UNIT – 1
PURE ARITHMETIC

CHAPTER 1

NUMBER SYSTEM

1.1 REVIEW

1. Natural numbers (N)	(i) Each of 1, 2, 3, 4, ..., etc., is a **natural number.** (ii) **The first** and **the smallest** natural number is **one** (1); whereas **the last** and **the largest** natural number **cannot be obtained.** (iii) Consecutive natural numbers **differ by one** (1).
2. Whole numbers (W)	(i) Each of 0, 1, 2, 3, 4, ..., etc., is a **whole number.** (ii) **The first** and **the smallest** whole number is **zero** (0); whereas **the last** and **the largest** whole number **cannot be obtained.** (iii) Consecutive whole numbers **differ by one** (1). (iv) Except zero (0) every whole number is a natural number.
3. Integers (I or Z)	(i) Each of ...., –4, –3, –2, –1, 0, 1, 2, 3, 4, ... is **an integer.** (ii) Every integer is either : (a) negative of a natural number, such as –1, –2, –3, –4, ..., etc. (b) zero *i.e.* 0, or (c) a natural number *i.e.* 1, 2, 3, 4, ..., etc. (iii) **The smallest** (the first) and **the largest** (the last) integers **cannot be obtained.** (iv) Consecutive integers **differ by one** (1).
4. Rational numbers (Q)	(i) It is a number which can be expressed as $\frac{a}{b}$, where **a** and **b** both **are integers** and $b \neq 0$. (ii) **Every fraction** such as $\frac{3}{8}, \frac{7}{15}, -\frac{6}{11}$, etc. is **a rational number.** (iii) Since, $8 = \frac{8}{1}, -3 = \frac{-3}{1}, 23 = \frac{23}{1}$, etc., therefore **each integer is a rational number.** For the same reason, each natural number and each whole number is also a rational number. (iv) Zero (0) can be written as : $\frac{0}{5}, \frac{0}{-8}, \frac{0}{10}$, etc., therefore zero is also a rational number. (v) Every decimal number can be expressed as a fraction; so it is also a rational number. *e.g.* $0{\cdot}7 = \frac{7}{10}, 2{\cdot}5 = \frac{25}{10} = \frac{5}{2}$, etc.
5. Irrational numbers ($\overline{Q}$)	(i) The number which cannot be expressed as $\frac{a}{b}$; where $a \in I$, $b \in I$ and $b \neq 0$; is called an **irrational number.** Infact a number, which is not rational, is irrational.

	(ii) Each of $\sqrt{2}$, $\sqrt{7}$, $\sqrt{15}$, $3\sqrt{7}$, $3-\sqrt{2}$, $\sqrt{5}+\sqrt{3}$, etc., is an irrational number. (iii) Each non-terminating and non-recurring number is an irrational number.
6. **Real numbers (R)**	(i) Rational numbers and irrational numbers, taken together, are called **real numbers.** *i.e.* $R = Q \cup \overline{Q}$ (ii) Every real number is either rational or irrational.

TEST YOURSELF

1. 5 is an ...., 9 is an .... and $\frac{5}{9}$ is a ....
2. -8 is an ...., 15 is an .... and $\frac{-8}{15}$ is a ....
3. 5 is a .... and $\sqrt{7}$ is an ...., then each of $5+\sqrt{7}$, $5-\sqrt{7}$, $\sqrt{7}-5$ and $5\sqrt{7}$ is an ....
4. $\sqrt{13}$ is an .... and 8 is a .... then each of : $8\sqrt{13}$, $\sqrt{13}+8$, $8-\sqrt{13}$, etc., is an ....

1.2 PRIME AND COMPOSITE NUMBERS

1. **Prime number :** A natural number, which is greater than 1 and is divisible by one (1) and itself only, is called a **prime number.**

   (i) 5 is a prime number as it is greater than one and is divisible by one (1) and itself only.

   For the same reason, each of the following is a prime number :

   2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, etc.

   (ii) Two (2) is the smallest prime number and the largest prime number cannot be obtained. In other words, there are an infinite number of prime numbers.

   (iii) Except 2, every prime number is an odd natural number.

2. **Composite number :** A natural number, which is greater than 1 and is not prime, is called a **composite number.**

   A composite number is divisible by 1 (one), by itself and atleast by one more number.

   6 is divisible by 1, by itself and by 2 and 3 also; therefore 6 is a composite number.

   (i) Each of 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, etc. is a composite number.

   (ii) The smallest composite number is 4 and the largest composite number cannot be obtained *i.e.* there are an infinite number of composite numbers.

   (iii) A composite number can be even or odd.

One (1) is neither prime nor composite.

1.3 MORE ABOUT NUMBERS

1. **Twin primes :** Prime numbers, which differ by two, are said to be *twin prime* numbers.

   e.g. 5 and 7 ; 11 and 13, etc.

2. **Prime triplet :** The set of three consecutive prime numbers with a difference of 2 is called the *prime triplet*. Note that there is only one set {3, 5, 7} of the prime triplet.

3. **Co-primes :** The pairs of numbers, which are not divisible by a common number other than 1, are called *co-primes*.

   e.g. 10 and 21 are co-primes, since these numbers cannot be divided by the same number other than 1.

   Similarly, 7 and 16 ; 8 and 25 ; 16 and 25 are some more pairs of co-primes.

1.4 TESTS OF DIVISIBILITY

1. **Division by 2, 4 and 8 :**

   (i) **A number is divisible by 2,** if its *last* (unit) digit is divisible by 2 (*e.g.* 78, 50, 146, etc.)

   (ii) **A number is divisible by 4,** if the number formed by its last *two* digits is divisible by 4.

   e.g. 172 is divisible by 4 as the number 72 formed by its last two digits is divisible by 4. Similarly, 392, 500, 29320, etc. are also divisible by 4.

   A number is divisible by 8, if the number formed by its last *three* digits is divisible by 8 (*e.g.* 33176, 436000, etc.)

2. **Division by 3 and 9**

   (i) **A number is divisible by** 3, if the *sum* of its digits is divisible by 3 (*e.g.* 27, 192, 54924, etc.)

   Since the sum of digits of 192 = 1 + 9 + 2 = 12, *which is divisible by* 3, therefore 192 is divisible by 3.

   (ii) **A number is divisible by** 9, if the *sum* of its digits is divisible by 9 (*e.g.* 27, 198, etc.).

3. **Division by 6**

   If a number is divisible by 3 as well as by 2, it is divisible by 6.

4. **Division by 5 and 10**

   (i) **A number is divisible by** 5, if its last (unit) digit is 5 or 0 (*e.g.* 345, 240, etc.).

   (ii) **A number is divisible by** 10, if its last digit is 0 (*e.g.* 310, 4000, etc.)

5. **Division by 11**

Find the sum of the digits in the even places of the given number and the sum of the digits in its odd places. If the difference between the two sums is 0 or a multiple of 11, then the given number is divisible by 11.

e.g. consider the number 72512. The sum of its digits at even places = 2 + 1 = 3 and the sum of its digits at odd places = 7 + 5 + 2 = 14.

The difference between the two sums = 14 – 3 = 11; *which is a multiple of 11.*

∴ 72512 is divisible by 11.

In the same way, each of 957, 1496, 68772 is divisible by 11.

EXERCISE 1 (A)

1. State, **true** or **false** :
   (i) Every natural number is a whole number.
   (ii) Every whole number is an integer.
   (iii) The negative of every natural number is an integer.
   (iv) Every integer is a natural number.
   (v) Zero (0) is the smallest integer.
   (vi) If x is a whole number, then each of x – 1, x – 2, x – 3, etc., is also a whole number.
   (vii) If x is an integer, then each of x + 1, x + 2, x + 3, etc., is also an integer.
   (viii) If x is an integer, x – 100 is also an integer.
   (ix) Every integer is a real number.
   (x) Some real numbers are whole numbers.
   (xi) Every whole number is a natural number.
   (xii) Every integer is a rational number.
   (xiii) Some rational numbers are integers.
2. Read each of the following numbers carefully.
   87, 54, 0, –13, – 4·7, $\sqrt{5}$, $2\frac{1}{7}$, $\sqrt{15}$, $-\frac{8}{7}$, $3\sqrt{2}$, 4·807, 0·002, $\sqrt{16}$ and $2 + \sqrt{3}$.
   From the given numbers, write which are :
   (i) natural numbers (ii) whole numbers
   (iii) real numbers (iv) rational numbers
   (v) irrational numbers
   (vi) integers.
3. Give three examples of rational numbers which are not natural numbers.
4. Give four examples of rational numbers which are not integers.
5. Give four examples of rational numbers which are not whole numbers.
6. Write all prime numbers between 20 and 50.
7. Write all composite numbers between 10 and 35.
8. Write two pairs of twin primes between 20 and 50.
9. Write three pairs of co-primes between 15 and 30.
10. Find which of the numbers (2, 3, 4, 5, 6, 8, 9,10, 11) will divide the following numbers exactly :
    (i) 5232 (ii) 255 (iii) 3465
    (iv) 2057 (v) 693 (vi) 1800
    (vii) 1044 (viii) 5700 (ix) 2360
11. (i) Is zero a rational number ?
    (ii) Can zero be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$?
    (iii) Give five examples in support of your answer for part (ii).
12. If the three digit number $4x2$ is divisible by 9, find the value of the digit x.

> $4x2$ is divisible by 9
> $\Rightarrow 4 + x + 2$ is divisible by 9
> *i.e.* $6 + x$ is divisible by 9
> $\Rightarrow 6 + x = 9$ or 18 or 27, ...
> *i.e.* $x = 9 - 6$ or $18 - 6$ or $27 - 6$, ...
> $= 3$ or 12 or 21, ...
> Since, x is a digit, therefore $x = 3$ **(Ans.)**

13. Each of the following four digit numbers is divisible by 9. Find the value of the digit x.
    (i) 15 x 2 (ii) 4 x 23
14. Each of the following three digit numbers is divisible by 3. Find the values of the digit x.
    (i) 4 x 3 (ii) 4 x 5

1.5 MORE ABOUT IRRATIONAL NUMBERS

Remember :

1. $\sqrt{3} + \sqrt{2} \neq \sqrt{5}$; $\sqrt{3} - \sqrt{2} \neq \sqrt{1}$
2. $\sqrt{3} + \sqrt{3} \neq \sqrt{6}$; but $\sqrt{3} + \sqrt{3} = 2\sqrt{3}$
3. $\sqrt{5} \times \sqrt{5} = 5$; $2\sqrt{3} \times \sqrt{3} = 2 \times 3 = 6$; $5\sqrt{3} \times 2\sqrt{7} = 5 \times 2\ \sqrt{3 \times 7} = 10\sqrt{21}$ and so on.
4. $\frac{3}{\sqrt{3}} = \sqrt{3}$; $\frac{\sqrt{8}}{\sqrt{4}} = \sqrt{\frac{8}{4}} = \sqrt{2}$; $\frac{\sqrt{12}}{\sqrt{3}} = \sqrt{4} = 2$ and so on.
5. $\sqrt{12} = \sqrt{2 \times 2 \times 3} = 2\sqrt{3}$; $2\sqrt{18} = 2\sqrt{3 \times 3 \times 2} = 2 \times 3\sqrt{2} = 6\sqrt{2}$ and so on.

1.6 RATIONALISING FACTOR (R.F.)

If the product of two irrational quantities is rational; each is called the rationalising factor of the other.

e.g. (i) $\sqrt{3} \times \sqrt{3} = 3$; therefore $\sqrt{3}$ is the rationalising factor of $\sqrt{3}$.

(ii) $(2+\sqrt{3}) \times (2-\sqrt{3}) = (2)^2 - (\sqrt{3})^2 = 4 - 3 = 1$ [$\because (a+b)(a-b) = a^2 - b^2$]

$\Rightarrow$ $2+\sqrt{3}$ and $2-\sqrt{3}$ are rationalising factors of each other.

(iii) Similarly, $\sqrt{5} - \sqrt{3}$ and $\sqrt{5} + \sqrt{3}$ are rationalising factors of each other as :

$(\sqrt{5} - \sqrt{3}) \times (\sqrt{5} + \sqrt{3}) = (\sqrt{5})^2 - (\sqrt{3})^2 = 5 - 3 = 2$

1.7 RATIONALISATION

The process of multiplying an irrational number by its rationalising factor is called *rationalisation.*

Rationalisation of the denominator of a fraction :

Steps : 1. Divide and multiply the given fraction by the rationalising factor of its denominator.

2. Simplify, if necessary.

> When a fraction is divided and multiplied by the same number, rational or irrational, its value remains the same.

Example 1 :

Rationalise the denominators of :

(i) $\frac{1}{\sqrt{2}}$ (ii) $\frac{3\sqrt{5}}{\sqrt{6}}$ (iii) $\frac{1}{\sqrt{3}-\sqrt{2}}$ (iv) $\frac{\sqrt{5}}{3-\sqrt{2}}$

Solution :

(i) $\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$ [Rationalising factor of $\sqrt{2}$ is $\sqrt{2}$. So, multiply and divide by $\sqrt{2}$]

$= \frac{\sqrt{2}}{2}$ **(Ans.)**

(ii) $\frac{3\sqrt{5}}{\sqrt{6}} = \frac{3\sqrt{5}}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$ [Rationalising factor of $\sqrt{6} = \sqrt{6}$]

$= \frac{3\times\sqrt{30}}{6}$ [$\sqrt{5} \times \sqrt{6} = \sqrt{5\times6} = \sqrt{30}$ and $\sqrt{6} \times \sqrt{6} = 6$]

$= \frac{1}{2}\sqrt{30}$ **(Ans.)**

(iii) $\frac{1}{\sqrt{3}-\sqrt{2}} = \frac{1}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ [Rationalising factor of $\sqrt{3} - \sqrt{2} = \sqrt{3}+\sqrt{2}$]

$= \frac{\sqrt{3}+\sqrt{2}}{3-2}$ [$(\sqrt{3} - \sqrt{2})(\sqrt{3}+\sqrt{2}) = (\sqrt{3})^2 - (\sqrt{2})^2 = 3 - 2$]

$= \frac{\sqrt{3}+\sqrt{2}}{1} = \sqrt{3}+\sqrt{2}$ **(Ans.)**

(iv) $\frac{\sqrt{5}}{3-\sqrt{2}} = \frac{\sqrt{5}}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}}$ [Rationalising factor of $3-\sqrt{2} = 3+\sqrt{2}$]

$= \frac{\sqrt{5}(3+\sqrt{2})}{9-2} = \frac{3\sqrt{5}+\sqrt{10}}{7}$ **(Ans.)**

TEST YOURSELF

5. Rationalising factor (R.F.) of $2\sqrt{7}$ is ....

6. R.F. of $5+\sqrt{13}$ is .... and of $7-\sqrt{3}$ is ....

7. $(\sqrt{7}-\sqrt{2})(\sqrt{7}+\sqrt{2})$ = ... = ... = ...

8. $(5\sqrt{2}+3\sqrt{3})(5\sqrt{2}-3\sqrt{3})$ = ... = ... = ...

EXERCISE 1 (B)

1. State, **true** or **false** :

(i) $\sqrt{4}+\sqrt{5}=\sqrt{9}$ (ii) $3\sqrt{9}+3=9$

(iii) $5\sqrt{3}+2\sqrt{3}=7\sqrt{6}$ (iv) $5\sqrt{3}-2\sqrt{3}=3\sqrt{3}$

2. Write the smallest rationalising factor of :

(i) $\sqrt{5}$ (ii) $2\sqrt{3}$ (iii) $\sqrt{3}+\sqrt{2}$

(iv) $3-\sqrt{5}$ (v) $3\sqrt{2}-1$

3. Rationalise the denominator of :

(i) $\frac{2}{\sqrt{3}}$ (ii) $\frac{4}{\sqrt{6}}$ (iii) $\frac{2\sqrt{3}}{\sqrt{2}}$

(iv) $\frac{3}{\sqrt{5}}$ (v) $\frac{2\sqrt{2}}{\sqrt{3}}$

4. Rationalise the denominator of :

(i) $\frac{1}{2-\sqrt{3}}$ (ii) $\frac{2}{\sqrt{5}+\sqrt{3}}$ (iii) $\frac{6}{\sqrt{10}-2}$

(iv) $\frac{5}{\sqrt{7}+\sqrt{2}}$ (v) $\frac{16}{\sqrt{3}-1}$

5. Rationalise the denominator of :

(i) $\frac{22}{2\sqrt{3}+1}$ (ii) $\frac{17}{3\sqrt{2}-1}$ (iii) $\frac{18}{3\sqrt{2}-2\sqrt{3}}$

(iv) $\frac{\sqrt{2}}{\sqrt{6}-\sqrt{2}}$ (v) $\frac{\sqrt{3}}{\sqrt{6}+\sqrt{3}}$

6. Simplify each of the following and then state whether each of the following is rational or irrational :

(i) $(2-\sqrt{3})^2$ (ii) $(6+\sqrt{6})(6-\sqrt{6})$

(iii) $8 \div 2\sqrt{2}$ (iv) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})$

(v) $(\sqrt{7}+3)^2$ (vi) $4\sqrt{18}$

7. Simplify :

(i) $\frac{3+\sqrt{5}}{3-\sqrt{5}}+\frac{3-\sqrt{5}}{3+\sqrt{5}}$

(ii) $\frac{\sqrt{15}-2}{\sqrt{15}+2}+\frac{\sqrt{15}+2}{\sqrt{15}-2}$

(iii) $\frac{4+\sqrt{13}}{4-\sqrt{13}}-\frac{4-\sqrt{13}}{4+\sqrt{13}}$

(iv) $\frac{\sqrt{7}+2}{\sqrt{7}-2}-\frac{\sqrt{7}-2}{\sqrt{7}+2}$

1.8 PROPERTIES OF NUMBERS

In Arithmetic; ***addition*** (+), ***subtraction*** (–), ***multiplication*** (×) and ***division*** (÷) are known as **four fundamental operations**. Here, we shall try to explore some properties of these operations on the different types of numbers studied so far.

First Property : Closure

If *a* and *b* belong to a certain type of numbers such that *a* + *b* also belongs to the same type of numbers, we say the type of ***numbers***, under consideration, is ***closed for addition*** (+).

In the same way, if for a and b belonging to a certain type of numbers;

(i) $a - b$ belongs to the same type of numbers, the numbers are said to be ***closed for subtraction***.

(ii) $a \times b$ belongs to the same type of numbers, the numbers are said to be ***closed for multiplication***.

(iii) $a \div b$ belongs to the same type of numbers, the numbers are said to be ***closed for division***.

Now we shall discuss the closure property, as stated above, for all the four fundamental operations on different types of numbers.

1. Natural numbers (N) :

Consider two natural numbers, 5 and 15.

(i) $5 + 15 = 20$ *is a natural number and* $15 + 5 = 20$ *is also a natural number.*

(ii) $5 - 15 = -10$ *is not a natural number but* $15 - 5 = 10$ *is a natural number.*

(iii) $5 \times 15 = 75$ *is a natural number and* $15 \times 5 = 75$ *is also a natural number.*

(iv) $5 \div 15 = \frac{5}{15} = \frac{1}{3}$ *is not a natural number but* $15 \div 5 = \frac{15}{5} = 3$ *is a natural number.*

Thus, if a and b are any two natural numbers, then :

(i) $a + b$ is **always** a natural number, so **natural numbers are closed for addition.**

(ii) $a - b$ is **not always** a natural number, so **natural numbers are not closed for subtraction.**

(iii) $a \times b$ is **always** a natural number, so **natural numbers are closed for multiplication.**

(iv) $a \div b$ is **not always** a natural number, so **natural numbers are not closed for division.**

2. Whole numbers (W)

On checking for whole numbers, it can easily be shown that the whole numbers are:

(i) closed for addition and multiplication

(ii) not closed for subtraction and division.

3. Integers (I)

For any two integers a and b, it can easily be shown that :

(i) $a + b$ is always an integer e.g. $12 + (-4) = 8$; $(-5) + (-8) = -13$ and so on.

(ii) $a - b$ is always an integer e.g. $12 - (-4) = 16$; $(-5) - (-8) = 3$ and so on.

(iii) $a \times b$ is always an integer e.g. $12 \times (-4) = -48$; $(-5) \times (-8) = 40$ and so on.

(iv) $a \div b$ is not always an integer e.g. $12 \div (-4) = -3$ is an integer but $(-5) \div (-8) = \frac{5}{8}$; *not an integer.*

Thus, integers are closed for addition, subtraction and multiplication but not for division.

4. Rational numbers (Q)

A number, which can be expressed as $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is called a **rational number**.

(i) Each of $-\frac{3}{5}, \frac{8}{15}, \frac{2}{11}$, etc., is a rational number.

(ii) Since, –12, –5, 0, 3, 8, etc., can be expressed as $\frac{p}{q}$, so each of these numbers is also a rational number.

(iii) $\sqrt{3}, 2\sqrt{5}, 5+\sqrt{3}, 7-\sqrt{5}$, etc., cannot be expressed as $\frac{p}{q}$ where p and q are integers and $q \neq 0$, so each of these numbers is not a rational number. In fact, these numbers are called irrational numbers.

Consider two rational numbers $\frac{5}{8}$ and $-\frac{3}{10}$.

(i) $\frac{5}{8} + \left(-\frac{3}{10}\right) = \frac{25+(-12)}{40} = \frac{13}{40}$, *a rational number.*

(ii) $\frac{5}{8} - \left(-\frac{3}{10}\right) = \frac{25-(-12)}{40} = \frac{25+12}{40} = \frac{37}{40}$, *a rational number.*

(iii) $\frac{5}{8} \times \left(-\frac{3}{10}\right) = \frac{5}{8} \times -\frac{3}{10} = -\frac{3}{16}$, *a rational number.*

(iv) $\frac{5}{8} \div \left(-\frac{3}{10}\right) = \frac{5}{8} \times -\frac{10}{3} = -\frac{25}{12}$, *a rational number.*

Now, consider the rational numbers $\frac{5}{8}$ and 0.

(i) $\frac{5}{8} + 0 = \frac{5}{8}$, *a rational number.* (ii) $\frac{5}{8} - 0 = \frac{5}{8}$, *a rational number.*

(iii) $\frac{5}{8} \times 0 = 0$, *a rational number.* (iv) $\frac{5}{8} \div 0 = \frac{5}{8\times 0} = \frac{5}{0}$, *not a rational number.*

For any rational number a, $a \div 0$ is not defined and so it is not a rational number. For the same reason, each of $\frac{5}{0}, \frac{0}{0}, \frac{-8}{0}$, etc., is not a rational number.

Thus, for any two rational numbers a and b; each of $a + b$, $a - b$ and $a \times b$ is always a rational number but $a \div b$ is not always a rational number. Therefore, *rational numbers are closed for addition, subtraction and multiplication, but not closed for division.*

Second Property : Commutativity

For any *two* numbers a and b, (Natural numbers, Whole numbers, Integers or Rational numbers) it can be shown that :

(i) $a + b = b + a$ *i.e.* $3 + 5 = 5 + 3$, $7 + \left(\frac{-4}{15}\right) = \left(\frac{-4}{15}\right) + 7$, etc.

(ii) $a - b \neq b - a$ i.e. $3 - 5 \neq 5 - 3$, $7 - \left(\frac{-4}{15}\right) \neq \left(\frac{-4}{15}\right) - 7$, etc.

(iii) $a \times b = b \times a$ i.e. $3 \times 5 = 5 \times 3$, $7 \times \left(\frac{-4}{15}\right) = \left(\frac{-4}{15}\right) \times 7$, etc.

(iv) $a \div b \neq b \div a$ i.e. $3 \div 5 \neq 5 \div 3$, $7 \div \left(\frac{-4}{15}\right) \neq \left(\frac{-4}{15}\right) \div 7$, etc.

For these facts, we say,

(i) **addition** is **commutative.**

(ii) **subtraction** is **not commutative.**

(iii) **multiplication** is **commutative.**

(iv) **division** is **not commutative.**

Third Property : Associativity

For any *three* numbers *a*, *b* and *c* (Natural numbers, Whole numbers, Integers or Rational numbers), it can be shown that :

(i) $a + (b + c) = (a + b) + c$

(ii) $a - (b - c) \neq (a - b) - c$

(iii) $a \times (b \times c) = (a \times b) \times c$

(iv) $a \div (b \div c) \neq (a \div b) \div c$

For these facts, we say :

(i) **addition** is **associative.**

(ii) **subtraction** is **not associative.**

(iii) **multiplication** is **associative.**

(iv) **division** is **not associative.**

Examples :

Consider *three* numbers -5, $\frac{2}{3}$ and 4.

(i) Since, $-5 + \left(\frac{2}{3} + 4\right) = -5 + \frac{14}{3} = \frac{-15+14}{3} = -\frac{1}{3}$

and, $\left(-5 + \frac{2}{3}\right) + 4 = -\frac{13}{3} + 4 = \frac{-13+12}{3} = -\frac{1}{3}$

$\therefore$ $-5 + \left(\frac{2}{3} + 4\right) = \left(-5 + \frac{2}{3}\right) + 4 \Rightarrow$ **Addition is associative**

(ii) Since, $-5 - \left(\frac{2}{3} - 4\right) = -5 - \left(-\frac{10}{3}\right) = -5 + \frac{10}{3} = \frac{-15+10}{3} = -\frac{5}{3}$

and, $\left(-5 - \frac{2}{3}\right) - 4 = \frac{-17}{3} - 4 = \frac{-17-12}{3} = -\frac{29}{3}$

$\therefore$ $-5 - \left(\frac{2}{3} - 4\right) \neq \left(-5 - \frac{2}{3}\right) - 4 \Rightarrow$ **Subtraction is not associative**

(iii) Since, $-5 \times \left(\frac{2}{3} \times 4\right) = -5 \times \frac{8}{3} = \frac{-40}{3}$

and, $\left(-5 \times \frac{2}{3}\right) \times 4 = -\frac{10}{3} \times 4 = \frac{-40}{3}$

$\therefore \quad -5 \times \left(\frac{2}{3} \times 4\right) = \left(-5 \times \frac{2}{3}\right) \times 4 \Rightarrow$ **Multiplication is associative**

(iv) Since, $-5 \div \left(\frac{2}{3} \div 4\right) = -5 \div \left(\frac{2}{3} \times \frac{1}{4}\right) = -5 \div \frac{1}{6} = -5 \times 6 = -30$

and, $\left(-5 \div \frac{2}{3}\right) \div 4 = \left(-5 \times \frac{3}{2}\right) \div 4 = \frac{-15}{2} \div 4 = \frac{-15}{2} \times \frac{1}{4} = \frac{-15}{8}$

$\therefore \quad -5 \div \left(\frac{2}{3} \div 4\right) \neq \left(-5 \div \frac{2}{3}\right) \div 4 \Rightarrow$ **Division is not associative**

Fourth Property : Distributivity

For any *three* numbers a, b and c, the following results are always true.

(i) $a \times (b + c) = a \times b + a \times c$ and

(ii) $a \times (b - c) = a \times b - a \times c$

For $a \times (b + c) = a \times b + a \times c$; we say, **distributivity of multiplication over addition.**

And, for $a \times (b - c) = a \times b - a \times c$; we say, **distributivity of multiplication over subtraction.**

Examples :

Consider *three* numbers 5, 8 and 4.

Since, $5 \times (8 + 4) = 5 \times 12 = 60$ and $5 \times 8 + 5 \times 4 = 40 + 20 = 60$

$\therefore \quad 5 \times (8 + 4) = 5 \times 8 + 5 \times 4 \Rightarrow$ **Multiplication is distributive over addition**

Since, $5 \times (8 - 4) = 5 \times 4 = 20$ and $5 \times 8 - 5 \times 4 = 40 - 20 = 20$

$\therefore \quad 5 \times (8 - 4) = 5 \times 8 - 5 \times 4 \Rightarrow$ **Multiplication is distributive over subtraction**

The role of zero (0)

If zero is added to any number or any number is added to zero, in each case, the result is the number itself.

That is, for any number a, $a + 0 = a$ and $0 + a = a$

e.g. $5 + 0 = 5$ and $0 + 5 = 5$, $(-7) + 0 = -7$ and $0 + (-7) = 0$ and so on

For this property of ***zero***, it is called ***the identity for addition***.

The role of one (1) :

If one is multiplied with any number or any number is multiplied with one, in each case, the result is the number itself.

That is, for any number a, $a \times 1 = a$ and $1 \times a = a$

For this property of ***one*** (1), it is called ***the identity for multiplication***.

EXERCISE 1 (C)

1. Fill in the blanks in the following table :
[Write '**yes**' if the given numbers are closed for given operation (addition, subtraction, multiplication or division), otherwise write '**no**'].

Numbers	**Closed for :**			
	addition	*subtraction*	*multiplication*	*division*
(a) Natural numbers	....	....	....	....
(b) Whole numbers	....	....	....	....
(c) Integers	....	....	....	....
(d) Rational numbers	....	....	....	....

2. Fill in the blanks in the following table :
[Write '**yes**' if the given operation is commutative for given numbers, otherwise write '**no**'].

Numbers	**Commutative for :**			
	addition	*subtraction*	*multiplication*	*division*
(a) Natural numbers	....	....	....	....
(b) Whole numbers	....	....	....	....
(c) Integers	....	....	....	....
(d) Rational numbers	....	....	....	....

3. Fill in the blanks in the following table :
[Write '**yes**' if the given operation is associative for given numbers, otherwise write '**no**'].

Numbers	**Associative for :**			
	addition	*subtraction*	*multiplication*	*division*
(a) Natural numbers	....	....	....	....
(b) Whole numbers	....	....	....	....
(c) Integers	....	....	....	....
(d) Rational numbers	....	....	....	....

4. Take *two* examples, for each case, to show that :
   (i) whole numbers are closed for addition.
   (ii) integers are closed under subtraction.
   (iii) natural numbers are not closed for subtraction
   (iv) rational numbers are closed for multiplication.
   (v) rational numbers are closed for subtraction.
   (vi) integers are not closed for division.
   (vii) rational numbers are not closed for division.

5. For any *three* numbers *a*, *b* and *c*; write (in words), the property satisfied by each of the followings :
   (i) $a + b = b + a$
   (ii) $a - b \neq b - a$
   (iii) $a + (b + c) = (a + b) + c$
   (iv) $a - (b - c) \neq (a - b) - c$
   (v) $a \times (b + c) = a \times b + a \times c$
   (vi) $a \times (b - c) = a \times b - a \times c$
   (vii) $a \times b = b \times a$
   (viii) $a \div b \neq b \div a$
   (ix) $a \div (b \div c) \neq (a \div b) \div c$
   (x) $a \times (b \times c) = (a \times b) \times c$

ANSWERS

TEST YOURSELF

1. integer, integer, rational number **2.** integer, integer, rational number **3.** rational number, irrational number, irrational number **4.** irrational number, rational number, irrational number **5.** $\sqrt{7}$ **6.** $5-\sqrt{13}, 7+\sqrt{3}$ **7.** $(\sqrt{7})^2-(\sqrt{2})^2=7-2=5$ **8.** $(5\sqrt{2})^2-(3\sqrt{3})^2=50-27=23$

EXERCISE 1(A)

1. (i) True (ii) True (iii) True (iv) False (v) False (vi) False [**Reason :** Let $x = 1 \in W$; then $x-2=1-2=-1 \notin W$, $x-3=1-3=-2 \notin W$ and so on] (vii) True (viii) True (ix) True (x) True (xi) False (xii) True (xiii) True **2.** (i) 87, 54 and $\sqrt{16}$ (ii) 87, 54, 0 and $\sqrt{16}$ (iii) all the given numbers (iv) 87, 54, 0, –13, – 4·7, $2\frac{1}{7}$, $-\frac{8}{7}$, 4·807, 0·002 and $\sqrt{16}$ (v) $\sqrt{5}$, $\sqrt{15}$, $3\sqrt{2}$ and $2+\sqrt{3}$ (vi) 87, 54, 0, – 13 and $\sqrt{16}$ **3.** – 8, $\frac{5}{9}$ and 0 [Answer is not unique] **4.** $\frac{3}{5}$, $-\frac{4}{7}$, $\frac{8}{15}$ and $\frac{7}{16}$ [Answer is not unique] **5.** – 8, – 3, $\frac{2}{3}$ and $\frac{-5}{7}$ [Answer is not unique] **6.** 23, 29, 31, 37, 41, 43 and 47 **7.** 12, 14, 15, 16, 18, 20, 21, 22, 24, 25, 26, 27, 28, 30, 32, 33 and 34 **8.** 29 and 31, 41 and 43 **9.** 16 and 21, 18 and 25, 20 and 27 **10.** (i) 2, 3, 4, 6 and 8 (ii) 3 and 5 (iii) 3, 5, 9 and 11 (iv) 11 (v) 3, 9 and 11 (vi) 2, 3, 4, 5, 6, 8, 9 and 10 (vii) 2, 3, 4, 6 and 9 (viii) 2, 3, 4, 5, 6 and 10 (ix) 2, 4, 5, 8 and 10. **11.** (i) yes (ii) yes (iii) $\frac{0}{5}, \frac{0}{8}, \frac{0}{15}, \frac{0}{-3}, \frac{0}{-6}$ **13.** (i) 1 (ii) 0 or 9 **14.** (i) 2, 5 or 8 (ii) 0, 3, 6 or 9.

EXERCISE 1(B)

1. (i) False (ii) False (iii) False (iv) True **2.** (i) $\sqrt{5}$ (ii) $\sqrt{3}$ (iii) $\sqrt{3}-\sqrt{2}$ (iv) $3+\sqrt{5}$ (v) $3\sqrt{2}+1$ **3.** (i) $\frac{2\sqrt{3}}{3}$ (ii) $\frac{2\sqrt{6}}{3}$ (iii) $\sqrt{6}$ (iv) $\frac{3\sqrt{5}}{5}$ (v) $\frac{2\sqrt{6}}{3}$ **4.** (i) $2+\sqrt{3}$ (ii) $\sqrt{5}-\sqrt{3}$ (iii) $\sqrt{10}+2$ (iv) $\sqrt{7}-\sqrt{2}$ (v) $8(\sqrt{3}+1)$ **5.** (i) $2(2\sqrt{3}-1)$ (ii) $3\sqrt{2}+1$ (iii) $3(3\sqrt{2}+2\sqrt{3})$ (iv) $\frac{\sqrt{3}+1}{2}$ (v) $\sqrt{2}-1$ **6.** (i) $7-4\sqrt{3}$; irrational (ii) 30; rational (iii) $2\sqrt{2}$; irrational (iv) 3; rational (v) $16+6\sqrt{7}$; irrational (vi) $12\sqrt{2}$; irrational **7.** (i) 7 (ii) $3\frac{5}{11}$ (iii) $\frac{16\sqrt{13}}{3}$ (iv) $\frac{8\sqrt{7}}{3}$

EXERCISE 1(C)

1. (a) yes, no, yes, no (b) yes, no, yes, no (c) yes, yes, yes, no (d) yes, yes, yes, no **2.** (a) yes, no, yes, no (b) yes, no, yes, no (c) yes, no, yes, no (d) yes, no, yes, no **3.** (a) yes, no, yes, no (b) yes, no, yes, no (c) yes, no, yes, no (d) yes, no, yes, no **5.** (i) addition of numbers is commutative. (ii) subtraction of numbers is not commutative. (iii) addition of numbers is associative. (iv) subtraction of numbers is not associative. (v) multiplication is distributive over addition. (vi) multiplication is distributive over subtraction. (vii) multiplication of numbers is commutative. (viii) division of numbers is not commutative. (ix) division of numbers is not associative. (x) multiplication of numbers is associative.