24 GRAPHS

- Graph
- Abscissa
- Ordinate
- Truth Table

Graphs

A graph is a diagrammatical representation displaying the relation between two or more variables.

Graphs are plotted on a graph paper that is divided into small squares that make counting distances from the coordinate axes easier. The X axis and Y axis divide the graph paper into four quadrants as shown in Figure 24.1.

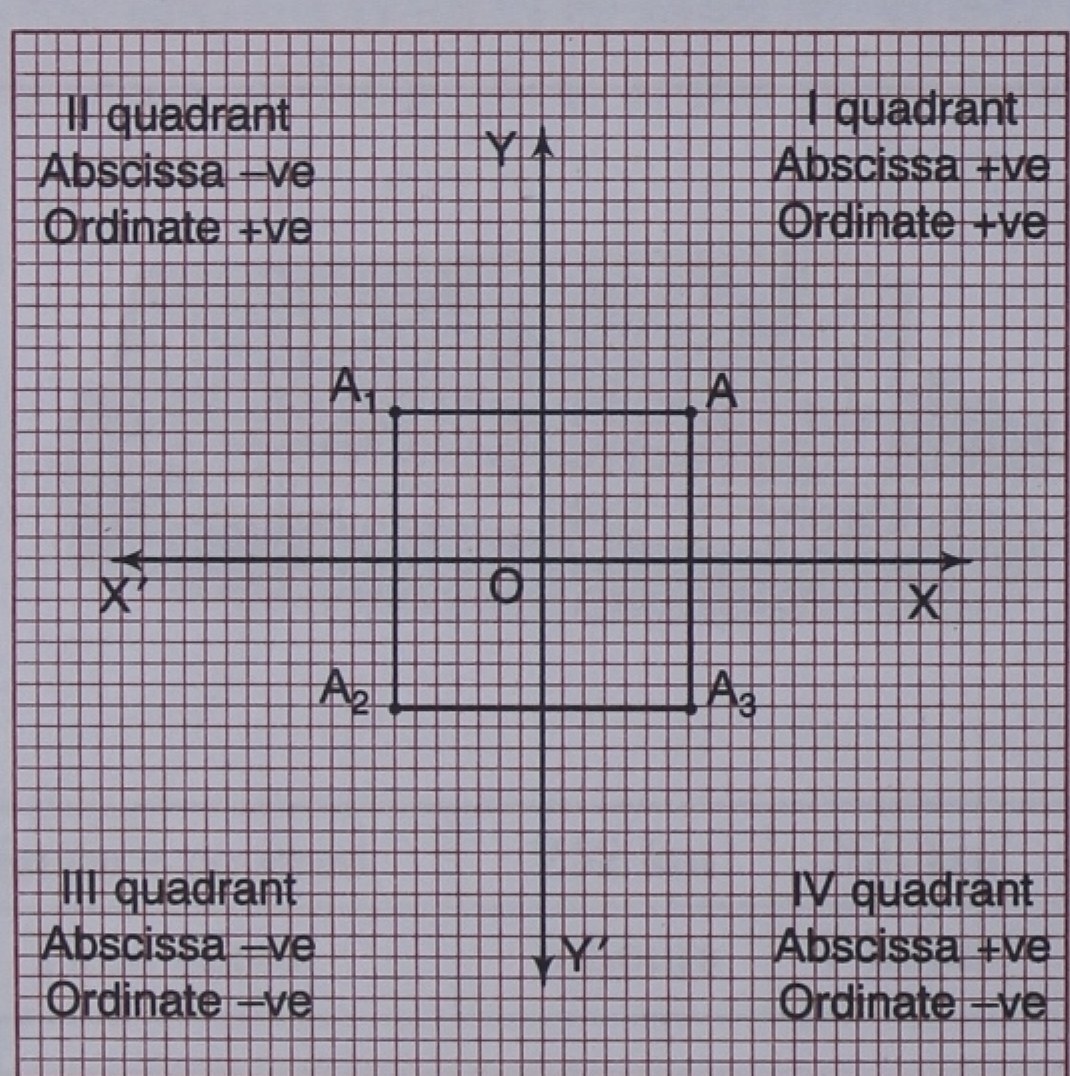

Fig. 24.1

The position of a point on the graph paper is described by an ordered pair (x, y). It represents its distance x along the X axis from the Y axis called the **abscissa** (horizontal distance), followed by its distance y along the Y axis from the X axis called the **ordinate** (vertical distance). The two distances plot the position of a point with respect to the origin of the graph. O (0, 0).

The position of a point is determined by dropping perpendiculars on the X axis and the Y axis and measuring the abscissa and ordinate respectively. In Figure 24.1, the position of the points are: A (7, 7), A_1 (–7, 7), A_2 (–7, –7) and A_3 (7, –7). Notice that the abscissae of all points to the left of the Y axis are negative while the ordinates of all points below the X axis are negative.

Example1: Plot A (3, 7), B (–5, –4), C (3, –7), D (–6, 4), E (0, –4), F(0, 6), and G(6, 0), on a graph paper.

See Figure 24.2.

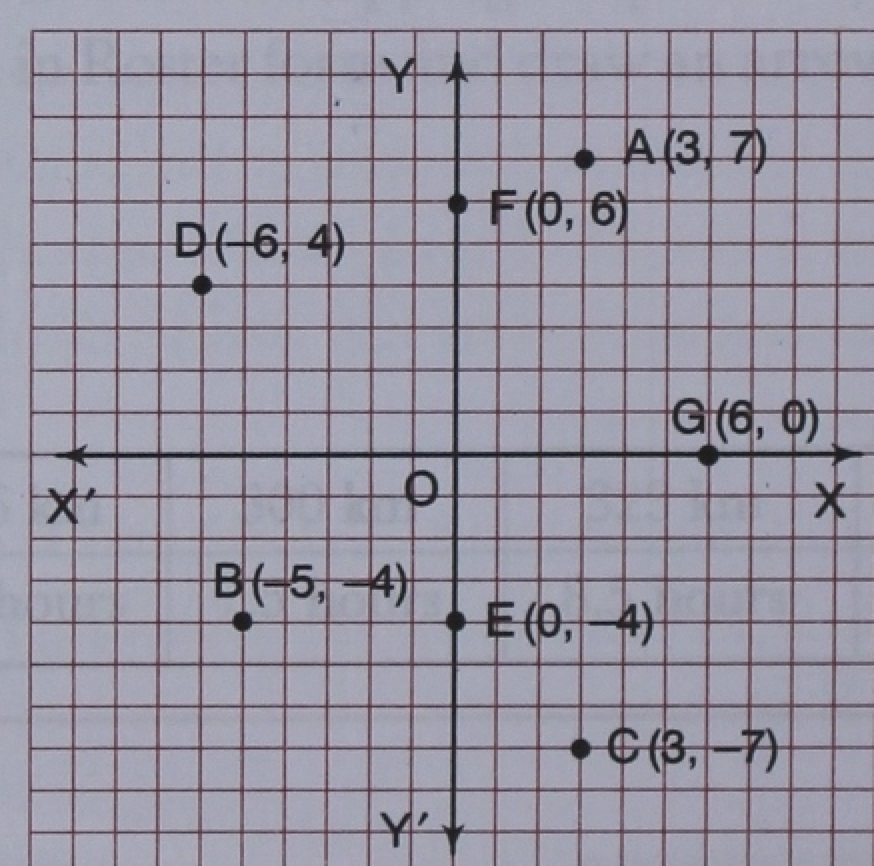

Fig. 24.2

Try this!

Plot A (1, 1), B (7, –3), C (–8, 5), and D (–9, –9) on a graph paper.

Graphical Representation of a Linear Equation in Two Variables

In order to represent a linear equation in two variables, say x and y, a truth table is constructed in which the values for y are calculated for at least 3 values of x (although a line can be drawn even with two points). The ordered pairs of values that satisfy the equation are then plotted on the graph paper and joined by a line that represents the equation.

Example 2: Draw a graph for $2x - 6y = 12$

$\Rightarrow \quad -6y = 12 - 2x$ (changing the subject to y)

$\Rightarrow \quad 6y = 2x - 12$

$\Rightarrow \quad y = \frac{2x}{6} - \frac{12}{6}$

$\Rightarrow \quad y = \frac{x}{3} - 2$

Truth table

x	3	6	9
Substituting x	$\frac{3}{3} - 2$	$\frac{6}{3} - 2$	$\frac{9}{3} - 2$
y	−1	0	1

Graphically,

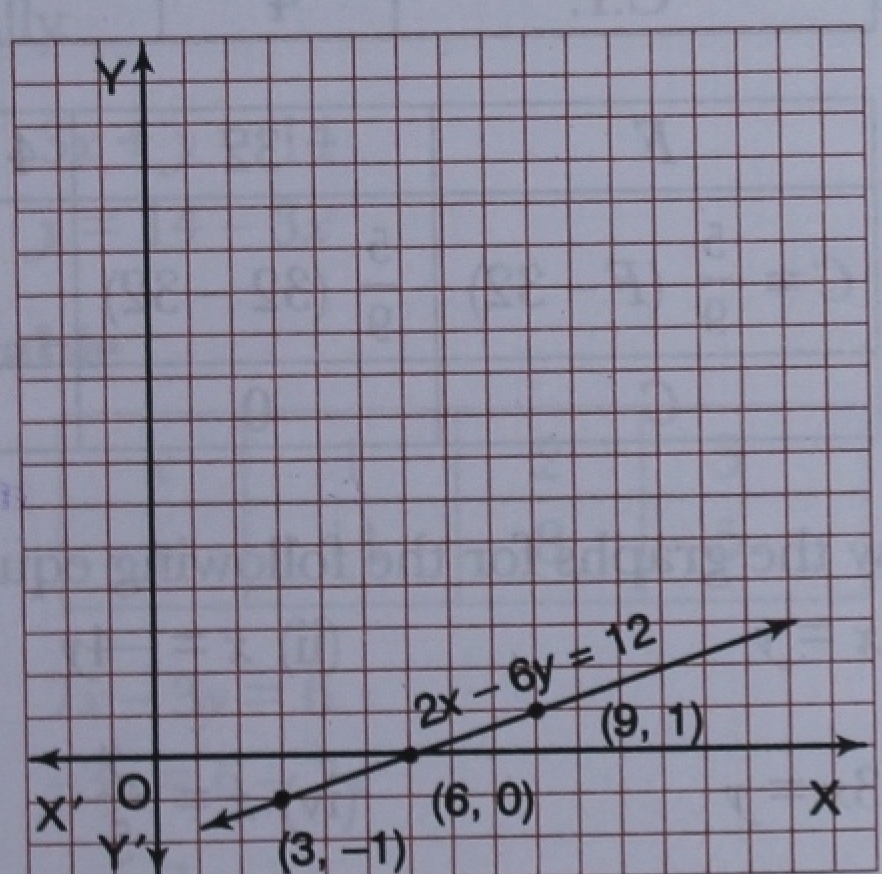

Fig. 24.3

Try this!
Prepare a truth table for $x + 2y = 10$

The Concept of a Linear Locus

The locus is a path traced by a point moving according to a given condition.

Think of a man entering a big office building, with a hundred rooms in 10 floors, through the entrance marked (0, 0). See Figure 24.4. From (0, 0) if he takes a lift straight up to the 8^{th} floor where does he reach? He reaches room (0, 8). Although his vertical distance (ordinate) from the entrance increased, his horizontal distance (abscissa) did not. In fact, he may keep going up or down the lift but his horizontal distance from the entrance will not change as his locus is described by the equation $x = 0$. Whatever the value of y, the value of x is 0.

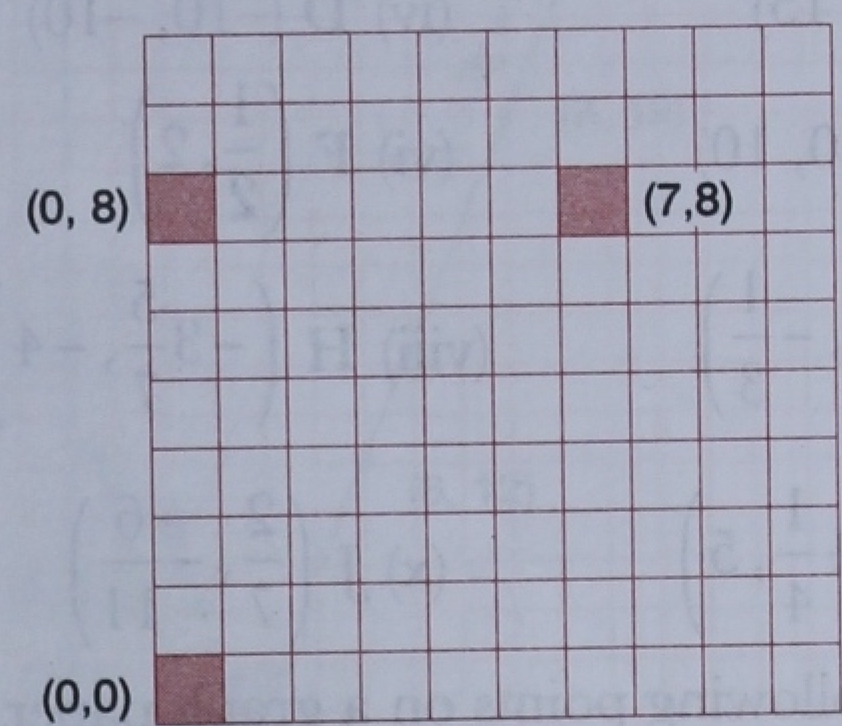

Fig. 24.4

Now if he steps out of the lift at the 8th floor and walks straight ahead to the 7^{th} room, where does he reach? He reaches room (7, 8). Although his horizontal distance (abscissa) from the entrance increased from 0 to 7, his vertical distance (ordinate) did not, as he remained on the 8^{th} floor. He could have gone right up to the 10^{th} room, but he would still be 8 floors up. This is because his locus is described by the equation $y = 8$. Whatever the value of x, the value of y is 8.

> A linear graph that is parallel to one axis is perpendicular to the other axis.

In Figure 24.5, $x = 9$ is parallel to the Y axis and perpendicular to the X axis.

Similarly, $y = -5$ is parallel to the X axis and perpendicular to the Y axis (Fig. 24.6).

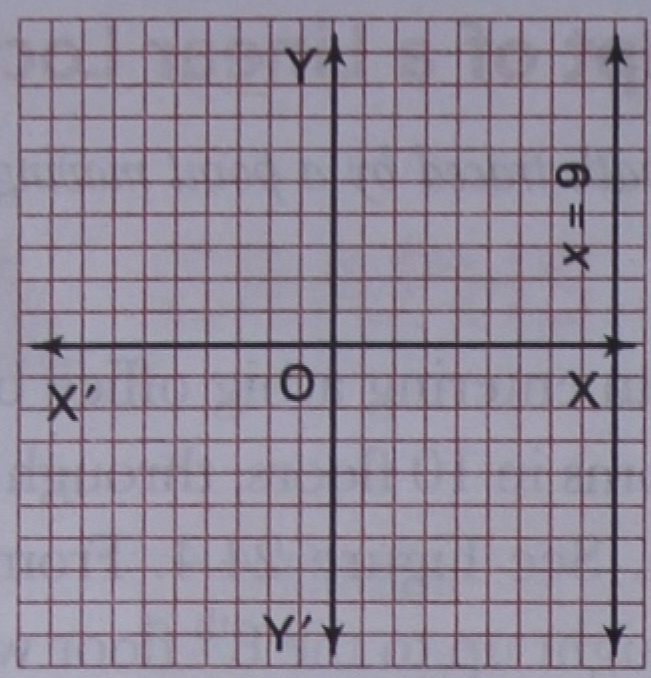

Fig. 25.5

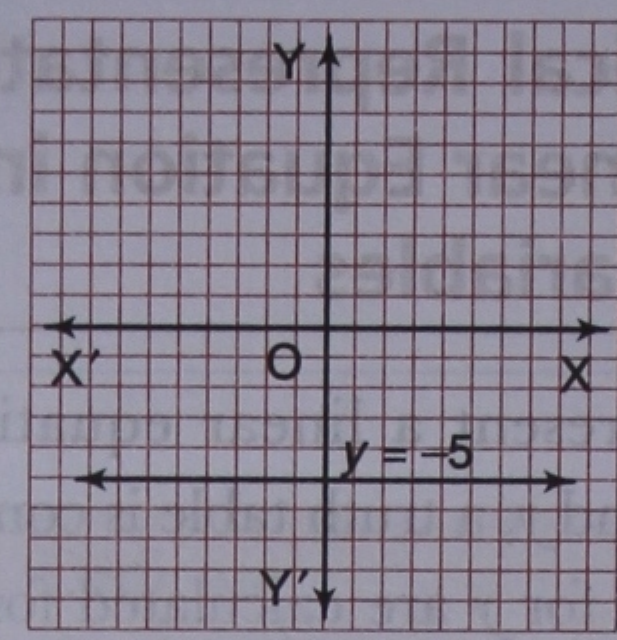

Fig. 25.6

Exercise 24.1

1. In which quadrants will the following points lie?
   (i) A (–5, 7) (ii) B (6, –8)
   (iii) C (7, 15) (iv) D (–10, –10)
   (v) E (–10, 10) (vi) F $\left(\frac{1}{2}, 2\right)$
   (vii) G $\left(3, -\frac{1}{3}\right)$ (viii) H $\left(-3\frac{5}{7}, -4\right)$
   (ix) I $\left(-4\frac{1}{4}, 5\right)$ (x) J $\left(\frac{2}{7}, -\frac{6}{11}\right)$
2. Plot the following points on a graph paper.
   (i) N (3, –6) (ii) P (5, 13)
   (iii) Q (–11, 6) (iv) R (–7, –9)
   (v) S (–5, 1) (vi) T (0, 7)
   (vii) U (–3, 0) (viii) V (9, 0)
   (ix) W (0, –11) (x) X (12, 12)
3. Plot four points A, B, C, and D on a graph paper such that the distance between A and B, B and C, C and D, D and A is 20 cm each and the lines connecting A with C and B with D intersect at the origin (0, 0).
4. Draw the graphs for the following equations. (***Note:*** These are equations involving only one variable.)
   (i) $x = 7$ (ii) $y = 9$
   (iii) $x + 3 = 0$ (iv) $y + 6 = 0$
   (v) $2x + 3 = 13$ (vi) $3y - 17 = 10$
5. Complete the following truth tables and draw the graphs.

(i)

t	1	2	3
$d = st$ where $s = 2$ m/s	1×2		
d	2		

(ii)

C.P.	10	20	30
S.P. = $\frac{110\text{ C.P.}}{100}$	$\frac{110 \times 10}{100}$		
S.P.	11		

(iii)

S.P.	5	10	15
C.P. = $\frac{100\text{ S.P.}}{125}$	$\frac{100 \times 5}{125}$		
C.P.	4		

(iv)

F	32	41	50
$C = \frac{5}{9}(F - 32)$	$\frac{5}{9}(32 - 32)$		
C	0		

6. Draw the graphs for the following equations.
   (i) $x = y$ (ii) $x = -4y$
   (iii) $3x = y$ (iv) $x = \frac{y}{5}$
   (v) $x = y + 6$ (vi) $x = y - 4$
   (vii) $x + y = 15$ (viii) $x - y = 6$
   (ix) $2x + y = 20$ (x) $3x - y = 1$
   (xi) $x + 3y = 27$ (xii) $x - 4y = 16$
   (xiii) $15x + 5y = 45$ (xiv) $8x - 2y = 6$

Graphical Solution of Simultaneous Equations

All points that lie on the graph of a linear equation satisfy the equation. All points that lie on the graph of another linear equation satisfy that equation. But there can be *only one point*, the coordinates of which satisfy both equations simultaneously. That is the point of intersection of the two linear graphs. Thus, *the coordinates of the point of intersection is the solution of a pair of simultaneous equations.*

Given a pair of simultaneous equations, the solution can be found graphically by following these steps:

Steps 1: Draw the graphs for both equations on the same graph paper.

Steps 2: If necessary, extend the linear graphs to intersect at a point.

Steps 3: Drop perpendiculars from the point of intersection on the X and the Y axes.

Steps 4: Read the coordinates of the point of intersection, which is the solution of the simultaneous equations.

Example 3: Solve $3x + y = 14;\ 7x - 3y = 6$ graphically.

$$3x + y = 14$$

$$\Rightarrow \quad y = 14 - 3x$$

Truth table

x	1	2	3
y	11	8	5

$$7x - 3y = 6$$

$$\Rightarrow \quad -3y = 6 - 7x$$

$$\Rightarrow \quad y = \frac{7x}{3} - 2$$

Truth table

x	3	6	9
y	5	12	19

Observe how some thought has gone behind choosing the values of x in order to calculate the corresponding values of y. In the first equation, x could be any natural number as then y will be a positive or negative integer. However, the values of x have been kept low so as to have the linear graph as close to the origin as possible. In the second equation, x has been assigned values which are multiples of 3 so as to obtain an integral value for y.

Thus, the solution to $3x + y = 14$;
$7x - 3y = 6$ is $(3, 5)$ or $x = 3$ and $y = 5$

(See Figure 24.7)

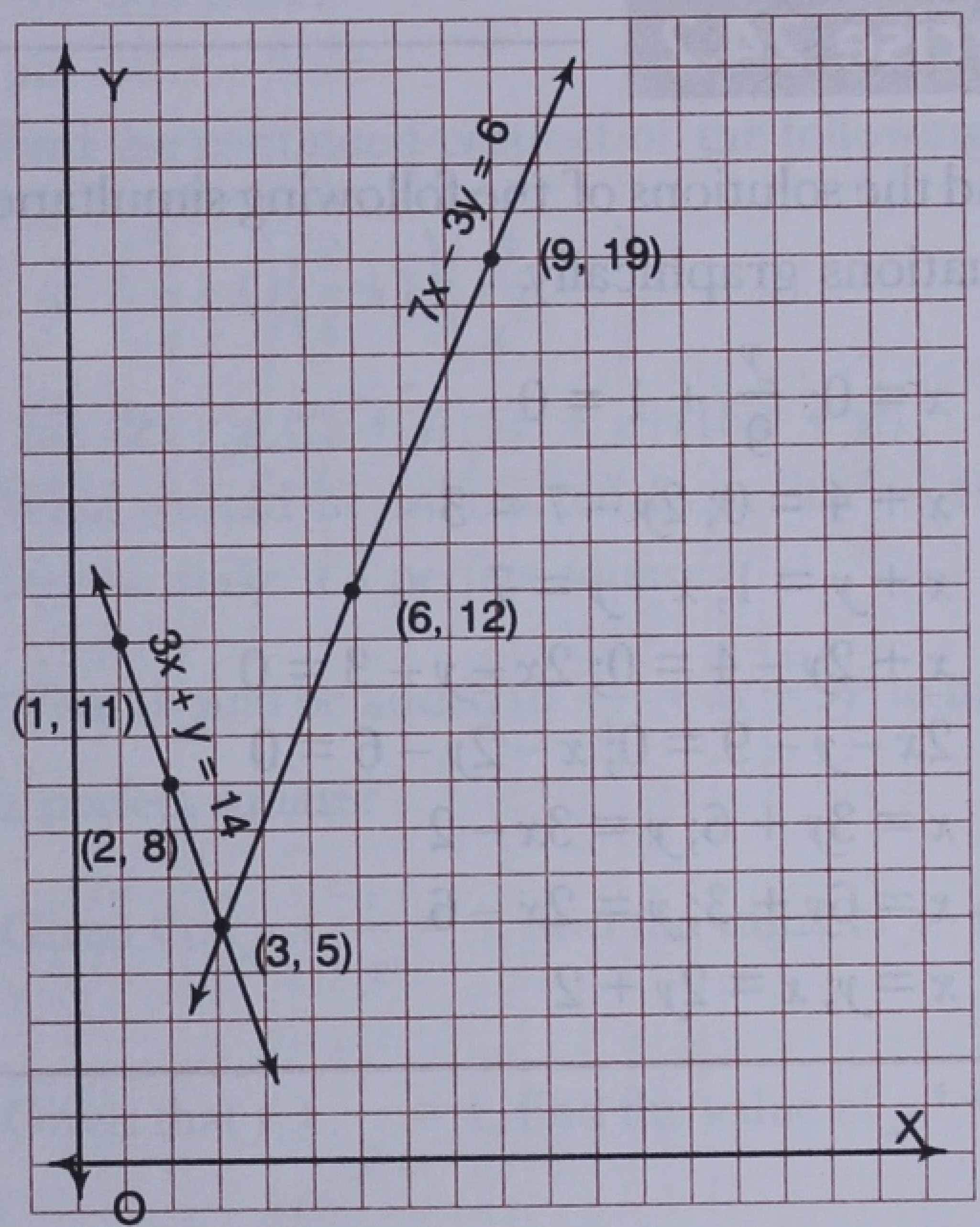

Fig. 24.7

Example 4: Solve $3x + y = 9;\ x + 2y = 8$ graphically.

$$3x + y = 9$$

$$\Rightarrow \quad y = 9 - 3x$$

Truth table

x	−1	0	+1
y	12	9	6

$$x + 2y = 8$$

$$\Rightarrow \quad 2y = 8 - x$$

$$\Rightarrow \quad y = 4 - \frac{x}{2}$$

Truth table

x	–2	0	+2
y	5	4	3

The linear graph (Fig. 25.8) for $3x + y = 9$ has been extended to interesect the linear graph for $x + 2y = 8$ at point (2, 3).

Thus, the solution to $3x + y = 9$; $x + 2y = 8$ is (2, 3) or $x = 2$ and $y = 3$.

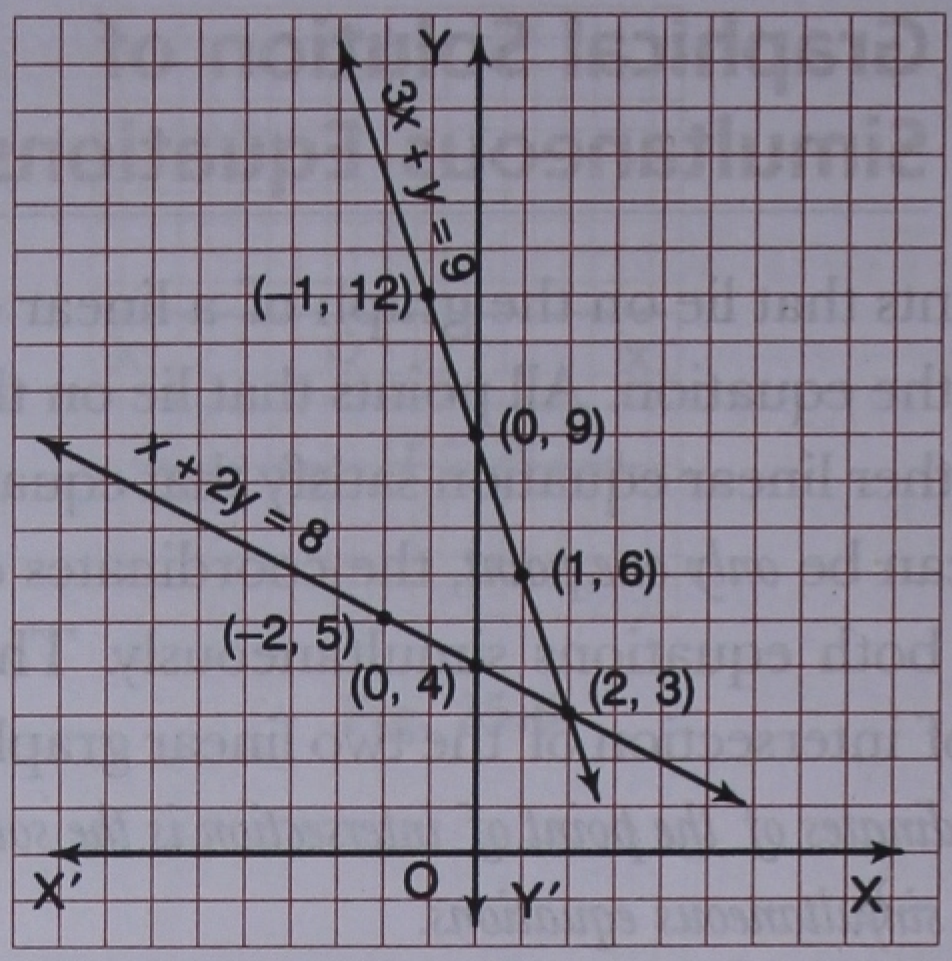

Fig. 24.8

Exercise 24.2

Find the solutions of the following simultaneous equations graphically.

(i) $x = 0;\ \frac{y}{9} + 1 = 0$

(ii) $x + 4 = 0;\ 2y - 7 = 3$

(iii) $x + y = 1;\ x - y = 7$

(iv) $x + 2y - 4 = 0;\ 2x - y - 3 = 0$

(v) $2x - y - 9 = 0;\ x - 2y - 6 = 0$

(vi) $x = 3y + 6;\ y = 3x - 2$

(vii) $x = 6y + 3;\ y = 2x - 6$

(viii) $x = y;\ x = 2y + 2$

(ix) $3x - 6y = 3;\ x + y + 5 = 0$

(x) $5x + 3y = 12;\ 7x - 3y = 24$

(xi) $x = 4$ and $2x - 3y + 1 = 0$

(xii) $2x - 3y = -6$ and $x - \frac{y}{2} = 1$

(xiii) $3x - 2y = 0$ and $y + 3 = 0$

(xiv) $4x + 3y = 1$ and $2x - y = 3$

(xv) $x + y = 7$ and $x - y = 3$

(xvi) $\frac{x}{2} - \frac{y}{3} = 3$ and $x + y = 1$

Revision Exercise

1. In which quadrants will the following points lie ?
   (i) A(–8, 5) (ii) B(7, –2)
   (iii) $C\left(-2\frac{4}{9}, -1\right)$ (iv) D(2, 10)
2. Draw the graphs for the following equations.
   (i) $x + 4 = 0$ (ii) $x = y + 1$
   (iii) $2x - 2y = 4$ (iv) $3x + y = 12$.
3. Find the solutions of the following simultaneus equations graphically.
   (i) $x + 2 = 0;\ 3y - 5 = 1$
   (ii) $2x + 3y = 6;\ 3x - 2y = 3$
   (iii) $x + 5y = 5;\ 4x + 3y = 6$
   (iv) $x - y = 3;\ x + y = 5$