of proper subsets of $A = 3 = 2^2 - 1$.

(*iii*) Let $A = \{1, 2, 3\}$, then the subsets of A are
ϕ, $\{1\}$, $\{2\}$, $\{3\}$, $\{1, 2\}$, $\{2, 3\}$, $\{1, 3\}$, A.
Note that $n(A) = 3$, number of subsets of $A = 8 = 2^3$ and number of proper subsets of $A = 7 = 2^3 - 1$.

Example 1. Match each of the sets on the left hand side described in roster form with the same set on the right hand side described in set builder form :

(*i*) {3, 4, 5, 6}	(*a*) $\{x : x$ is a letter in the word TEACHER$\}$
(*ii*) {1, 3, 5, 7, 9}	(*b*) $\{x : x \in \mathbf{N}$ and $2 < x < 7\}$
(*iii*) {T, C, H, A, E, R}	(*c*) $\{x : x$ is an odd natural number less than 10$\}$.

Solution. Write the sets on the right in the roster form which are described in set builder form.

(*a*) Given set $= \{x : x$ is a letter in the word TEACHER$\}$.
As there are seven letters in the word TEACHER and the letter E is

repeated, so the given set in the roster form = {T, E, A, C, H, R}.
Hence, (*iii*) matches (*a*).

(*b*) Given set = $\{x : x \in \mathbf{N}$ and $2 < x < 7\}$.
As $x \in \mathbf{N}$ and $2 < x < 7$, the values of x are 3, 4, 5, 6.
Thus, the given set in roster form = {3, 4, 5, 6}.
Hence, (*i*) matches (*b*).

(*c*) Given set = $\{x : x$ is an odd natural number less than 10$\}$.
As x is an odd natural number less than 10, so the values of x are 1, 3, 5, 7, 9. Thus, the given set in the roster form = {1, 3, 5, 7, 9}.
Hence, (*ii*) matches (*c*).

Example 2. State whether each of the following statement is true or false for the sets A and B where
A = {letters of word LAXMAN REKHA} and
B = {letters of word ULTRA FRESH}
(*i*) A and B are disjoint sets
(*ii*) A = B
(*iii*) A ↔ B.

Solution. The given sets in the roster form are
A = {L, A, X, M, N, R, E, K, H} and
B = {U, L, T, R, A, F, E, S, H}

Write each element of the set once and only once

(*i*) False (∵ letters L, R, A, E, H are common in both sets)
(*ii*) False (∵ the letter K belongs to set A and it does not belong to set B)
(*iii*) True (∵ $n(A) = 9 = n(B)$)

Example 3. Consider the following sets :
ϕ, A = {1, 3}, B = {1, 5, 9} and C = {1, 2, 3, 5, 7, 9}.
State whether each of the following statement is true or false for the above given sets:
(*i*) A and B are disjoint sets
(*ii*) $\phi \subset B$
(*iii*) $A \subseteq B$
(*iv*) $A \subset C$
(*v*) $C \supset B$
(*vi*) $C \not\subseteq A$.

Solution. (*i*) False, because the element 1 is common in both sets A and B.
(*ii*) True, because empty set is a proper subset of every set except itself.
(*iii*) False, because $3 \in A$ but $3 \notin B$.
(*iv*) True, because every element of A is an element of C and there exists an element 2 in C which is not in A.
(*v*) True, because every element of B is an element of C and there exists an element 3 in C which does not belong to B.
(*vi*) True, because $2 \in C$ but $2 \notin A$.

Example 4. State whether each of the following statements is true or false for the sets A, B, and C where
A = {letters of BOWL}, B = {letters of BELLOW}
and C = {letters of ELBOW}.

(*i*) $A \subseteq B$ (*ii*) $B \supseteq C$

(*iii*) $B = C$ (*iv*) $A \leftrightarrow B$

(*v*) $A \subset B$ (*vi*) $C \supset B$.

Solution. The given sets in the roster form are :

$A = \{B, O, W, L\}$

$B = \{B, E, L, O, W\}$, and

$C = \{E, B, L, O, W\}$

Note that $n(A) = 4$, $n(B) = 5$ and $n(C) = 5$. Clearly,

(*i*) true (*ii*) true (*iii*) true

(*iv*) false (*v*) true (*vi*) false.

Example 5. Let $\xi = \{1, 2, 3, ..., 50\}$, $A = \{x : x \text{ is divisible by 2 and 3}\}$, $B = \{x, x = n^2, n \in \mathbf{N}\}$ and $C = \{x : x \text{ is a factor of } 42\}$.

State whether each of the following statement is true or false :

(*i*) A and B are disjoint sets

(*ii*) B and C are joint sets

(*iii*) $A \leftrightarrow B$

(*iv*) $B \leftrightarrow C$.

Solution. Here $\xi = \{1, 2, 3, ..., 50\}$

It is understood that A, B and C are subsets of ξ, so the members of these sets are to be taken only from ξ.

The sets A, B and C in roster form are :

Write sets A, B, C in roster form. Choose members of these sets from ξ

$A = \{6, 12, 18, 24, 30, 36, 42, 48\}$,

$B = \{1, 4, 9, 16, 25, 36, 49\}$, and

$C = \{1, 2, 3, 6, 7, 14, 21, 42\}$

Note that $n(A) = 8$, $n(B) = 7$ and $n(C) = 8$

(*i*) False ($\because$ the element 36 is common)

(*ii*) True ($\because$ the element 1 is common)

(*iii*) False ($\because n(A) \neq n(B)$)

(*iv*) False ($\because n(B) \neq n(C)$)

Example 6. If $P = \{x : x$ is a letter in the word GEORGE CANTOR$\}$ and $Q = \{x : x$ is a vowel in the word GEORGE CANTOR$\}$, then

(*i*) write the sets P and Q in roster form

(*ii*) write $n(P)$ and $n(Q)$

(*iii*) write all the subsets of Q

(*iv*) write the number of proper subsets of P.

Solution. (*i*) The sets P and Q in the roster form are:

Write each element of the set once and only once

$P = \{G, E, O, R, C, A, N, T\}$, and

$Q = \{E, O, A\}$.

(*ii*) $n(P) = 8$ and $n(Q) = 3$.

(*iii*) The subsets of Q are

ϕ, {E}, {O}, {A}, {E, O}, {O, A}, {E, A}, Q.

(*iv*) The number of proper subsets of $P = 2^8 - 1 = 256 - 1 = 255$.

Exercise 1.2

1. State whether the following sets are finite sets or infinite sets. In case of finite sets, mention the cardinal number.
   (*i*) A = $\{x : x \in \mathbf{I}, x < 5\}$
   (*ii*) A = $\{x : x \in \mathbf{W}, x$ is divisible by 4 and 9$\}$
   (*iii*) P = $\{x : x$ is an even prime number $> 2\}$
   (*iv*) F = $\{x : x \in \mathbf{N}$ and x is a factor of 84$\}$
   (*v*) B = $\{x : x$ is a two digit number, sum of whose digits is 12$\}$
   (*vi*) C = $\{x : x \in \mathbf{W}, 3x - 7 \leq 8\}$
   (*vii*) {all circles drawn in a plane}
   (*viii*) $\{x : x = 5n, n \in \mathbf{I}$ and $x < 20\}$
   (*ix*) $\{x : x = \frac{n}{n+1}, n \in \mathbf{W}$ and $n \leq 10\}$.
2. Match each of the sets on the left hand side described in roster form with the same set on the right hand side described in set builder form :
   (*i*) {2, 3} — (*a*) $\{x : x \in \mathbf{N}$ and x is a divisor of 6$\}$
   (*ii*) {1, 2, 3, 6} — (*b*) $\{x : x$ is an odd natural number less than 6$\}$
   (*iii*) {T, E, I, L} — (*c*) $\{x : x$ is a prime factor of 6$\}$
   (*iv*) {1, 3, 5} — (*d*) $\{x : x$ is a letter in the word LITTLE$\}$.
3. If $\xi = \{1, 2, 3, ..., 30\}$ and A = $\{x \mid x \in \mathbf{N}$ and $x^2 \in \xi\}$, then list the elements of A.
4. If $\xi = \{1, 2, 3, ..., 10\}$ and A = $\{x : x$ is a prime factor of 66$\}$, then list the elements of A.
5. If $\xi = \{1, 2, 3, ..., 25\}$, A = $\{x : x \leq 5\}$ and B = $\{x : x^2 \in \xi\}$, find whether A = B or A ≠ B.
6. State whether each of the following statements is true or false :
   (*i*) $2 \subseteq \{1, 2, 3\}$ (*ii*) $\{2\} \in \{1, 2, 3\}$ (*iii*) $\phi \in \{1, 2, 3\}$
   (*iv*) $0 \in \phi$ (*v*) $\{1, 2\} \subset \{1, 2, 3\}$ (*vi*) $\phi \subseteq \{\phi\}$
   (*vii*) For any two sets A and B, either A ⊆ B or B ⊆ A.
   (*viii*) Every set has a proper subset.
   (*ix*) Every subset of a finite set is finite.
   (*x*) Every subset of an infinite set is infinite.
7. If A = {1, 2, 3, 4, 5}, B = {2, 3, 4} and C = {2, 4, 5}, which of the following statements are true?
   (*i*) A ⊆ B (*ii*) A ⊆ C (*iii*) B ⊂ A (*iv*) C ⊂ A
   (*v*) B ⊆ C (*vi*) B ↔ C (*vii*) B = C (*viii*) $\phi \subset$ B
8. In the following sets, determine whether A and B are equivalent sets, and if so, whether A = B :
   (*i*) A = {1, 2, 3, 4} and B = $\{x : x \in \mathbf{W}, x^2 < 10\}$
   (*ii*) A = {letters of ANKITA} and B = $\{x : 2x - 3 < 7, x \in \mathbf{W}\}$
   (*iii*) A = $\{x : x \in \mathbf{N}, 3x - 2 < 2\}$ and B = {1}
   (*iv*) A = {letters of word RAMAN} and B = {letters of word ABBAS}
9. Given $\xi = \{0, 1, 2, ..., 50\}$, A = $\{x : x = 6n, n \in \mathbf{W}\}$, B = $\{x : x = 7n, n \in \mathbf{W}\}$ and C = $\{x : x$ is a factor of 36$\}$.
   State whether each of the following statement is true or false :
   (*i*) A and B are disjoint sets (*ii*) B and C are disjoint sets
   (*iii*) A and C are disjoint sets (*iv*) A ↔ B
   (*v*) B ↔ C (*vi*) A ↔ C
10. Write all the subsets of the set A = {0, 5, 10}. Which of these are proper subsets and which are improper subsets?

Summary

- Any well defined collection of objects is called a set.
- A set may be described by listing all its members within braces and separating these by commas. This is called roster (or tabular) form.
- A set may be described as $\{x \mid x \text{ has property } p\}$. This is called rule method or set builder form.
- A finite set has limited (countable) number of elements.
- An infinite set has unlimited (uncountable) number of elements.
- A set containing one element is called a singleton set.
- Empty (null or void) set has no elements. It is a finite set.
- A set that contains all the elements under consideration in a given problem is called universal set. Choice of universal set is not unique.
- The number of different elements in a finite set A is called its cardinal number. Cardinal number of A is written as $n(A)$.
- The cardinal number of empty set is zero.
- Cardinal number of an infinite set is not defined.
- Two sets are called equal sets if they have same elements.
- Two finite sets A and B are called equivalent sets if $n(A) = n(B)$. It is written as $A \leftrightarrow B$.
- Two equal sets must be equivalent sets but two equivalent sets may not be equal.
- Two sets are called overlapping (or joint) sets if they have atleast one element in common.
- Two sets are called disjoint sets if they have no element in common.
- Set A is called a subset of B if every element of A is also a member of B. It is written as $A \subseteq B$. If $A \subseteq B$, we may say that B is a super set of A and it is written as $B \supseteq A$.
- Set A is called a proper subset of B if every element of A is also a member of B and there exists atleast one element in B which is not a member of A. It is written as $A \subset B$.
- If A is a finite set with $n(A) = m$, then the number of subsets of A = 2^m and the number of proper subsets of A = $2^m - 1$.

Check Your Progress

1. Write the following sets in roster form :
   (*i*) $\{x : x = 5n - 1, n \in \mathbf{W} \text{ and } x < 40\}$ (*ii*) $\{x : x \in \mathbf{I} \text{ and } x^2 < 25\}$
   (*iii*) $\{x : x = \frac{2n-1}{n+2}, n \in \mathbf{W} \text{ and } n < 4\}$ (*iv*) $\{x : 2x + 3 < 1, x \in \mathbf{I}\}$
2. Write the following sets in set builder form :
   (*i*) {even integers that lie between –9 and 12}
   (*ii*) {11, 13, 17, 19, 23, 29, 31}
3. If A = {3, 5, 7, 9, 11}, then write which of the following statements are correct and which are incorrect :
   (*i*) $3 \in A$ (*ii*) $5, 9, 11 \in A$ (*iii*) $8 \notin A$
   (*iv*) $7 \notin A$ (*v*) $\{3\} \in A$ (*vi*) $\{5, 7\} \in A$
   (*vii*) $5 \subseteq A$ (*viii*) $\{5, 7\} \subset A$ (*ix*) $\phi \subset A$

4. State whether the following statements are true or false. Justify your answer.

(*i*) $\phi = \{0\}$ (*ii*) The empty set has no subsets

(*iii*) $\phi \subset \{0\}$

(*iv*) Collection of all hard working students of your school is a set

(*v*) The set $\{x : x \in \mathbf{I}, x < 3\}$ is finite

(*vi*) If A = $\{x : x$ is a digit in the numeral 5025711138$\}$, then $n(A) = 10$

(*vii*) If A = $\{x : x$ is a two digit number, sum of whose digits is 18$\}$, then $n(A) = 1$

(*viii*) If P = $\{x : x$ is a two digit number, sum of whose digits is 19$\}$, then $n(P) = 0$

(*ix*) Every non-empty set has a proper subset.

5. If A and B are two finite sets, then state whether the following statements are true or false:

(*i*) $A \leftrightarrow B$ implies A = B (*ii*) A = B implies $A \leftrightarrow B$.

6. If A = {letters of word JACOB} and B = {letters of word ABDULLA}, then state whether each of the following statement is true or false :

(*i*) A and B are disjoint sets (*ii*) $A \subseteq B$ (*iii*) $B \subseteq A$ (*iv*) $A \leftrightarrow B$

7. Given $\xi = \{x : x \in \mathbf{W}, x < 15\}$, A = {multiples of 2}, B = {multiples of 3}, C = {multiples of 5} and D = {multiples of 6}.

State whether each of the following statements is true or false :

(*i*) C and D are disjoint sets (*ii*) $C \leftrightarrow D$ (*iii*) C = D (*iv*) $A \leftrightarrow B$

(*v*) $A \supseteq B$ (*vi*) $C \subseteq A$ (*vii*) $D \subseteq A$ (*viii*) $D \subseteq B$

(*ix*) $C \subseteq B$ (*x*) $D \subset A$.

8. Let P = $\{x : x$ is a vowel in the word CHENNAI$\}$. Write all the subsets of P. Which of these are proper subsets and which are improper subsets?