

## 1

## Ratio and Proportion

## Ratio

We can compare the size of two quantities of the same kind in many different ways. One of them is **ratio**. The ratio of two quantities  $a$  and  $b$  of the **same kind** and in the **same unit of measurement** is the fraction  $\frac{a}{b}$ . It is also denoted as  $a : b$  (read as “ $a$  to  $b$ ” or “ $a$  is to  $b$ ”).

In the ratio  $a : b$ ,  $a$  is the first term or the **antecedent** and  $b$  is the second term or the **consequent**.

## A few points

1. A ratio is a number, it has no unit.
2. There can be no ratio between quantities of different kinds, for example, there can be no ratio between 3 kg and 4 m.
3. The quantities that are compared must be expressed in the same unit.

**Example** To work out the ratio of Rs 2 and 75 p, we must convert Rs 2 into paise. Thus, the ratio of Rs 2 and 75 p is not  $\frac{2}{75}$ , but  $\frac{200}{75} = \frac{8}{3}$ .

4. A ratio does not change if its antecedent and consequent are multiplied or divided by the same number.

**Example**  $12 \text{ m} : 3.6 \text{ m} = \frac{12}{3.6} = \frac{12 \times 10}{3.6 \times 10} = \frac{120}{36} = \frac{120 \div 12}{36 \div 12} = \frac{10}{3} = 10 : 3$ .

5. To express a ratio in the simplest form, take these steps:

- (i) If the antecedent and the consequent are integers then divide both by their HCF.
- (ii) If the antecedent or the consequent or both are fractions then multiply both by the LCM of their denominators.

**EXAMPLE**

**Express the ratios (i) 36 g : 48 g and (ii)  $2\frac{1}{4} \text{ m} : 7\frac{2}{5} \text{ m}$  in the simplest form.**

**Solution**

- (i) The HCF of 36 and 48 = 12.

$$\therefore 36 \text{ g} : 48 \text{ g} = 36 : 48 = (36 \div 12) : (48 \div 12) = 3 : 4.$$

- (ii) The LCM of the denominators 4 and 5 = 20.

$$\therefore 2\frac{1}{4} \text{ m} : 7\frac{2}{5} \text{ m} = \frac{9}{4} : \frac{37}{5} = \left(\frac{9}{4} \times 20\right) : \left(\frac{37}{5} \times 20\right) = (9 \times 5) : (37 \times 4) = 45 : 148.$$

6. A ratio can be expressed as a fraction. So, the rules for comparing fractions apply to ratios as well.

**EXAMPLE**

**Arrange the given ratios in descending order.**

- (i)  $4 : 7, 3 : 7$     (ii)  $5 : 8, 5 : 11$     (iii)  $3 : 5, 4 : 7$     (iv)  $3 : 4, 2 : 5, 7 : 8$

**Solution**

(i) The ratios have the same consequent. So, the ratio with the greater antecedent will be greater. Thus,  $4 : 7 > 3 : 7$ .

(ii) Here, the ratios have the same antecedent. So the ratio with the smaller consequent will be greater. Thus,  $5 : 8 > 5 : 11$ .

(iii) Since the ratios have different consequents and antecedents, we change them into like fractions.

The LCM of the consequents 5 and 7 = 35.

$$\therefore 3 : 5 = \frac{3}{5} = \frac{3 \times (\text{LCM} \div 5)}{5 \times (\text{LCM} \div 5)} = \frac{3 \times 7}{5 \times 7} = \frac{21}{35} = 21 : 35$$

$$\text{and } 4 : 7 = \frac{4}{7} = \frac{4 \times (\text{LCM} \div 7)}{7 \times (\text{LCM} \div 7)} = \frac{4 \times 5}{7 \times 5} = \frac{20}{35} = 20 : 35.$$

$$\therefore 21 > 20, 21 : 35 > 20 : 35, \text{ that is, } 3 : 5 > 4 : 7.$$

(iv) The LCM of the consequents 4, 5 and 8 = 40.

$$\therefore 3 : 4 = \frac{3}{4} = \frac{3 \times (\text{LCM} \div 4)}{4 \times (\text{LCM} \div 4)} = \frac{3 \times 10}{4 \times 10} = \frac{30}{40} = 30 : 40,$$

$$2 : 5 = \frac{2}{5} = \frac{2 \times (\text{LCM} \div 5)}{5 \times (\text{LCM} \div 5)} = \frac{2 \times 8}{5 \times 8} = \frac{16}{40} = 16 : 40,$$

$$7 : 8 = \frac{7}{8} = \frac{7 \times (\text{LCM} \div 8)}{8 \times (\text{LCM} \div 8)} = \frac{7 \times 5}{8 \times 5} = \frac{35}{40} = 35 : 40.$$

$$\therefore 35 > 30 > 16, 35 : 40 > 30 : 40 > 16 : 40, \text{ that is, } 7 : 8 > 3 : 4 > 2 : 5.$$

**Continued ratio**

If three quantities be such that the ratio of the first two is  $a : b$  and the ratio of the last two is  $b : c$  then the three quantities are said to be in the **continued ratio**  $a : b : c$ .

**Examples** (i) If  $a : b = 1 : 3$  and  $b : c = 3 : 7$  then  $a : b : c = 1 : 3 : 7$ .

(ii) Let  $m, n$  and  $p$  be such that  $m : n = 1 : 2$  and  $n : p = 3 : 5$ .

Then,  $m : n = (1 \times 3) : (2 \times 3) = 3 : 6$  and  $n : p = (3 \times 2) : (5 \times 2) = 6 : 10$ .

$$\therefore m : n : p = 3 : 6 : 10.$$

**Note** 3 is the antecedent in the second ratio and 2 is the consequent in the first ratio.

It would be useful to remember that if the ratio of two quantities be  $a : b$  then the two quantities can be expressed as  $ak$  and  $bk$ . Also, if the continued ratio of three quantities be  $a : b : c$  then the three quantities can be expressed as  $ak, bk$  and  $ck$ .

**Dividing a quantity in a given ratio**

If a number  $n$  is divided into two parts in the ratio  $a : b$ ,

$$\text{first part} = \frac{a}{a+b} \times n, \quad \text{second part} = \frac{b}{a+b} \times n$$

If a number  $n$  is divided into three parts in the continued ratio  $a : b : c$ ,

$$\text{first part} = \frac{a}{a+b+c} \times n, \quad \text{second part} = \frac{b}{a+b+c} \times n, \quad \text{third part} = \frac{c}{a+b+c} \times n$$

**EXAMPLE**

**Divide 2.4 kg of sweets between Arun and Ranju in the ratio 7 : 5.**

**Solution**

Given, Arun's share : Ranju's share = 7 : 5.

$$\therefore \text{Arun's share} = \frac{7}{7+5} \times 2.4 \text{ kg} = \frac{7}{12} \times 2.4 \text{ kg} = 1.4 \text{ kg}$$

$$\text{and Ranju's share} = \frac{5}{7+5} \times 2.4 \text{ kg} = \frac{5}{12} \times 2.4 \text{ kg} = 1 \text{ kg.}$$

$\therefore$  Arun and Ranju will get 1.4 kg and 1 kg of sweets respectively.

**EXAMPLE**

**A rod of length 3.24 m is broken into three pieces such that the ratio of their lengths is 4 : 2 : 3. Find the length of each piece.**

**Solution**

$$\text{The length of the first piece} = \frac{4}{4+2+3} \times 3.24 \text{ m} = \frac{4}{9} \times 3.24 \text{ m} = 1.44 \text{ m.}$$

$$\text{The length of the second piece} = \frac{2}{4+2+3} \times 3.24 \text{ m} = \frac{2}{9} \times 3.24 \text{ m} = 0.72 \text{ m.}$$

$$\begin{aligned} \therefore \text{the length of the third piece} \\ &= \text{the length of the rod} - (\text{the sum of the lengths of the first two pieces}) \\ &= 3.24 \text{ m} - (1.44 \text{ m} + 0.72 \text{ m}) = 3.24 \text{ m} - 2.16 \text{ m} = 1.08 \text{ m.} \end{aligned}$$

Alternatively,

$$\text{the length of the third piece} = \frac{3}{4+2+3} \times 3.24 \text{ m} = \frac{1}{3} \times 3.24 \text{ m} = 1.08 \text{ m.}$$

Hence, the lengths of the pieces are 1.44 m, 0.72 m and 1.08 m respectively.

**Multiplying ratio**

The fraction by which a quantity (the original quantity) has to be multiplied to make it equal to another quantity (the final quantity) is called the multiplying ratio.

$$\text{Multiplying ratio} = \frac{\text{final quantity}}{\text{original quantity}}$$

**Example** The multiplying ratio which changes 36 m into 42 m  
 $= \frac{\text{final quantity}}{\text{original quantity}} = \frac{42 \text{ m}}{36 \text{ m}} = \frac{7}{6} = 7 : 6.$

**Solved Examples****EXAMPLE 1**

**Express (i)  $2\frac{1}{2} : 1.5$ , (ii) 650 g is to 2 kg in the simplest form.**

**Solution**

$$(i) \quad 2\frac{1}{2} : 1.5 = \frac{5}{2} : \frac{15}{10} = \frac{5}{2} : \frac{3}{2} = \frac{5}{2} \times \frac{2}{3} = \frac{5}{3} = 5 : 3.$$

$$(ii) \quad 650 \text{ g is to } 2 \text{ kg} = 650 \text{ g} : 2000 \text{ g} = \frac{650}{2000} = \frac{13}{40} = 13 : 40.$$

**EXAMPLE 2****(i) Which of the following ratios is greater?**

$$1\frac{1}{3} : 2\frac{1}{2}, \quad 6\frac{3}{4} : 7\frac{1}{2}$$

**(ii) Arrange the following ratios in ascending order.**

$$9 : 10, \quad 4 : 5, \quad 5 : 18, \quad 2 : 3$$

**Solution**

$$(i) \quad 1\frac{1}{3} : 2\frac{1}{2} = \frac{4}{3} : \frac{5}{2} = \left(\frac{4}{3} \times 6\right) : \left(\frac{5}{2} \times 6\right) = 8 : 15$$

$$\text{and } 6\frac{3}{4} : 7\frac{1}{2} = \frac{27}{4} : \frac{15}{2} = \left(\frac{27}{4} \times 4\right) : \left(\frac{15}{2} \times 4\right) = 27 : 30 = 9 : 10.$$

Thus, the ratios are 8 : 15 and 9 : 10.

The LCM of 15 and 10 = 30.

$$\therefore 8 : 15 = (8 \times 2) : (15 \times 2) = 16 : 30 \text{ and } 9 : 10 = (9 \times 3) : (10 \times 3) = 27 : 30.$$

Thus, the ratios are 16 : 30 and 27 : 30.

$$\therefore 27 > 16, \quad 27 : 30, \text{ that is, } 6\frac{3}{4} : 7\frac{1}{2} \text{ is the greater ratio.}$$

**(ii) The LCM of the consequents 10, 5, 18 and 3 = 90.**

$$\therefore 9 : 10 = \frac{9}{10} = \frac{9 \times (\text{LCM} \div 10)}{10 \times (\text{LCM} \div 10)} = \frac{9 \times 9}{10 \times 9} = \frac{81}{90} = 81 : 90,$$

$$4 : 5 = \frac{4}{5} = \frac{4 \times (\text{LCM} \div 5)}{5 \times (\text{LCM} \div 5)} = \frac{4 \times 18}{5 \times 18} = \frac{72}{90} = 72 : 90,$$

$$5 : 18 = \frac{5}{18} = \frac{5 \times (\text{LCM} \div 18)}{18 \times (\text{LCM} \div 18)} = \frac{5 \times 5}{18 \times 5} = \frac{25}{90} = 25 : 90$$

$$\text{and } 2 : 3 = \frac{2}{3} = \frac{2 \times (\text{LCM} \div 3)}{3 \times (\text{LCM} \div 3)} = \frac{2 \times 30}{3 \times 30} = \frac{60}{90} = 60 : 90.$$

$$\therefore 25 < 60 < 72 < 81, \quad 25 : 90 < 60 : 90 < 72 : 90 < 81 : 90,$$

that is, 5 : 18 &lt; 2 : 3 &lt; 4 : 5 &lt; 9 : 10.

**EXAMPLE 3**

**If  $a$ ,  $b$  and  $c$  are three numbers such that  $a : b = 1\frac{1}{2} : 2\frac{1}{3}$  and  $b : c = 1\frac{1}{4} : \frac{1}{2}$  then find (i)  $a : b : c$  and (ii)  $c : a$ .**

**Solution**

$$\text{Here, } a : b = \frac{3}{2} : \frac{7}{3} = \left(\frac{3}{2} \times 6\right) : \left(\frac{7}{3} \times 6\right) = 9 : 14$$

$$\text{and } b : c = \frac{5}{4} : \frac{1}{2} = \left(\frac{5}{4} \times 4\right) : \left(\frac{1}{2} \times 4\right) = 5 : 2.$$

In the ratio  $a : b$ ,  $b = 14$  and in the ratio  $b : c$ ,  $b = 5$ . We have to make  $b$  equal in the two ratios. The LCM of 5 and 14 =  $5 \times 14$ . So, we multiply the antecedent and consequent of the first ratio by 5 and the antecedent and consequent of the second ratio by 14.

$$\text{Thus, } a : b = 9 : 14 = (9 \times 5) : (14 \times 5) = 45 : 70$$

$$\text{and } b : c = 5 : 2 = (5 \times 14) : (2 \times 14) = 70 : 28.$$

$$(i) \therefore a : b : c = 45 : 70 : 28.$$

$$(ii) \therefore a : c = 45 : 28 \text{ and } c : a = 28 : 45.$$

**EXAMPLE 4** If  $2A = 3B = 4C$ , find  $A : B : C$ .

**Solution** Let  $2A = 3B = 4C = x$  then  $A = \frac{x}{2}$ ,  $B = \frac{x}{3}$ ,  $C = \frac{x}{4}$ .

$$\therefore A : B : C = \frac{x}{2} : \frac{x}{3} : \frac{x}{4} = \frac{1}{2} : \frac{1}{3} : \frac{1}{4} = \left(\frac{1}{2} \times 12\right) : \left(\frac{1}{3} \times 12\right) : \left(\frac{1}{4} \times 12\right) = 6 : 4 : 3.$$

**EXAMPLE 5** Mr Bose divided Rs 100 between his son and daughter in the ratio  $2\frac{3}{4} : 2\frac{1}{4}$ . How much money did his son get?

**Solution** The son's share =  $\frac{2\frac{3}{4}}{2\frac{3}{4} + 2\frac{1}{4}} \times \text{Rs } 100 = \frac{11}{5} \times \text{Rs } 100$

$$= \frac{11}{20} \times \text{Rs } 100 = 11 \times \text{Rs } 5 = \text{Rs } 55.$$

**EXAMPLE 6** A certain quantity of rice is put into two bags in the ratio 9 : 7. If the bigger bag contains 27 kg of rice, what is the total quantity of rice?

**Solution** Let the rice in the smaller bag be  $x$  kg.  
Then  $27 \text{ kg} : x \text{ kg} = 9 : 7$  or  $\frac{27}{x} = \frac{9}{7}$  or  $x = \frac{27 \times 7}{9} = 21$ .

$\therefore$  the smaller bag contains 21 kg of rice.  
 $\therefore$  the total quantity of rice =  $27 \text{ kg} + 21 \text{ kg} = 48 \text{ kg}$ .

**EXAMPLE 7** Divide Rs 19,000 among Jack, George and Payal in the ratio  $2\frac{2}{3} : 1\frac{3}{4} : 3\frac{1}{2}$ .

**Solution** The ratio =  $\frac{8}{3} : \frac{7}{4} : \frac{7}{2} = \left(\frac{8}{3} \times 12\right) : \left(\frac{7}{4} \times 12\right) : \left(\frac{7}{2} \times 12\right) = 32 : 21 : 42$ .

$\therefore$  Jack's share =  $\frac{32}{32+21+42} \times \text{Rs } 19000 = \frac{32}{95} \times \text{Rs } 19000 = \text{Rs } 6400$ .

George's share =  $\frac{21}{32+21+42} \times \text{Rs } 19000 = \frac{21}{95} \times \text{Rs } 19000 = \text{Rs } 4200$ .

Payal's share =  $\frac{42}{32+21+42} \times \text{Rs } 19000 = \frac{42}{95} \times \text{Rs } 19000 = \text{Rs } 8400$ .

**EXAMPLE 8** Three friends X, Y and Z share Rs 840. If their shares  $x$ ,  $y$ ,  $z$  are such that  $x : y = 2 : 1$  and  $y : z = 2 : 1$  then find each share.

**Solution** Let  $z = \text{Rs } k$ . Since  $y : z = 2 : 1$ ,  $\frac{y}{z} = \frac{2}{1}$  or  $y = 2z = 2 \times \text{Rs } k$ ; so,  $y = \text{Rs } 2k$ .

Also, since  $x : y = 2 : 1$ ,  $x = 2y = 2 \times \text{Rs } 2k = \text{Rs } 4k$ .

$\therefore$  the sum of the shares =  $x + y + z = \text{Rs } 840$  or  $4k + 2k + k = 840$  or  $7k = 840$ .

$\therefore k = \frac{840}{7} = 120$ .

$\therefore$  X's share =  $\text{Rs } 4k = \text{Rs } 4 \times 120 = \text{Rs } 480$ ,  
Y's share =  $\text{Rs } 2k = \text{Rs } 2 \times 120 = \text{Rs } 240$   
and Z's share =  $\text{Rs } k = \text{Rs } 120$ .

**EXAMPLE 9** Divide Rs 216 among A, B and C such that B's share : C's share = 5 : 1 and A's share : B's share =  $1\frac{1}{2} : 1$ .

**Solution** Let C's share = Rs  $x$ . Then, B's share = Rs  $5x$  and A's share = Rs  $\left(\frac{3}{2} \times 5x\right)$   
 $= \text{Rs } \frac{15x}{2}$ .

Given, sum of their shares = Rs 216.

$$\therefore \frac{15x}{2} + 5x + x = 216 \quad \text{or} \quad \frac{27x}{2} = 216 \quad \text{or} \quad x = \frac{216 \times 2}{27} = 16.$$

$$\therefore \text{A's share} = \text{Rs } \frac{15 \times 16}{2} = \text{Rs } 120, \text{ B's share} = \text{Rs } 5 \times 16 = \text{Rs } 80 \text{ and}$$

$$\text{C's share} = \text{Rs } 16.$$

Hence, A, B and C get Rs 120, Rs 80 and Rs 16 respectively.

**EXAMPLE 10** A donation box contains 5-rupee, 2-rupee and 50-paisa coins in the ratio 5 : 3 : 9 by number. If the total value of all the coins is Rs 142, find the number of each type of coin.

**Solution** The number of 5-rupee, 2-rupee and 50-paisa coins are in the ratio 5 : 3 : 9. We can assume that the number of 5-rupee, 2-rupee and 50-paisa coins are  $5k$ ,  $3k$  and  $9k$  respectively.

Thus, the value of the 5-rupee coins =  $5k \times \text{Rs } 5 = \text{Rs } 25k$ ,

the value of the 2-rupee coins =  $3k \times \text{Rs } 2 = \text{Rs } 6k$

and the value of the 50-paisa coins =  $\text{Rs } \frac{1}{2} \times 9k = \text{Rs } \frac{9k}{2}$ .

$$\text{Given, total value of the coins} = \text{Rs } 142 \quad \text{or} \quad 25k + 6k + \frac{9k}{2} = 142$$

$$\text{or} \quad \frac{71k}{2} = 142 \quad \text{or} \quad k = 4.$$

Thus, the number of 5-rupee coins =  $5k = 5 \times 4 = 20$ ,

the number of 2-rupee coins =  $3k = 3 \times 4 = 12$  and

the number of 50-paisa coins =  $9k = 9 \times 4 = 36$ .

**EXAMPLE 11** Two numbers are in the ratio 2 : 3. If the first number is decreased by 4 and the second number is increased by 2, the ratio changes to 2 : 5. Find the numbers.

**Solution** Given,  $\frac{\text{the first number}}{\text{the second number}} = \frac{2}{3}$ .

Let the first number =  $2x$ , then the second number =  $3x$ .

$$\text{Also given that } \frac{2x - 4}{3x + 2} = \frac{2}{5} \quad \text{or} \quad 5(2x - 4) = 2(3x + 2)$$

$$\text{or} \quad 10x - 20 = 6x + 4 \quad \text{or} \quad 10x - 6x = 4 + 20 \quad \text{or} \quad 4x = 24 \quad \text{or} \quad x = 6.$$

$$\therefore \text{the first number} = 2 \times 6 = 12 \text{ and the second number} = 3 \times 6 = 18.$$

**EXAMPLE 12** A school has 210 students. The ratio of the number of boys to the number of girls is 3 : 2. The ratio changes to 7 : 5 after the admission of 30 new students. Find the number of newly admitted girls.

**Solution** Given,  $\frac{\text{the number of boys}}{\text{the number of girls}} = \frac{3}{2} = 3 : 2$  and the total number of students = 210.

$$\therefore \text{the number of boys} = \frac{3}{3+2} \times 210 = \frac{3}{5} \times 210 = 126$$

$$\text{and the number of girls} = \frac{2}{3+2} \times 210 = \frac{2}{5} \times 210 = 84.$$

Let the number of newly admitted girls =  $x$ .

Then the number of newly admitted boys =  $30 - x$ .

$\therefore$  the total number of girls =  $84 + x$

and the total number of boys =  $126 + 30 - x = 156 - x$ .

$$\text{Given, } \frac{156 - x}{84 + x} = \frac{7}{5} \quad \text{or} \quad 5(156 - x) = 7(84 + x)$$

$$\text{or } 780 - 5x = 588 + 7x \quad \text{or} \quad 7x + 5x = 780 - 588$$

$$\text{or } 12x = 192 \quad \text{or} \quad x = \frac{192}{12} = 16.$$

Hence, the number of newly admitted girls = 16.

### Remember These

1. A ratio exists only between quantities of the same kind measured in the same unit.
2. In a ratio  $a : b$ ,  $a$  = first term (antecedent),  $b$  = second term (consequent).
3.  $a : b = (a \times k) : (b \times k)$  and  $a : b = (a \div k) : (b \div k)$ , where  $k \neq 0$  is any number.
4. If the ratio of (i) two quantities be  $a : b$  then the quantities may be expressed as  $ak$  and  $bk$ .  
(ii) three quantities be  $a : b : c$  then the quantities are  $ak$ ,  $bk$  and  $ck$ .
5. If a number  $n$  is divided into two parts in the ratio  $a : b$  then  
the first part =  $\frac{a}{a+b} \times n$  and the second part =  $\frac{b}{a+b} \times n$ .
6. If a number  $n$  is divided into three parts in the ratio  $a : b : c$  then  
the first part =  $\frac{a}{a+b+c} \times n$ , the second part =  $\frac{b}{a+b+c} \times n$  and the third part =  $\frac{c}{a+b+c} \times n$ .
7. Multiplying ratio =  $\frac{\text{final quantity}}{\text{original quantity}}$ .

### EXERCISE

### 1A

1. Express the following ratios in the simplest form.

(i) 39 weeks to 182 days

(ii) 20 min : 3 h

(iii) 2 m : 1 m 70 cm

(iv) 5 cm to 1.25 km

(v) 80 paise : Rs 5

(vi) 9 kg :  $2\frac{2}{5}$  kg

(vii) 26 : 11.7

(viii)  $1\frac{1}{2} : 2\frac{1}{3}$

(ix)  $6\frac{2}{7} : 0.88$

(x)  $23\frac{4}{7} : 20\frac{5}{8}$

(xi)  $\frac{1}{2} : \frac{2}{3} : \frac{4}{5}$

(xii)  $1\frac{5}{6} : 4\frac{1}{3} : 2\frac{1}{4}$

2. Which of the following ratios is greater?

(i) 3 : 4 or 5 : 6

(ii) 5 : 7 or 13 : 14

(iii)  $2\frac{1}{2} : 3\frac{3}{4}$  or  $1\frac{2}{3} : 1\frac{5}{6}$

(iv)  $\frac{1}{5} : \frac{1}{6}$  or 1.8 : 2.4

3. Write the following in ascending order of magnitude.

(i)  $2 : 3, 7 : 12, 13 : 19$       (ii)  $5 : 12, 11 : 18, 13 : 24$       (iii)  $1 : 2, 3 : 4, 8 : 15, 11 : 20$

4. (i) If  $a : b = 4 : 5$  and  $b : c = 5 : 9$ , find  $a : b : c$  and  $a : c$ .

(ii) If  $a : b = \frac{1}{2} : \frac{1}{3}$  and  $c : b = \frac{1}{5} : \frac{1}{4}$ , find  $a : b : c$  and  $c : a$ .

(iii) If  $a : c = 3 : 4$  and  $b : c = 5 : 16$ , find  $a : b : c$  and  $a : b$ .

5. Find  $a : b : c$  when (i)  $2a = 3b = 5c$       (ii)  $3a = 4b = 6c$

6. (i) Divide Rs 3240 between A and B in the ratio  $11 : 7$ .

(ii) Divide Rs 1720 between Ram and Rita in the ratio  $2\frac{1}{5} : 2\frac{1}{10}$ .

7. A sum of money is divided between A and B in the ratio  $2\frac{1}{2} : 3\frac{1}{2}$ . If A's share is ₹ 5525, find the sum of money.

8. Ram covered a certain distance by bicycle and then covered three fourths of that distance by car. If he covered a total distance of 1400 m, what distance did he cover by bicycle?

9. A board of length 2.1 m is cut into two pieces in the ratio  $2 : 5$ . Find the lengths of the two pieces.

10. Brass is an alloy of copper and zinc in the ratio  $3 : 2$ . Find the quantities of copper and zinc in 25 kg of brass.

11. (i) Divide 940 g into three parts in the ratio  $\frac{1}{3} : \frac{1}{4} : \frac{1}{5}$ .

(ii) Divide Rs 2472 into three parts in the ratio  $3\frac{1}{4} : 1\frac{2}{5} : \frac{1}{2}$ .

12. Divide ₹ 1800 among A, B and C such that A's share : B's share =  $3 : 1$  and B's share : C's share =  $2 : 1$ .

13. Divide Rs 13,600 among Jaya, Vijaya and Sudha such that Jaya's share : Vijaya's share =  $\frac{1}{2} : 1$  and Vijaya's share : Sudha's share =  $\frac{3}{4} : 1$ .

14. Divide Rs 1216 into three parts such that the first part is  $\frac{1}{5}$  of the second part and the ratio of the second part to the third part is  $3 : 4$ .

15. A certain sum of money is divided into three parts in the ratio  $3 : 5 : 9$ . If the third part is Rs 450, find the total amount and the other two parts.

16. A bag contains 5-rupee, 2-rupee and 1-rupee coins in the ratio  $1 : 2 : 5$  by number. If the total value of the coins is Rs 140, find the number of each type of coin.

17. Find the multiplying ratio that will change 26 g to 39 g.

18. Decrease 64 cm in the ratio  $3 : 4$ .

19. Ajit used to save Rs 8400 per month. His monthly saving has decreased in the ratio  $6 : 5$ . What is his monthly saving now?

20. Two numbers are in the ratio  $4 : 7$ . When 3 is added to each number, the ratio changes to  $3 : 5$ . Find the numbers.



21. What must be added to each term of the ratio 4 : 9 so that the resultant ratio becomes 2 : 3?
22. Two numbers are in the ratio 5 : 6. When 3 is subtracted from the first number and 2 is added to the second number, the ratio changes to 3 : 5. Find the numbers.
23. Two numbers are in the ratio 5 : 7. When 5 is added to the first number and 2 is subtracted from the second number, the new numbers are in the ratio 10 : 11. Find the numbers.
24. In a school of 600 students the number of boys is 360. When 60 new students are admitted in the school, the ratio of the number of boys to the number of girls becomes 13 : 9. Find the number of newly admitted boys.

## ANSWERS

1. (i) 3 : 2 (ii) 1 : 9 (iii) 20 : 17 (iv) 1 : 25000 (v) 4 : 25 (vi) 15 : 4 (vii) 20 : 9 (viii) 9 : 14  
(ix) 50 : 7 (x) 8 : 7 (xi) 15 : 20 : 24 (xii) 22 : 52 : 27
2. (i) 5 : 6 (ii) 13 : 14 (iii)  $1\frac{2}{3} : 1\frac{5}{6}$  (iv)  $\frac{1}{5} : \frac{1}{6}$
3. (i) 7 : 12 < 2 : 3 < 13 : 19 (ii) 5 : 12 < 13 : 24 < 11 : 18 (iii) 1 : 2 < 8 : 15 < 11 : 20 < 3 : 4
4. (i) 4 : 5 : 9, 4 : 9 (ii) 15 : 10 : 8, 8 : 15 (iii) 12 : 5 : 16, 12 : 5
5. (i) 15 : 10 : 6 (ii) 4 : 3 : 2
6. (i) Rs 1980, Rs 1260 (ii) Rs 880, Rs 840
7. ₹ 13,260
8. 800 m
9. 60 cm, 1.5 m
10. 15 kg, 10 kg
11. (i) 400 g, 300 g, 240 g (ii) Rs 1560, Rs 672, Rs 240
12. ₹ 1200, ₹ 400, ₹ 200
13. Rs 2400, Rs 4800, Rs 6400
14. Rs 96, Rs 480, Rs 640
15. Rs 850; Rs 150, Rs 250
16. 10, 20, 50
17. 3 : 2
18. 48 cm
19. Rs 7000
20. 24, 42
21. 6
22. 15, 18
23. 25, 35
24. 30

## Proportion

If two ratios  $a : b$  and  $c : d$  are equal, that is,  $a : b = c : d$  then the four quantities  $a$ ,  $b$ ,  $c$  and  $d$  are said to be in proportion. This is also written as  $a : b :: c : d$ . The quantities  $a$ ,  $b$ ,  $c$ ,  $d$ , which are called the **terms of the proportion**, need not all be of the same kind. However, the terms of the first ratio must be of the same kind, and the terms of the second ratio must also be of the same kind.

**Examples** (i) The numbers 28, 42, 52 and 78 are in proportion because

$$28 : 42 = \frac{28}{42} = \frac{2}{3} = 2 : 3 \text{ and } 52 : 78 = \frac{52}{78} = \frac{2}{3} = 2 : 3.$$

(ii) The quantities 3 h, 6 h, 5 L, 10 L are in proportion because

$$3 \text{ h} : 6 \text{ h} = \frac{3 \text{ h}}{6 \text{ h}} = \frac{1}{2} = 1 : 2 \text{ and } 5 \text{ L} : 10 \text{ L} = \frac{5 \text{ L}}{10 \text{ L}} = \frac{1}{2} = 1 : 2.$$

The first and the fourth terms of a proportion are called the **extremes**, while the second and the third terms are called the **means**. The fourth term is also called the **fourth proportion** to the other three terms.

Now, if  $a : b = c : d$  then  $\frac{a}{b} = \frac{c}{d}$  or  $ad = bc$ , i.e.,

the product of the extremes = the product of the means

This is known as the **cross product rule**.

### Continued proportion

Three quantities  $a, b, c$  are said to be in **continued proportion** if  $a : b = b : c$ . The three quantities must be of the **same kind**. The quantity  $b$  is called the **mean proportional** between  $a$  and  $c$ , while the quantity  $c$  is called the **third proportional** to  $a$  and  $b$ .

Now, if  $\frac{a}{b} = \frac{b}{c}$  then  $b^2 = a \times c$  or  $b = \sqrt{a \times c}$ .

Also,  $c = \frac{b^2}{a}$  and  $a = \frac{b^2}{c}$ .

**Example** The numbers 3, 9 and 27 are in continued proportion because

$$3 : 9 = \frac{3}{9} = \frac{1}{3} = 1 : 3 \text{ and } 9 : 27 = \frac{9}{27} = \frac{1}{3} = 1 : 3. \text{ So, } 3 : 9 = 9 : 27.$$

### Solved Examples

**EXAMPLE 1** Do the two ratios  $2\frac{1}{2} : 3\frac{1}{2}$  and  $\frac{1}{21} : \frac{1}{15}$  form a proportion?

**Solution**  $2\frac{1}{2} : 3\frac{1}{2} = \frac{5}{2} : \frac{7}{2} = 5 : 7$ ,  $\frac{1}{21} : \frac{1}{15} = \frac{\frac{1}{21}}{\frac{1}{15}} = \frac{15}{21} = \frac{5}{7} = 5 : 7$ .

$\therefore 2\frac{1}{2} : 3\frac{1}{2} = \frac{1}{21} : \frac{1}{15}$ . So, the given ratios form a proportion.

**EXAMPLE 2** Find  $x$  if (i)  $\frac{1}{2} : x :: 9 : 6$  (ii)  $1\frac{3}{4} : 2\frac{2}{3} :: x : 5\frac{1}{3}$ .

**Solution** (i) Given,  $\frac{1}{x} = \frac{9}{6}$  or  $9x = 6 \times \frac{1}{2}$  or  $x = \frac{3}{9}$  or  $x = \frac{1}{3}$ .

Alternatively,  $\frac{1}{2} : x :: 9 : 6$ , so  $9x = \frac{1}{2} \times 6$  or  $x = \frac{3}{9} = \frac{1}{3}$ .

(ii) Given,  $\frac{7}{8} = \frac{x}{16}$  or  $\frac{7}{4} \times \frac{16}{3} = x \times \frac{8}{3}$  or  $x = \frac{28}{3} \times \frac{3}{8} = \frac{7}{2} = 3\frac{1}{2}$ .

Alternatively,  $2\frac{2}{3} \times x = 1\frac{3}{4} \times 5\frac{1}{3}$ , so  $\frac{8}{3} \times x = \frac{7}{4} \times \frac{16}{3}$

or  $x = \frac{7}{4} \times \frac{16}{3} \times \frac{3}{8} = \frac{7}{2} = 3\frac{1}{2}$ .

**EXAMPLE 3** Find the third proportional to  $\frac{3}{5}$  and  $\frac{2}{5}$ .

**Solution** Let  $x$  be the third proportional. So,  $\frac{3}{5} : \frac{2}{5} = \frac{2}{5} : x$ .

$$\text{Then, } \frac{\frac{3}{5}}{\frac{2}{5}} = \frac{\frac{2}{5}}{x} \quad \text{or} \quad \frac{3}{5} \times x = \frac{2}{5} \times \frac{2}{5} \quad \text{or} \quad x = \frac{2 \times 2}{3 \times 5} = \frac{4}{15}$$

Hence, the third proportional =  $\frac{4}{15}$ .

**EXAMPLE 4** Find the mean proportional between  $5\frac{1}{4}$  m and  $9\frac{1}{3}$  m.

**Solution** Let the mean proportional =  $x$  m.

$$\text{Then, } 5\frac{1}{4} : x = x : 9\frac{1}{3} \quad \text{or} \quad \frac{21}{4} = \frac{x}{\frac{28}{3}} \quad \text{or} \quad x^2 = \frac{21}{4} \times \frac{28}{3} = 49 \quad \text{or} \quad x = 7.$$

Hence, the mean proportional = 7 m.

**EXAMPLE 5** The areas of two fields are in the ratio 4 : 5. If the area of the smaller field is  $324 \text{ m}^2$ , find the area of the larger one.

**Solution** Let the area of the larger field be  $A \text{ m}^2$ .

From the question,  $324 \text{ m}^2 : A \text{ m}^2 = 4 : 5$  or  $324 : A = 4 : 5$ .

$$\therefore A \times 4 = 324 \times 5 \quad \text{or} \quad A = \frac{324 \times 5}{4} = \frac{1620}{4} = 405.$$

$\therefore$  the area of the larger field =  $405 \text{ m}^2$ .

### Remember These

- If  $\frac{a}{b} = \frac{c}{d}$  then the quantities  $a$ ,  $b$ ,  $c$  and  $d$  are said to be in proportion. The quantities  $a$  and  $b$  must be of the same kind and the quantities  $c$  and  $d$  must be of the same kind.
- If  $\frac{a}{b} = \frac{c}{d}$  then  $ad = bc$ , i.e., the product of the extremes = the product of the means.
- Three quantities  $a$ ,  $b$ ,  $c$  of the same kind are said to be in continued proportion when  $\frac{a}{b} = \frac{b}{c}$ , that is,  $b^2 = ac$ . The quantity  $b$  is called the mean proportional and the quantity  $c$  is called the third proportional.

### EXERCISE

### 1B

1. Identify the true and false statements.

(i)  $\frac{2}{3} : \frac{5}{7} = \frac{8}{9} : \frac{20}{21}$

(ii)  $0.3 : 1.5 :: 0.06 : 0.21$

(iii) Rs 6.50, Rs 16.25, 4.8 kg, 12 kg are in proportion.

(iv) 6 weeks, 24 days, 5.6 m, 3.2 m are in proportion.

2. Find  $x$  such that

(i)  $x : 25 = 16 : 80$

(ii)  $16 \text{ g} : x \text{ g} :: 3.8 \text{ m} : 5.7 \text{ m}$

(iii)  $5\frac{3}{5} \text{ L} : 3\frac{1}{2} \text{ L} = \text{Rs } 5\frac{1}{5} : x$

(iv)  $1 \text{ m } 60 \text{ cm} : 1 \text{ m } 45 \text{ cm} :: x \text{ kg} : 4.35 \text{ kg}$

3. Find the fourth proportional to the given quantities.

(i) 10, 12, 25

(ii)  $5, \frac{1}{3}, 4\frac{2}{7}$

(iii) Rs 80, Rs 112, 2.5 kg

(iv)  $2\frac{1}{2} \text{ m}, 3\frac{1}{5} \text{ m}, 75 \text{ g}$

4. Find the quantity to which Rs 45 bears the same ratio as 1.5 kg bears to 1.25 kg.

5. A certain quantity bears the same ratio to  $3\frac{1}{5} \text{ m}$  as Rs 4.5 bears to Rs 6. Find the quantity.

6. Are the following quantities in continued proportion?

(i) 16, 8, 4

(ii)  $2\frac{2}{5} \text{ m}, 9\frac{3}{5} \text{ m}, 38\frac{2}{5} \text{ m}$

(iii) 3.8 g, 5.7 g, 9.5 g

(iv) 2.4 mL, 3.6 mL, 5.4 mL

7. Find the third proportional to each of the following.

(i) 16, 6

(ii)  $1\frac{1}{3}, 2\frac{2}{5}$

(iii) 3.6 cm, 1.2 cm      (iv) 3 g, 4.5 g

8. Find the mean proportional between the following.

(i) 32, 18

(ii)  $1\frac{1}{20}, \frac{16}{105}$

(iii)  $4\frac{4}{7} \text{ g}, 3\frac{1}{2} \text{ g}$

(iv) Rs 1.5, Rs 6

9. In a park, the ratio of the number of children playing cricket to the number of children playing football is the same as the ratio of the number of children playing football to the number of children cycling. If 45 children are playing cricket and 30 are playing football, find the number of children who are cycling.

10. The ratio of Aftab's monthly income and expenditure is 5 : 3. If his monthly income is Rs 27,000, find his expenditure.

11. A piece of wood is cut in the ratio 2 : 3. If the shorter piece is 1.44 m long, find the length of the longer piece.

## ANSWERS

1. (i) True    (ii) False    (iii) True    (iv) True

2. (i) 5    (ii) 24    (iii) Rs  $3\frac{1}{4}$     (iv) 4.8

3. (i) 30    (ii)  $\frac{2}{7}$     (iii) 3.5 kg    (iv) 96 g

4. Rs 37.5

5.  $2\frac{2}{5} \text{ m}$

6. (i) Yes    (ii) Yes    (iii) No    (iv) Yes

7. (i)  $2\frac{1}{4}$     (ii)  $4\frac{8}{25}$     (iii) 0.4 cm    (iv) 6.75 g

8. (i) 24    (ii)  $\frac{2}{5}$     (iii) 4 g    (iv) Rs 3

9. 20

10. Rs 16,200

11. 2.16 m

