3

Percentage

Let us briefly review what you have read about percentage in your previous classes. Per cent (or per centum or per centuri) means 'in a hundred'. Thus, when we say 15 per cent, we mean 15 out of 100. Symbolically, this is written as 15%.

Some conversion rules of percentage

Rule 1 To convert a percentage into a fraction, write the number as the numerator and 100 as the denominator and then reduce this fraction to its simplest form. You can also express the fraction as a ratio.

Examples (i) $12\% = \dfrac{12}{100} = \dfrac{3}{25}$ (or 3 : 25)   (ii) $39\% = \dfrac{39}{100}$ (or 39 : 100)

Rule 2 To convert a fraction into a percentage, multiply the fraction by 100 and add the % symbol. Since ratios are essentially fractions, you can also express a ratio as a percentage.

Examples (i) $\dfrac{4}{5}$ (or 4 : 5) $= \left(\dfrac{4}{5} \times 100\right)\% = 80\%$

(ii) $\dfrac{1}{8}$ (or 1 : 8) $= \left(\dfrac{1}{8} \times 100\right)\% = 12.5\%$

Rule 3 To convert a percentage into a decimal, remove the % sign and shift the decimal point in the number by two places to the left.

Examples (i) $3\% \left(= \dfrac{3}{100}\right) = 0.03$ (ii) $1.7\% \left(= \dfrac{1.7}{100}\right) = 0.017$

Rule 4 To convert a decimal into a percentage, shift the decimal point in the number by two places to the right and add the % sign.

Examples (i) $0.25 \left(= \dfrac{25}{100}\right) = 25\%$ (ii) $1.35 \left(= \dfrac{135}{100}\right) = 135\%$

(iii) $0.7 = 0.70 \left(= \dfrac{70}{100}\right) = 70\%$

Rule 5 To find a given percentage of a quantity, write the percentage as a fraction and multiply the quantity by this fraction.

$$x\% \text{ of } y = \dfrac{x}{100} \times y$$

Examples (i) $36\% \text{ of } 400 = \dfrac{36}{100} \times 400 = 144$

(ii) 25% of 40 L = $\dfrac{25}{100} \times 40$ L = 10 L

(iii) 56% of 20 kg = $\dfrac{56}{100} \times 20$ kg = 11.2 kg

Rule 6 To express a given quantity as a percentage of another quantity of the same kind, divide the given quantity by the other quantity, multiply the result by 100 and add the % sign.

$$x \text{ as percentage of } y = \left(\dfrac{x}{y} \times 100\right)\% \text{ of } y$$

Examples   (i) 25 as percentage of 80 = $\left(\dfrac{25}{80} \times 100\right)\%$ of 80 = 31.25% of 80

(ii) 46 g as percentage of 230 g = $\left(\dfrac{46}{230} \times 100\right)\%$ of 230 g = 20% of 230 g

Rule 7 To find a quantity from a given percentage of the quantity, express the given percentage as a fraction and divide the given quantity by this fraction.

Examples   (i) If 25% of a number is 136 then

$$\text{the number} = 136 \div \dfrac{25}{100} = 136 \times \dfrac{100}{25} = 136 \times 4 = 544.$$

(ii) If 30% of a quantity is 405 g then

$$\text{the quantity} = 405 \text{ g} \div \dfrac{30}{100} = 405 \text{ g} \times \dfrac{100}{30} = 1350 \text{ g} = 1.35 \text{ kg.}$$

Rule 8 To find the percentage change in a quantity, write the change in the quantity as the numerator and the original quantity as the denominator, and then multiply the fraction by 100.

$$\text{Percentage change} = \dfrac{\text{change in the quantity}}{\text{original quantity}} \times 100$$

Example   If the number of tigers in India was 1600 in the year 2008 and 1400 in year 2009 then decrease in the number of tigers = 1600 − 1400 = 200.

$$\therefore \quad \text{percentage change (or decrease)} = \dfrac{200}{1600} \times 100 = 12.5.$$

- If a quantity increases by x%,

$$\text{new quantity} = \left(1 + \dfrac{x}{100}\right) \times \text{original quantity, original quantity} = \dfrac{\text{new quantity}}{1 + \dfrac{x}{100}}$$

- If a quantity decreases by x%,

$$\text{new quantity} = \left(1 - \dfrac{x}{100}\right) \times \text{original quantity, original quantity} = \dfrac{\text{new quantity}}{1 - \dfrac{x}{100}}$$

EXAMPLE   (i) If 30 increases by 42%, what will be the new number?

(ii) If 56 decreases by 40%, what will be the new number?

Solution   (i) The new number $= \left(1 + \dfrac{42}{100}\right) \times 30 = \dfrac{142}{100} \times 30 = \dfrac{426}{10} = 42.6.$

(ii) The new number $= \left(1 - \dfrac{40}{100}\right) \times 56 = \dfrac{60}{100} \times 56 = \dfrac{336}{10} = 33.6.$

Alternatively

(i) The new number is 42% more than 30.

∴ the new number $= (100 + 42)\% \text{ of } 30 = 142\% \text{ of } 30 = 1.42 \times 30 = 42.6.$

(ii) The new number is 40% less than 56.

∴ the new number $= (100 - 40)\% \text{ of } 56 = 60\% \text{ of } 56 = 0.6 \times 56 = 33.6.$

EXAMPLE   **The price of petrol increased by 8% to Rs 54 per litre. Find the old price.**

Solution   The old price $= \dfrac{\text{Rs } 54}{1 + \dfrac{8}{100}} = \dfrac{\text{Rs } 54}{1 + \dfrac{2}{25}} = \dfrac{25}{27} \times \text{Rs } 54 = \text{Rs } 50 \text{ (per litre).}$

Solved Examples

EXAMPLE 1   **3224 students appeared in an examination and 75% passed. How many failed?**

Solution   The number of students who passed $= 75\% \text{ of } 3224 = \dfrac{75}{100} \times 3224 = 2418.$

Hence, the number of students who failed $= 3224 - 2418 = 806.$

Alternative method

Since 75% passed, $(100 - 75)\%$, i.e., 25% failed.

∴ the number of students who failed $= 25\% \text{ of } 3224 = \dfrac{25}{100} \times 3224 = 806.$

EXAMPLE 2   **Rakesh gets a salary of Rs 40,000 per month. His salary increases by 8% every year. Find (i) the increase in his (monthly) salary after a year and (ii) his monthly salary after two years.**

Solution   (i) The increase in his salary after a year $= 8\% \text{ of Rs } 40000$

$$= \dfrac{8}{100} \times \text{Rs } 40000 = \text{Rs } 3200.$$

(ii) His monthly salary after 1 year $= \text{Rs } 40000 + \text{Rs } 3200 = \text{Rs } 43,200.$

The next increase $= 8\% \text{ of Rs } 43200 = \dfrac{8}{100} \times \text{Rs } 43200 = \text{Rs } 3456.$

Hence, his monthly salary after 2 years $= \text{Rs } 43200 + \text{Rs } 3456 = \text{Rs } 46,656.$

EXAMPLE 3   **The price of a notebook was reduced by 10% in 2009. In 2010, it was increased by 10%. Find the percentage change in the price in two years.**

Solution   Let the price of the notebook $= \text{Rs } 100.$

Price after 10% reduction = Rs 100 – 10% of Rs 100 = Rs $100 - \frac{10}{100} \times$ Rs 100

$$= \text{Rs } 100 - \text{Rs } 10 = \text{Rs } 90.$$

Price after 10% increase = Rs 90 + 10% of Rs 90 = Rs $90 + \frac{10}{100} \times$ Rs 90 = Rs 99.

Thus, the price of the notebook decreased from Rs 100 to Rs 99, i.e., by Re 1 in two years.

∴ the percentage decrease in price after two years = $\frac{1}{100} \times 100 = 1.$

EXAMPLE 4 A farmer takes 250 mangoes to the market. In the first hour he sells 8% of the mangoes. In the second hour he sells 10% of the remaining mangoes.

    (i) How many mangoes does he sell?

    (ii) How many mangoes are left with him after two hours?

    (iii) What percentage of the total mangoes does he sell in two hours?

Solution   Mangoes sold in the first hour = 8% of 250 = $8 \times \frac{250}{100} = 20.$

∴ the number of mangoes left = 250 – 20 = 230.

Mangoes sold in the second hour = 10% of 230 = $10 \times \frac{230}{100} = 23.$

    (i) Total mangoes sold = 20 + 23 = 43.

    (ii) Total mangoes left after two hours = 250 – 43 = 207.

    (iii) Percentage of mangoes sold = $\frac{43}{250} \times 100 = 17.2.$

Hence, he sells 17.2% of the total mangoes in two hours.

EXAMPLE 5 Of the two candidates in an election, one got 40% of the votes and lost by 2980 votes. Find the total number of votes polled.

Solution   Let the total number of votes polled = x.

The loser got 40% of x = $\frac{40}{100}x = \frac{2x}{5}$ and the winner got 60% of x = $\frac{60}{100}x = \frac{3x}{5}.$

The difference between the votes of the winner and the loser = 2980

or $\frac{3x}{5} - \frac{2x}{5} = 2980$ or $\frac{x}{5} = 2980$

∴ $x = 2980 \times 5 = 14,900.$

Hence, the total number of votes polled = 14,900.

Alternative method

Since, the loser got 40% of the total votes polled, the winner got 60%.

Given, 60% of total votes polled – 40% of total votes polled = 2980.

∴ 20% of total votes polled = 2980.

Hence, the total number of votes polled = $2980 \div \frac{20}{100} = 2980 \div \frac{1}{5}$

$$= 2980 \times 5 = 14,900.$$

EXAMPLE 6 An umbrella is cheaper than a raincoat by 25%. By what per cent is the raincoat costlier than the umbrella?

Solution Let the price of the raincoat = Rs 100.

Then the price of the umbrella = Rs 100 – 25% of Rs 100

= Rs 100 – Rs 25 = Rs 75.

So, the price of the raincoat is Rs 25 more than the price of the umbrella.

We must find what per cent of Rs 75, Rs 25 is.

$\therefore$ the required percentage = $\dfrac{25}{75} \times 100 = 33\dfrac{1}{3}$.

Hence, the raincoat is costlier than the umbrella by $33\dfrac{1}{3}\%$.

EXAMPLE 7 **If A's income is 25% more than B's, by what per cent is B's income less than A's income?**

Solution Let B's income = Rs 100.

Then A's income = Rs 100 + 25% of Rs 100 = Rs 100 + Rs 25 = Rs 125.

So, B's income (Rs 100) is Rs 25 less than A's income (Rs 125).

$\therefore$ the required percentage = $\dfrac{25}{125} \times 100 = 20$.

Hence, B's income is 20% less than A's income.

EXAMPLE 8 **The price of a book was raised by $x\%$. Then it was slashed by 20% to bring it to the earlier level. Find the value of x.**

Solution Let the price of the book = Rs 100.

Then the price of the book after $x\%$ raise = Rs 100 + $x\%$ of Rs 100 = Rs $(100 + x)$.

So, the price of the book after 20% reduction = Rs $(100 + x) - 20\%$ of Rs $(100 + x)$

$$= Rs\,(100 + x) - \dfrac{20}{100} \times Rs\,(100 + x)$$

$$= Rs\left[\dfrac{4}{5}(100 + x)\right].$$

But, after 20% reduction, the price of the book is equal to its old price, i.e., Rs 100.

$\therefore$ $\dfrac{4}{5}(100 + x) = 100$ or $100 + x = 100 \times \dfrac{5}{4}$ or $100 + x = 125$ or $x = 125 - 100 = 25$.

EXAMPLE 9 **Parvez spent 20% of his pocket money on transport and Rs 60 on his breakfast. If he spent 15% of the remaining money on his lunch and had Rs 340 left, what was his pocket money?**

Solution Let his pocket money = Rs x.

Then money spent on transport = 20% of Rs x = $\dfrac{20}{100} \times Rs\,x$ = Rs $\dfrac{x}{5}$.

$\therefore$ money spent on transportation and breakfast = Rs $\left(\dfrac{x}{5} + 60\right)$.

$\therefore$ money left = Rs $\left[x - \left(\dfrac{x}{5} + 60\right)\right]$ = Rs $\left[\dfrac{4}{5}x - 60\right]$.

Now, 15% of Rs $\left(\dfrac{4x}{5} - 60\right)$ was spent on lunch.

Then the amount left = $(100\% - 15\%)$, that is, 85% of Rs $\left(\dfrac{4x}{5} - 60\right)$.

$$\therefore \quad 85\% \text{ of Rs}\left(\frac{4x}{5}-60\right)=\text{Rs } 340 \quad \text{or} \quad \frac{85}{100}\times\left(\frac{4x}{5}-60\right)=340$$

$$\text{or} \quad \frac{4x}{5}-60=\frac{340\times100}{85} \quad \text{or} \quad \frac{4x}{5}-60=400 \quad \text{or} \quad \frac{4x}{5}=460 \quad \text{or} \quad x=\frac{5}{4}\times460=575.$$

Hence, his pocket money = Rs 575.

EXAMPLE 10 In an examination, 30% of the students failed in English, 25% failed in mathematics and 12% failed in both. Find

(i) the percentage of students who failed in only English,

(ii) the percentage of students who failed in only mathematics,

(iii) the percentage of students who failed in either one or both the subjects,

(iv) the percentage of students who passed in both the subjects and

(v) the total number of students, if 171 passed in both the subjects.

Solution Let the total number of the students = 100.

Then, the number of students who failed in English = 30,

the number of students who failed in mathematics = 25

and the number of students who failed in English and mathematics = 12.

So, the number of students who failed in only English = 30 – 12 = 18,

and the number of students who failed in only mathematics = 25 – 12 = 13.

(i) Hence, the percentage of students who failed in only English = 18.

(ii) The percentage of students who failed in only mathematics = 13.

(iii) Now, the total number of students who failed

= the number of students who failed in only English

+ the number of students who failed in only mathematics

+ the number of students who failed in both English and mathematics

= 18 + 13 + 12 = 43.

$\therefore$ percentage of students who failed in either one or both the subjects = 43.

(iv) So, the total number of students who passed in both the subjects

= total number of students – total number of students who failed

= 100 – 43 = 57.

Hence, the percentage of students who passed in both subjects = 57.

(v) Given that 171 students passed in both the subjects.

$\therefore \quad$ 57% of the students = 171

$\therefore \quad$ the total number of students $=171\div\dfrac{57}{100}=\dfrac{100}{57}\times171=300.$

Alternative method (Venn diagram)

Students who failed in English = 30%, students who failed in mathematics = 25% and students who failed in both subjects = 12%.

(i) Students who failed in only English = 30% – 12% = 18%.

(ii) Students who failed in only mathematics
$$= 25\% - 12\% = 13\%.$$

(iii) Students who failed in either or both the subjects = (18 + 13 + 12)% = 43%.

18% 12% 13%

Failed in   Failed in
English   mathematics

Failed in both subjects
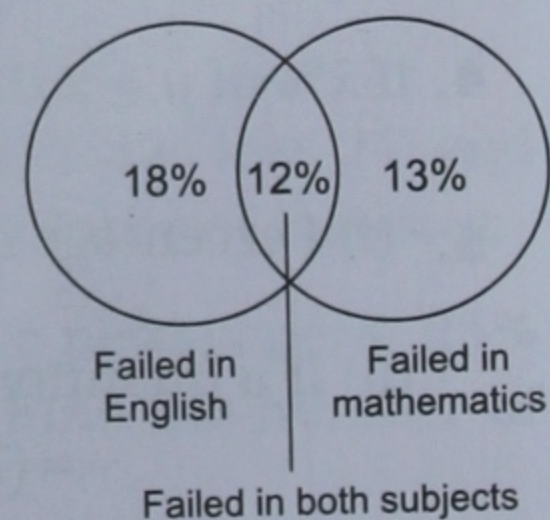
(iv) Students who passed in both subjects = 100% − 43% = 57%.

(v) Let the total number of students = x.

Given, 57% of $x = 171$ or $\dfrac{57}{100}x = 171$ or $x = \dfrac{100 \times 171}{57} = 300$.

Hence, the total number of students = 300.

EXAMPLE 11 In an examination, Jatinder secured 23 out of 25 in Sanskrit, 49 out of 50 in mathematics, 72 out of 75 in English and 91 out of 100 in science.

(i) Calculate his percentage marks in each subject.

(ii) In which subject did he perform the best and in which subject did he perform the worst?

(iii) Find his aggregate percentage of marks.

Solution   (i) Let us construct a table.

Subject	Sanskrit	Mathematics	English	Science
Marks obtained	23	49	72	91
Maximum marks	25	50	75	100
Percentage	$\left(\dfrac{23}{25} \times 100\right) = 92$	$\left(\dfrac{49}{50} \times 100\right) = 98$	$\left(\dfrac{72}{75} \times 100\right) = 96$	$\left(\dfrac{91}{100} \times 100\right) = 91$

(ii) He performed the best in mathematics while his performance was the worst in science.

(iii) He obtained 235 (= 23 + 49 + 72 + 91) out of 250 (= 25 + 50 + 75 + 100)

∴   his aggregate marks $= \left(\dfrac{235}{250} \times 100\right)\% = 94\%$.

Remember These

1. (i) To convert a percentage into a ratio or a fraction, divide the number by 100.

 (ii) To convert a ratio (or a fraction) into a percentage, multiply the fraction by 100 and add the % sign.

2. (i) To convert a percentage into a decimal, shift the decimal point by two places to the left.

 (ii) To convert a decimal into a percentage, shift the decimal point by two places to the right.

3. (i) $x\%$ of $y = \dfrac{x}{100} \times y$ (ii) x as a percentage of $y = \dfrac{x}{y} \times 100$

4. If $x\%$ of $y = z$ then $y = z \div \dfrac{x}{100} =$ given quantity ÷ percentage as a fraction.

5. (i) Percentage change $= \dfrac{\text{change in the quantity}}{\text{original quantity}} \times 100$

 (ii) If a quantity changes by some per cent then the new quantity

 $= (1 \pm \text{percentage as a fraction}) \times \text{original quantity}$.

EXERCISE 3

1. Convert the following into ratios.

   (i) 12%   (ii) $33\frac{1}{3}$%   (iii) 0.25%

2. Convert the following into percentages.

   (i) 2 : 5   (ii) 5 : 12   (iii) 21 : 50

3. Convert the following into fractions.

   (i) 2%   (ii) $6\frac{1}{6}$%   (iii) 0.008%

4. Express the following as percentages.

   (i) $\frac{3}{5}$ (ii) $1\frac{2}{3}$ (iii) $4\frac{1}{6}$

5. Express the following as decimals.

   (i) 24%   (ii) $2\frac{1}{5}$%   (iii) 0.02%

6. Express the following as decimals and percentages.

   (i) 1 : 2   (ii) $\frac{3}{5}$ (iii) 19 : 40

7. Increase the following by the given percentage.

   (i) 150 by 30%   (ii) 250 g by 7.5%   (iii) Rs 30 by 37.5%

8. Decrease the following by the given percentage.

   (i) 90 by 90%   (ii) 550 L by 36%   (iii) 80 kg by 37.5%

9. Express the first quantity as a percentage of the second.

   (i) 15, 45   (ii) 35 min, 2 hours   (iii) 540 g, 3 kg

10. Find the percentage change in each case.

    (i) 30 to 32   (ii) 18 g to 22.5 g   (iii) Rs 400 to Rs 840

    (iv) 20 to 18   (v) 4 m to 3 m   (vi) Rs 150 to Rs 142.5

11. If 30 of the 36 students of a class were present on a certain day, what percentage of students were absent?

12. Vikas spent 78% of his salary and saved Rs 5500. Find his salary.

13. The number of lions in a wildlife park was 160 in 2009. If it decreased by 15% in the year 2010, how many lions were left in 2010?

14. Due to the pulse polio campaign, the number of polio cases in Uttarakhand has come down by 85% to 135. Find the number of cases earlier.

15. The Indian cricket team played 60 one-day matches in a year. It won 32, lost 27 and drew 1. Express the wins, losses and draw as a percentage of the total number of matches.

16. Of the 200 items sold by a shopkeeper on a particular day, 46% were newspapers, 25% were pens, 12% were books and the rest were other assorted items. Find the number of each item sold.

17. Tickets sold for a school exhibition brought in Rs 10,500, $66\frac{2}{3}$% of which was given as scholarships to poor students. If the rest was used to buy books for the library, how much money was used to buy books?

18. A and B are business partners. A receives 38% of the profit. If B's share was Rs 15,500 in a particular year, find the total profit that year.

19. Out of a basket of 200 oranges, 15% were rejected being rotten and 10% of the rest were sold. What percentage of total number of oranges remained unsold?

20. A bookshop had 12,000 books. In the first week of April, it sold 15% of the books, and in the second week, it sold 15% less. How many books did it sell in the second week? Also, find the number of books left in the bookshop after the second week.

21. In a straight contest between two candidates, the loser got 38% of the votes polled and lost by 16,800 votes. Find the total number of votes polled.

22. Of the total runs made by the Indian team during a match, Sachin scored 44% and Dhoni scored 39%. If Sachin scored 30 runs more than Dhoni, find (i) the score of the Indian team (ii) the runs scored by Sachin.

23. A man's salary was raised by 10% and then reduced by 10%. Find the percentage change in his salary.

24. A's salary is 50% higher than B's. By what per cent is B's salary lower than A's?

25. A's salary is 5% lower than B's. By what per cent is B's salary higher than A's?

26. The price of a notebook was increased by x%. To bring the price to the earlier level, it had to be reduced by $16\frac{2}{3}$%. Find the value of x.

27. In a monthly test, Sumit scored 97 out of 100 in mathematics, 69 out of 75 in science, 123 out of 150 in Hindi and 42 out of 50 in English.
    (i) In which subjects was his performance the best?
    (ii) In which subject did he perform the worst?
    (iii) What was his aggregate percentage?

28. In an examination, 35% of the students failed in English, 30% failed in Sanskrit and 15% failed in both subjects. Find
    (i) the percentage of students who passed in both subjects and
    (ii) the total number of students if 400 students passed in both subjects.

ANSWERS

1. (i) 3 : 25   (ii) 1 : 3   (iii) 1 : 400   2. (i) 40%   (ii) $41\frac{2}{3}$%   (iii) 42%

3. (i) $\dfrac{1}{50}$ (ii) $\dfrac{37}{600}$ (iii) $\dfrac{1}{12500}$ 4. (i) 60%   (ii) $166\frac{2}{3}$%   (iii) $416\frac{2}{3}$%

5. (i) 0.24   (ii) 0.022   (iii) 0.0002   6. (i) 0.5, 50%   (ii) 0.6, 60%   (iii) 0.475, 47.5%

7. (i) 195   (ii) 268.75 g   (iii) Rs 41.25   8. (i) 9   (ii) 352 L   (iii) 50 kg

9. (i) $33\frac{1}{3}$%   (ii) $29\frac{1}{6}$%   (iii) 18%

10. (i) $6\frac{2}{3}$% increase   (ii) 25% increase   (iii) 110% increase   (iv) 10% decrease   (v) 25% decrease

   (vi) 5% decrease

11. $16\frac{2}{3}$ **12.** Rs 25,000   **13.** 136   **14.** 900

15. Won $53\frac{1}{3}$%, lost 45%, drew $1\frac{2}{3}$%   **16.** 92 newspapers, 50 pens, 24 books and 34 others

17. Rs 3500   **18.** Rs 25,000   **19.** 76.5   **20.** 1530, 8670

21. 70,000   **22.** (i) 600   (ii) 264   **23.** Reduced by 1%   **24.** $33\frac{1}{3}$

25. $5\frac{5}{19}$ **26.** 20   **27.** (i) Mathematics   (ii) Hindi   (iii) $88\frac{4}{15}$

28. (i) 50   (ii) 800

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