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23 RELATIONS AND MAPPINGS

- Relations
- Mappings

Relations

There are 7 articles in a shop's godown which had been bought for Rs {90, 100, 110, 120, 130, 140, 150} in that order. Let us call this Set C_P.

There are 7 articles on display in the showroom, the selling prices of which are Rs (95, 105, 115, 125, 135, 145, 155} respectively. Let us call this Set S_P.

If it is known that all articles in the shop are sold at a profit of Rs 15, we can easily pair off 6 articles from C_P and S_P that match as:

Set R = {(90, 105), (100, 115), (110, 125), (120, 135), (130, 145), (140, 155)}. Notice that one article in the godown that cost Rs 150 is not displayed and one article on display that is selling for Rs 95 is not in the godown. The set of **ordered pairs** R is such that there exists, between each **first component** and **second component**, a relation from C_P to S_P such that S = C + 15.

Similarly, in the set of ordered pairs R_1 = {(105, 90), (115, 100), (125, 110), (135, 120), (145, 130), (155, 140)} there exists a relation from S_P to C_P, such that C = S – 15.

The set of first components in a relation is known as its **domain**.

Domain of R = {90, 100, 110, 120, 130, 140},
Domain of R_1 = {105, 115, 125, 135, 145, 155}.

The set of second components in a relation is known as its **range**.

Range of R = {105, 115, 125, 135, 145, 155}
Range of R_1 = {90, 100, 110, 120, 130, 140}.
Although {95, 150} is an ordered pair, it does not belong to either R or R_1.

Example1: Given:

A = {–25, –15, –5, 0, 5, 15, 25}
B = {–30, –20, –10, 0, 10, 20, 30}

(i) Write relation R_1 from A to B describing 'is more than'.
We look at each term in A and make ordered pairs with a term in B that is less in value.

R_1 = {(–25, –30), (–15, –30), (–15, –20), (–5, –30), (–5, –20), (–5, –10), (0, –30), (0, –20), (0, –10), (5,–30), (5, –20), (5, –10), (5, 0), (15, –30), (15, –20), (15, –10), (15, 0), (15, 10), (25, –30), (25, –20), (25, –10), (25, 0), (25, 10), (25, 20)}

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(ii) Write relation R_2 from A to B describing 'is 5 more than.'

$R_2 = \{(-25, -30), (-15, -20), (-5, -10), (5, 0), (15, 10), (25, 20)\}$

(iii) Write relation R_3 from B to A describing 'is half of its sum with 5' and write its domain and range.

The first components of R_3 will come from Set B as the relation from B to A is $\frac{a+5}{2}$.

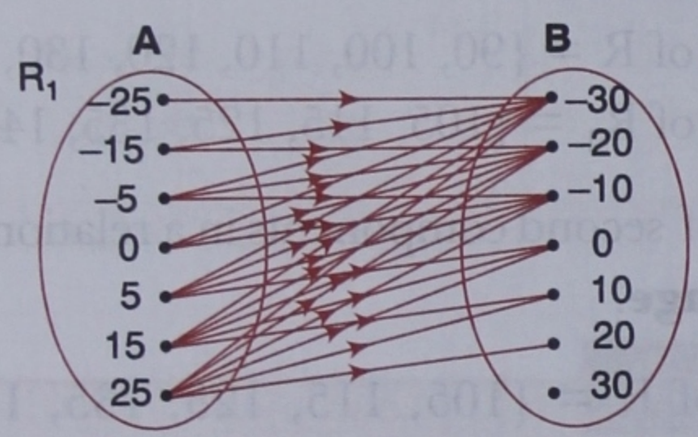
($R_3 = \{(-10, -25), (0, -5), (10, 15)\}$)

where domain = $\{-10, 0, 10\}$

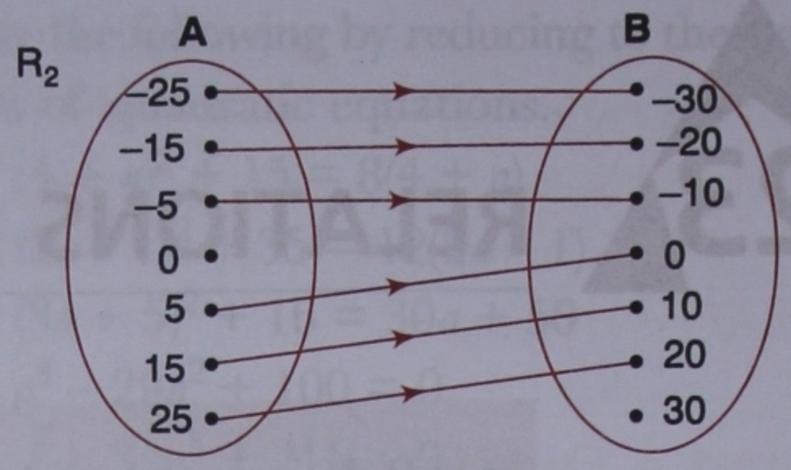
and range = $\{-25, -5, 15\}$

The relation between different sets is graphically represented by arrow diagrams. The three relations in the above example are described by the following arrow diagrams.

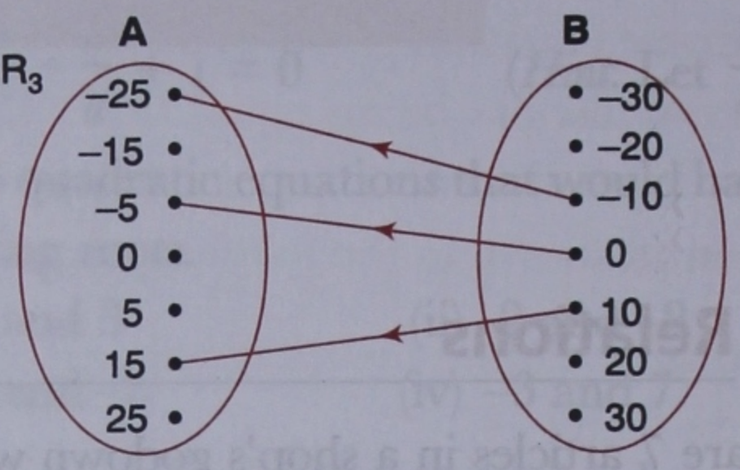
(i)



(ii)



(iii)



Notice that the arrows in R_3 point in the opposite direction as the relation is from B to A.

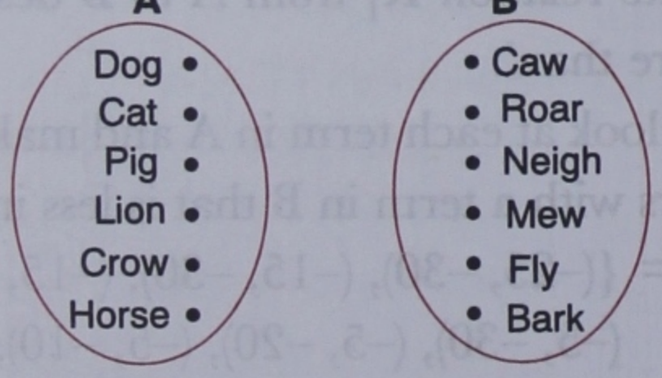
Try this!

Given: A = {2, 3, 4, 5} and B = {4, 8, 10, 12, 25, 40} write a relation R_1 from B to A describing 'is divisible by'.

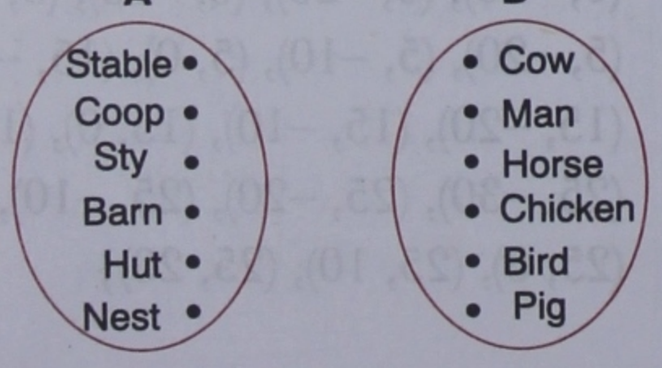
Exercise 23.1

1. Figure out a suitable relation between sets A and B in each of the following and draw arrows from A to B.

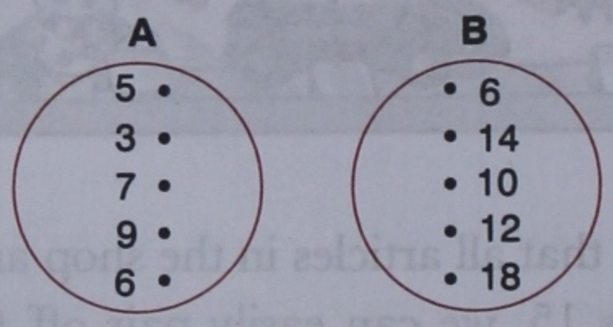
(i)



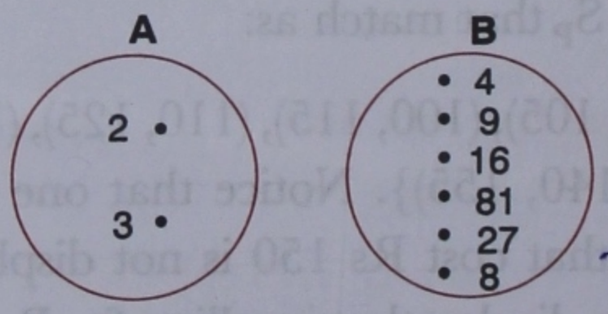
(ii)



(iii)



(iv)



2. Given A = {6, 8, 10, 12, 14} and B = {5, 7, 9, 11, 13}, does $R_1 = \{(5, 8), (7, 10), (9, 12), (11, 14)\}$ describe a relation from A to B or B to A? Describe the relation.

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3. Use the following ordered pairs to write the relation as indicated: (6, 18), (10, 2), (20, 4), (35, 7), (36, 9), (9, 81), (7, 21), (3, 9), (7, 49)
   (i) R_1 describing 'is a multiple of'
   (ii) R_2 describing 'is a factor of'
   (iii) R_3 describing 'is one-third of'
   (iv) R_4 describing 'is the square root of'
   (v) R_5 describing 'is five times'

4. Given A = {11, 9, 0, 25, 5} and B = {5, 2, –3, $-\frac{1}{2}$, $1\frac{4}{5}$}, write the following relations in Roster form and represent them by arrow diagrams.
   (i) R_1 describing $a = 5b$
   (ii) R_2 describing $a = 2b + 6$
   (iii) R_3 describing $a = 2b + 1$
   (iv) R_4 describing $a = 3b - 1$
   (v) R_5 describing $a = b^2$

Mapping

If every element in Set A is related to one and only one element in Set B, the relation from A to B is called a mapping.

Remember
For a relation from A to B to be a mapping,
- the domain of the relation must be equal to Set A.
- no two ordered pairs in the relations should have the same first component.

Thus, the relation between the cost price and selling price of the articles in the shop at the beginning of this chapter is not a mapping as the article costing Rs 150 was not related to any article displayed in the showroom.

Example 2: Given A = {2, 3, 4, 5} and B = {2, 3, 4, 5, 6, 7, 8, 9, 10}, which of the following relations from A to B are mappings?

(i) $R_1 = \{(3, 3), (4, 4), (5, 5)\}$
The element 2 is not present in the domain of R_1. Thus R_1 is not a mapping.

(ii) $R_2 = \{(2, 4), (3, 6), (4, 8), (5, 10)\}$
The domain of R_2 is equal to Set A and no two ordered pairs have the same first component. Thus R_2 is a mapping.

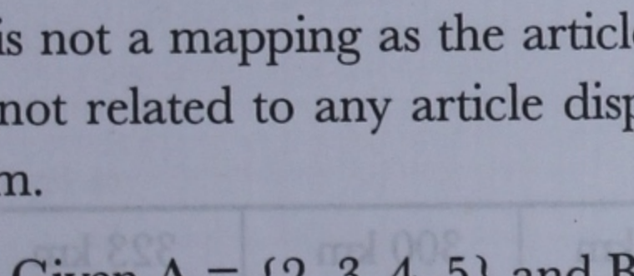
(iii) $R_3 = \{(2, 2), (2, 4), (2, 6), (2, 8), (2, 10), (3, 3), (3, 6), (3, 9), (4, 4), (4, 8), (5, 5), (5, 10)\}$
The domain of R_3 is equal to set A but the elements in the domain do not have unique second components. Also more than two ordered pairs have the same first components. Thus R_3 is not a mapping.

(iv) $R_4 = \{(2, 5) (3, 5), (4, 9), (5, 9)\}$
The domain of R_4 is equal to Set A and although the second components are repeated, no two ordered pairs have the same first component. Thus R_4 is a mapping.

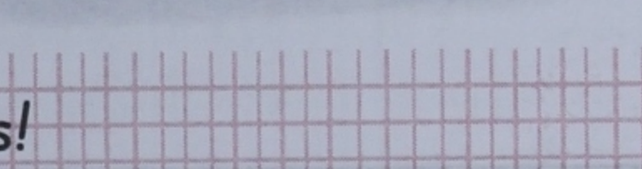
Try this!
Given: $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4, 5\}$
where $R = \{(1,1), (2, 2), (3, 3)\}$.
is R mapping from A to B?

Exercise 23.2

1. Which of the following arrow diagrams represent mappings?
   (i)



   (ii)



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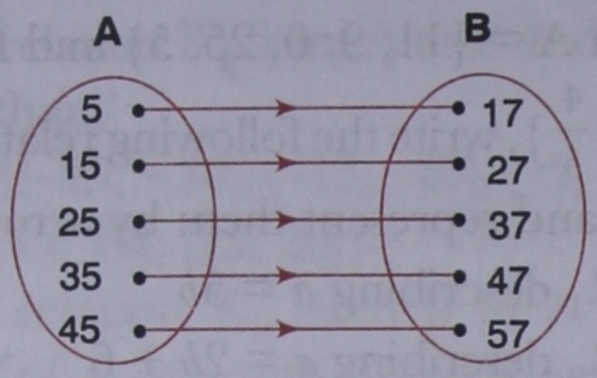
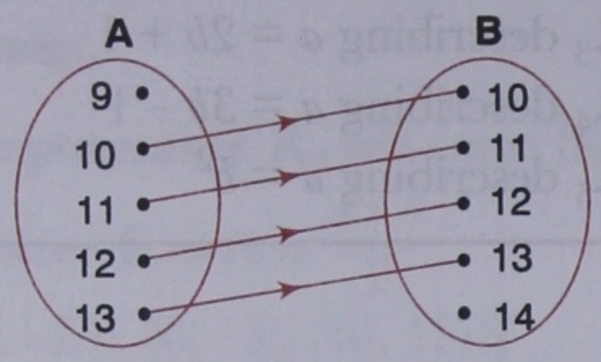
(iii)



(iv)



2. Given Set A = {a, b, c, d, e} and
Set B = {22, 23, 24, 25, 26},
   (i) write a relation R_1 from A to B that is not a mapping.
   (ii) write a relation R_2 from B to A that is not a mapping.
   (iii) write a relation R_3 from A to B that is a mapping.
   (iv) write a relation R_4 from B to A that is a mapping.

3. The distance covered by a bus driver over a week is given in Table 23.1. Is the relation between his average speed and the distance covered each day a mapping? Explain why. Write the relation in Roster form and draw an arrow diagram.

4. A man invested different amounts of money in various deposit schemes. Over a period of 1 year, the interest earned on them was as follows:

Principal	3500	4250	2750	3750	3250
Interest	343	391	308	360	338

Is the relation between the rates of interest on the various deposit schemes and the principal amounts invested in them a mapping? Explain why. Write the relation in Roster form and draw an arrow diagram.

Table 23.1

Distance covered	252 km	266 km	258 km	286 km	300 km	323 km	301 km
Time taken	6 hours	7 hours	6 hours	6.5 hours	7.5 hours	8.5 hours	7 hours

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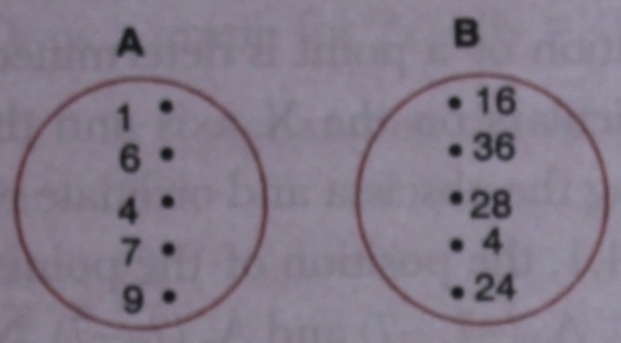
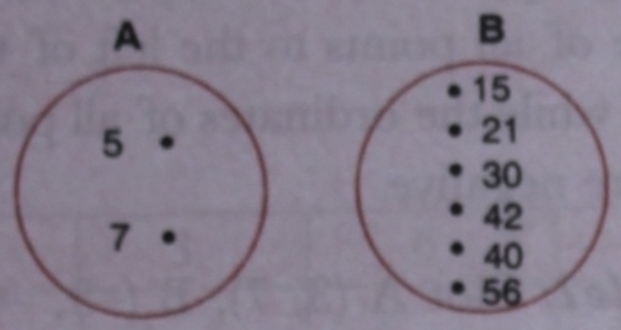
Revision Exercise

1. Given
   A = {–27, –17, –7, 0, 7, 17, 27}
   B = {–35, –25, –15, –5, 0, 5, 15, 25, 35}
   (i) Write relation R_1 from A to B describing "is more than".
   (ii) Write relation R_2 from A to B describing "is 5 more than".

2. Figure out a suitable relation between sets A and B in the following and draw arrows from A to B.
   (i)



   (ii)



3. Use the following ordered pairs to write the relation as indicated: (4, 12), (15, 3) (25, 5) (30, 6) (28, 7) (8, 64), (4, 12) (2, 6), (6, 36)
   (i) R_1 describing "is a multiple of"
   (ii) R_2 describing "is a factor of".
   (iii) R_3 describing "is the square root of".
   (iv) R_4 describing "is five times".

4. Given set A = {p, q, r, s, t} and
   set B = {8, 10, 12, 14, 16},
   (i) Write a relation R_1 from A to B that is not a mapping.
   (ii) Write a relation R_2 from B to A that is not a mapping.
   (iii) Write a relation R_3 from A to B that is a mapping.
   (iv) Write a relation R_4 relation from B to A that is a mapping.

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