Downloaded from https:// www.studiestoday.com

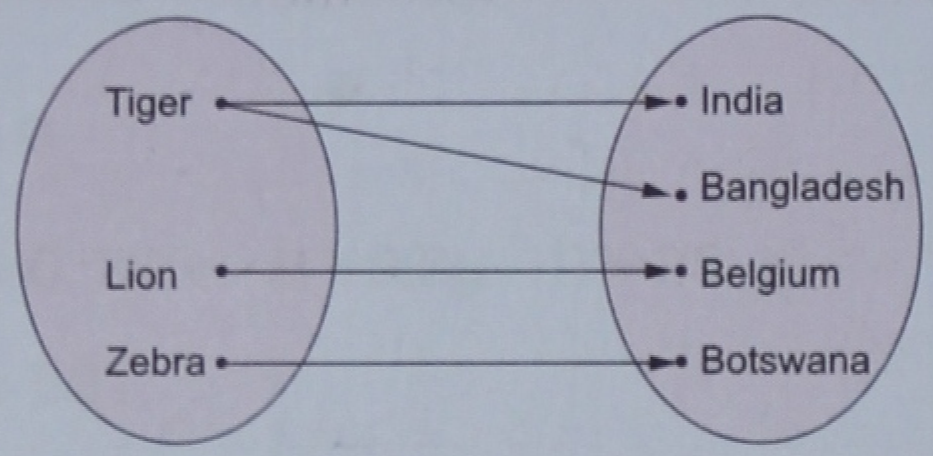
12

Relations and Mappings

Relations

Arrow diagrams

Let us consider the statement, "the tiger is the national animal of India and Bangladesh, the lion that of Belgium, and the Zebra that of Botswana". We can draw the following arrow diagram to represent it.



The animals (the elements of the first set) and the countries (the elements of the second set) are linked by a relation "is the national animal of". The arrows are drawn between the ordered pairs, or matching pairs, of elements that stisfy the relations.

A relation is a link between the elements of two sets based on some common property. In your previous class, you learnt two ways of representing relations—by making arrow diagrams and by expressing in the roster form.

Roster form

Let us call the set of animals A and the set of countries B. To express the relation between the two sets in the roster form, we write the matching, or ordered pairs (Tiger, India), (Tiger, Bangladesh), (Lion, Belgium), (Zebra, Botswana). Then the pair (Tiger, India) means "the tiger is the national animal of India". Thus, we get

$A = \{$Tiger, Lion, Zebra$\}$, $B = \{$India, Bangladesh, Belgium, Botswana$\}$ and

$R = \{$(Tiger, India), (Tiger, Bangladesh), (Lion, Belgium), (Zebra, Botswana)$\}$,

where R is the relation from the set A to the set B.

Notice that while writing the ordered pairs we have written the elements of the set A first. This is because the relation is from the set A to the set B.

> To express a relation from set A to set B in the roster form, write the elements of set A first. To express a relation from set B to set A, write the elements of set B first.

We could also write Tiger R India, Tiger R Bangladesh, Lion R Belgium, Zebra R Botswana. Here, R stands for "is the national animal of".

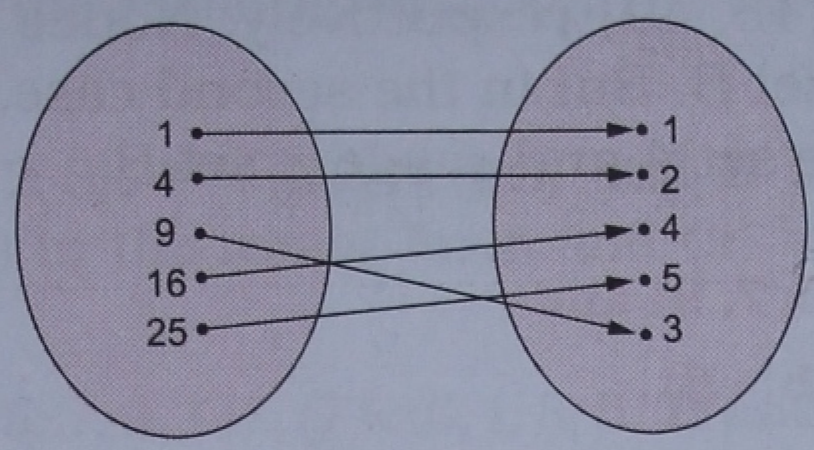
Thus $R = \{(a, b): a \in A, b \in B$ and a is the national animal of $b\}$.

Downloaded from https:// www.studiestoday.com

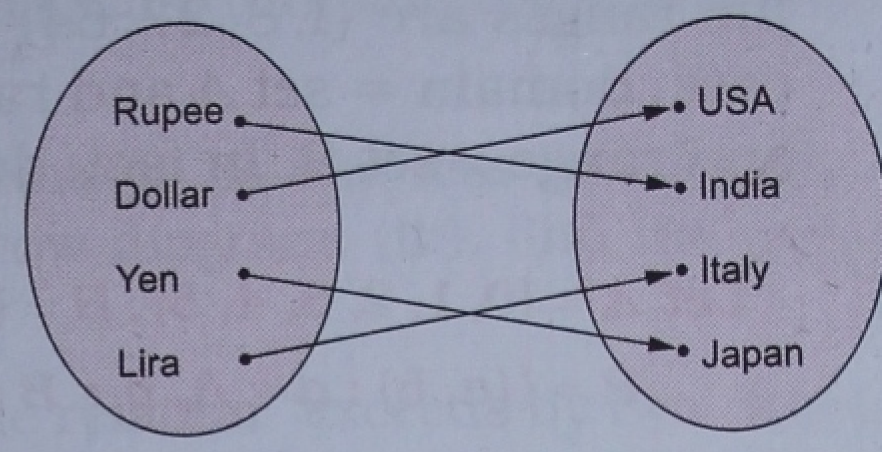
Downloaded from https:// www.studiestoday.com

EXAMPLE **Express the following relations in the roster form.**

(i)



(ii)



Solution (i) The matching pairs are (1, 1), (4, 2), (9, 3), (16, 4), (25, 5).

So, the relation $R = \{(1, 1), (4, 2), (9, 3), (16, 4), (25, 5)\}$ is from the set $A = \{1, 4, 9, 16, 25\}$ to the set $B = \{1, 2, 4, 5, 3)$. Also, in each pair the first element "is the square of" the second element.

We write: $1\,R\,1, 4\,R\,2, 9\,R\,3, 16\,R\,4, 25\,R\,5$

and $R = \{(a, b) : a \in A, b \in B \text{ and } a = b^2\}$.

(ii) The matching pairs are (Rupee, India), (Dollar, USA), (Yen, Japan) and (Lira, Italy).

So, the relation is R = {(Rupee, India), (Dollar, USA), (Yen, Japan), (Lira, Italy)}

from the set A = {Rupee, Dollar, Yen, Lira}

to the set B = {USA, India, Italy, Japan},

and it can be described as "is the currency of".

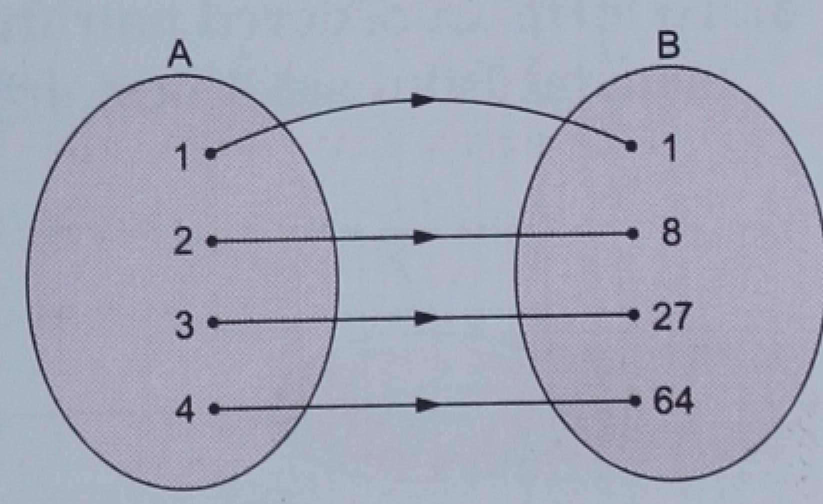
We can also write Rupee R India, Dollar R USA, Yen R Japan, Lira R Italy

and $R = \{(a, b) : a \in A, b \in B$ and a is the currency of $b\}$.

EXAMPLE **Let $A = \{1, 2, 3, 4\}$, $B = \{1, 8, 27, 64\}$ and R be the relation "is the cube root of" from the set A to set B. Represent R in the form of a arrow diagram and in the roster form.**

Solution Since $1 = \sqrt[3]{1}, 2 = \sqrt[3]{8}, 3 = \sqrt[3]{27}$ and $4 = \sqrt[3]{64}$ we write $1\,R\,1, 2\,R\,8, 3\,R\,27, 4\,R\,64$.

So, the ordered pairs are (1, 1), (2, 8), (3, 27) and (4, 64). The arrow diagram is shown along side.



In the roster form, R can be written as follows.

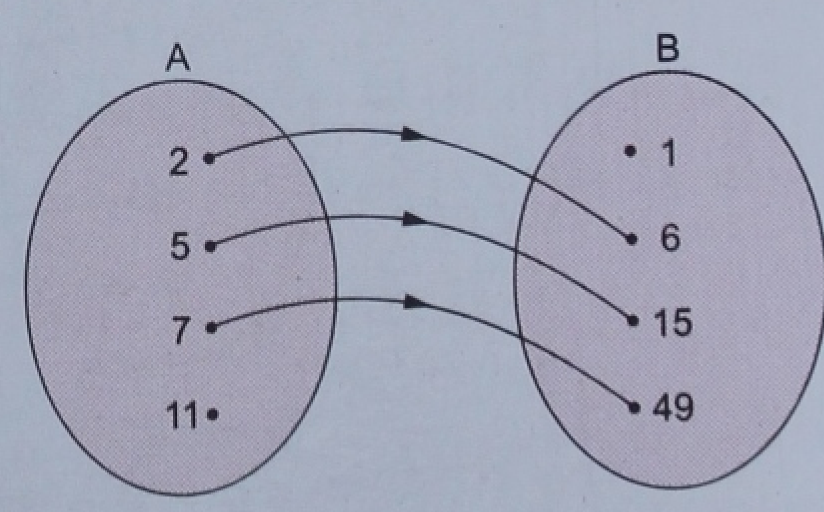
$R = \{(1, 1), (2, 8), (3, 27), (4, 64)\}$

i.e., $R = \{(a, b) : a \in A, b \in B \text{ and } a = \sqrt[3]{b}\}$.

EXAMPLE **Let $A = \{2, 5, 7, 11\}$, $B = \{1, 6, 15, 49\}$ and R be the relation "is a factor of" from set A to set B. Represent R with the help of an arrow diagram and in the roster form.**

Solution 2 is a factor of 6, 5 is a factor of 15, 7 is a factor of 49, i.e., $2\,R\,6, 5\,R\,15, 7\,R\,49$.

So, the ordered pairs are (2, 6), (5, 15) and (7, 49). The arrow diagram is shown alongside.



In roster form, R can be written as follows.

$R = \{(2, 6), (5, 15), (7, 49)\}$

i.e., $R = \{(a, b) : a \in A, b \in B$ and a is a factor of $b\}$.

Domain and range

The set of all the first elements (or components) of the ordered pairs of a relation is called the domain of the relation. The set of all the second elements (or components) of the ordered pairs of a relation is called the range of the relation.

Downloaded from https:// www.studiestoday.com

Downloaded from https:// www.studiestoday.com

In the preceding two examples the domains are {1, 2, 3, 4} and {2, 5, 7}, while the ranges are {1, 8, 27, 64} and {6, 15, 49} respectively. Notice that in the first case, domain = set A and range = set B. But in the second case, domain ≠ set A and range ≠ set B. In fact, domain ⊂ set A and range ⊂ set B.

EXAMPLE **Let $A = \{0, 1, 2, 3, 4, 5\}$, $B = \{1, 2, 3, 5, 7, 9\}$ and $R = \{(a, b) : a \in A, b \in B \text{ and } a + b = 8\}$.**

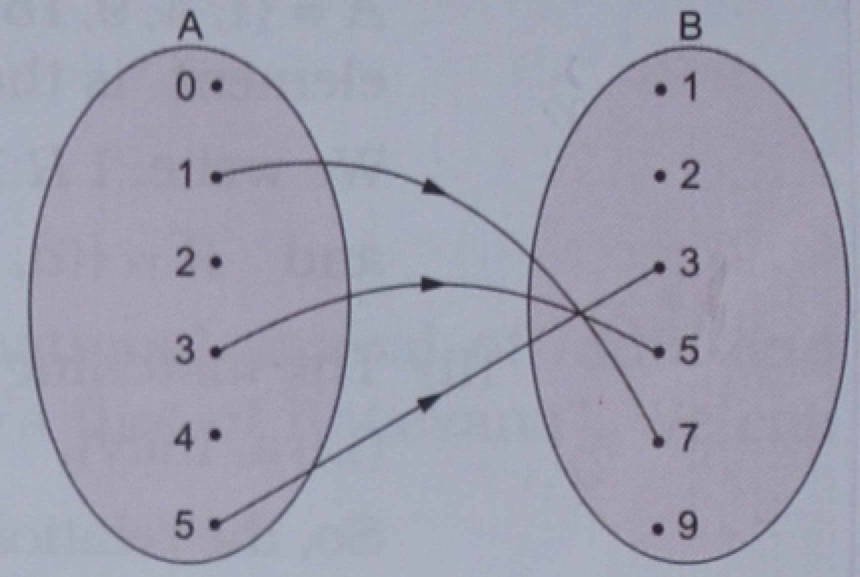
(i) Find R and represent it by an arrow diagram.

(ii) Find the domain and range of R.

Solution Clearly, $1 + 7 = 8$, $3 + 5 = 8$, $5 + 3 = 8$.
So, $(1, 7) \in R$, $(3, 5) \in R$ and $(5, 3) \in R$.

(i) $R = \{(1, 7), (3, 5)$ and $(5, 3)\}$.
The arrow diagram for R is shown alongside.

(ii) The domain of $R = \{(1, 3, 5\}$.
The range of $R = \{3, 5, 7\}$.

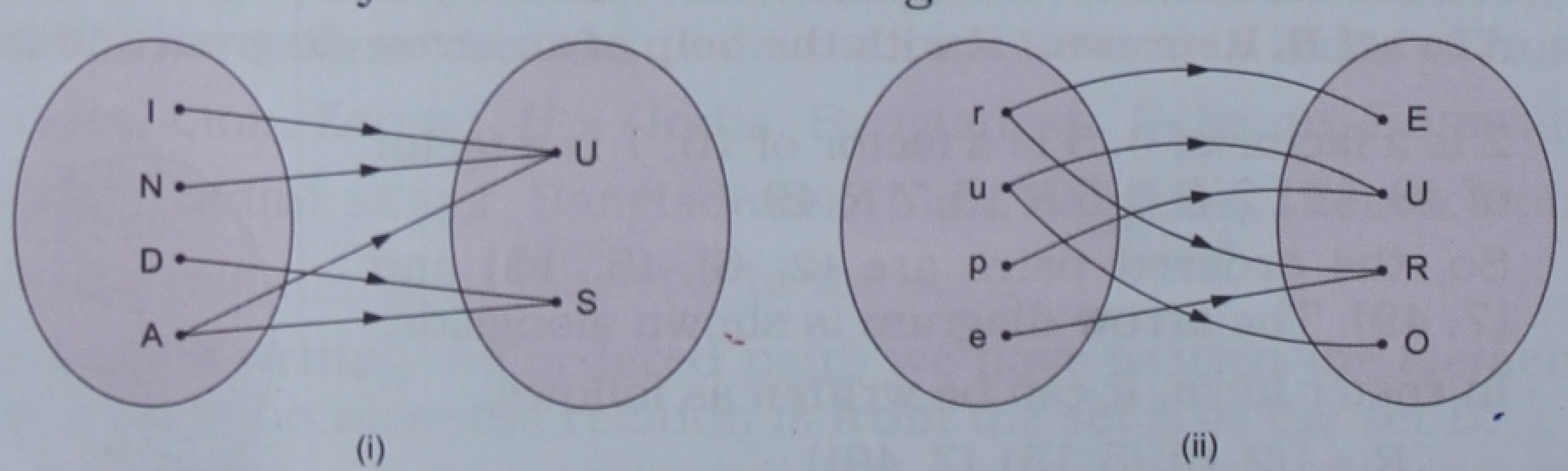


Remember These

1. A set R of matching pairs or ordered pairs (x, y) where $x \in A$, $y \in B$, is a relation from the set A to the set B.
Thus, $R = \{(x, y) : x \in A, y \in B\}$.
2. To represent an ordered pair (x, y) in an arrow diagram, draw an arrow from x to y.
4. To form an ordered pair from a set A to set B on the basis of an arrow diagram, write the element from set A first, followed by the element of the set B to which the arrow points.

EXERCISE 12A

1. Express the relation shown by each of the arrow diagrams in the roster form.



2. If $A = \{a, e, i, o, u\}$, $B = \{1, 3, 5, 7, 9\}$, which of the following are relations from A to B? Represent the relations by arrow diagrams.

(i) $R = \{(a, 1), (e, 5), (o, 7), (u, 9)\}$ (ii) $R = \{(a, 3), (a, 5), (e, 7), (o, 9), (u, 7)\}$

(iii) $R = \{(a, 7), (e, 9), (i, u)\}$

Downloaded from https:// www.studiestoday.com

Downloaded from https:// www.studiestoday.com

3. If $A = \{4, 6, 8, 12, 15\}$, $B = \{2, 3, 4, 5, 6, 7\}$ then express the relation R "is twice of" from set A to set B (i) in the roster form and (ii) by an arrow diagram. (iii), find the domain and range of R.

4. Let $X = \{0, 1, 2, 3\}$, $Y = \{1, 2, 3, 4, 5\}$ and R be the relation "is one less than" from X to Y. Express R (i) in the roster form and (ii) by an arrow diagram. (iii), find the domain and range of R.

5. Let $P = \{0, 2, 5, 7, 9, 11\}$, $Q = \{0, 1, 3, 5, 7\}$ and R be the relation "exceeds by two" from P to Q. Express R (i) in the roster form and (ii) by an arrow diagram. Also, find the domain and range of R.

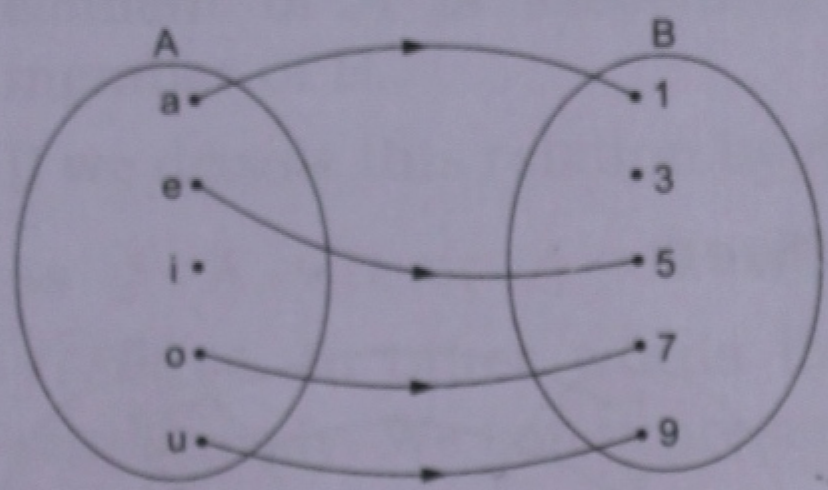
6. Let $A = \{-2, -1, 0, 1, 2\}$, $B = \{-2, 0, 2, 4\}$ and $R = \{(a, b) : a \in A, b \in B \text{ and } a \geq b\}$. Express R (i) in the roster form and (ii) by an arrow diagram. (iii) Find the domain and range of R.

7. Let $A = \{1, 4, 8, 9, 12\}$, $B = \{2, 7, 5, 9, 8\}$ and $R = \{(a, b) : a \in A, b \in B \text{ and } a + b \text{ is divisible by } 4\}$. Express R (i) in the roster form and (ii) by an arrow diagram. (iii) Find the domain and range of R.

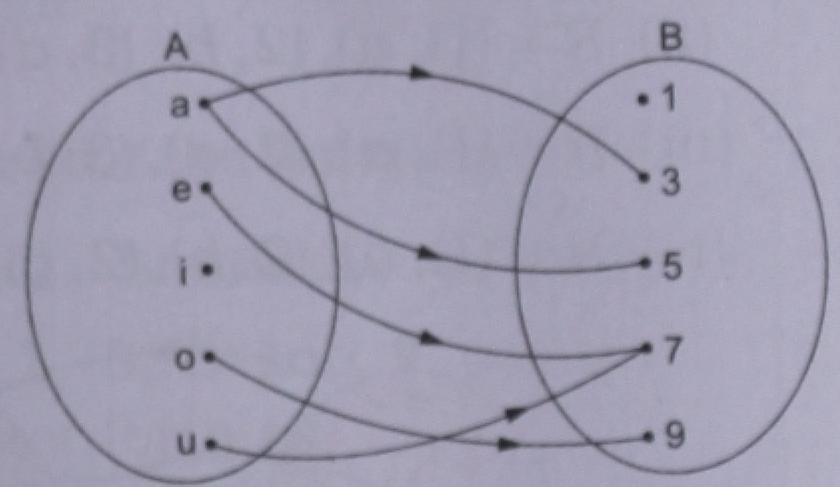
ANSWERS

1. (i) $\{(I, U), (N, U), (A, S), (D, S), (A, U)\}$ (ii) $\{(r, E), (r, R), (u, U), (u, O), (p, U), (e, R)\}$

2. (i) Yes



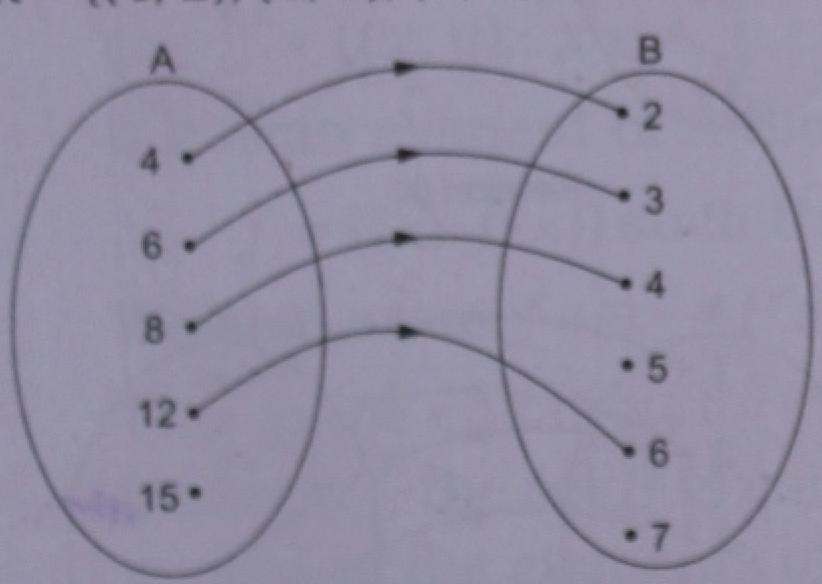
(ii) Yes



(iii) No

3. (i) $R = \{(4, 2), (6, 3), (8, 4), (12, 6)\}$

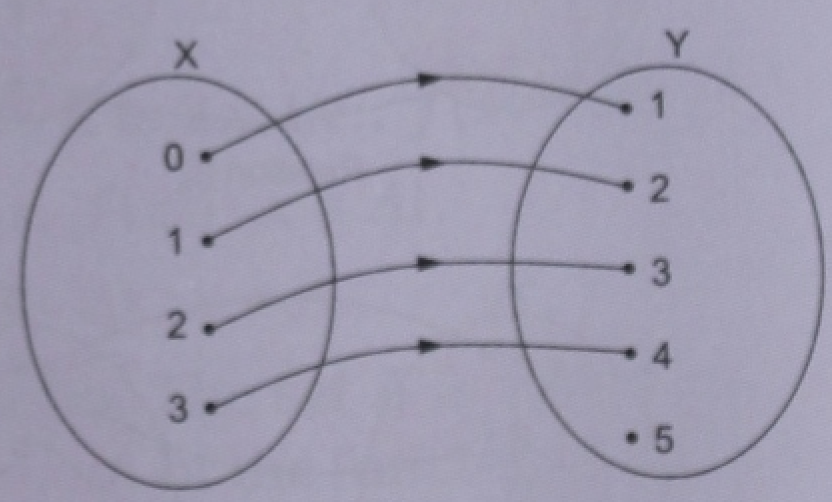
(ii)



(iii) Domain = $\{4, 6, 8, 12\}$, range = $\{2, 3, 4, 6\}$

4. (i) $R = \{(0, 1), (1, 2, (2, 3), (3, 4)\}$

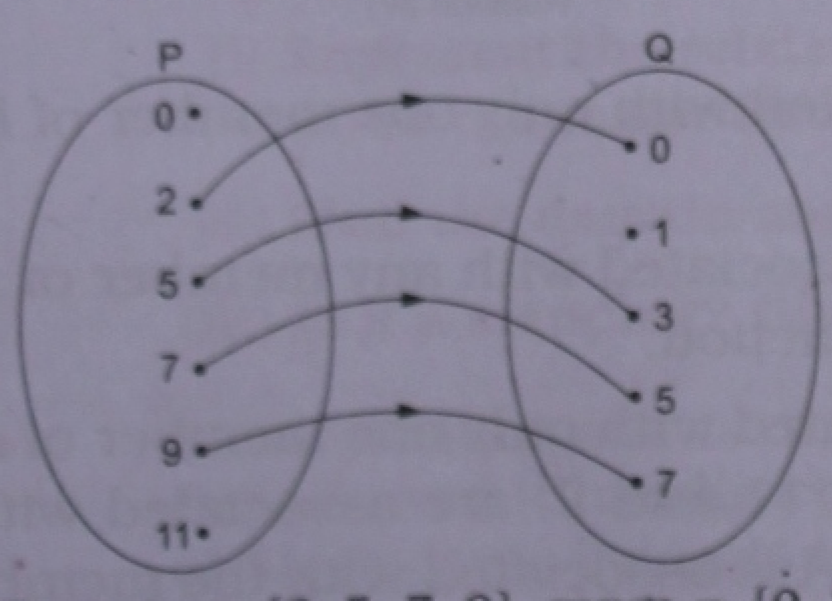
(ii)



(iii) Domain = $\{0, 1, 2, 3\}$, range = $\{1, 2, 3, 4\}$

5. (i) $R = \{(2, 0), (5, 3), (7, 5), (9, 7)\}$

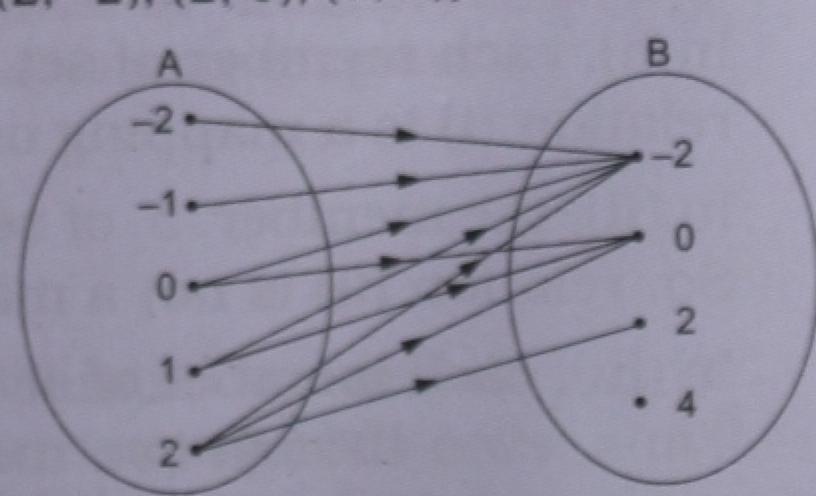
(ii)



Domain = $\{2, 5, 7, 9\}$, range = $\{0, 3, 5, 7\}$

6. (i) $R = \{(-2, -2), (-1, -2), (0, -2), (0, 0), (1, -2), (1, 0), (2, -2), (2, 0), (2, 2)\}$

(ii)

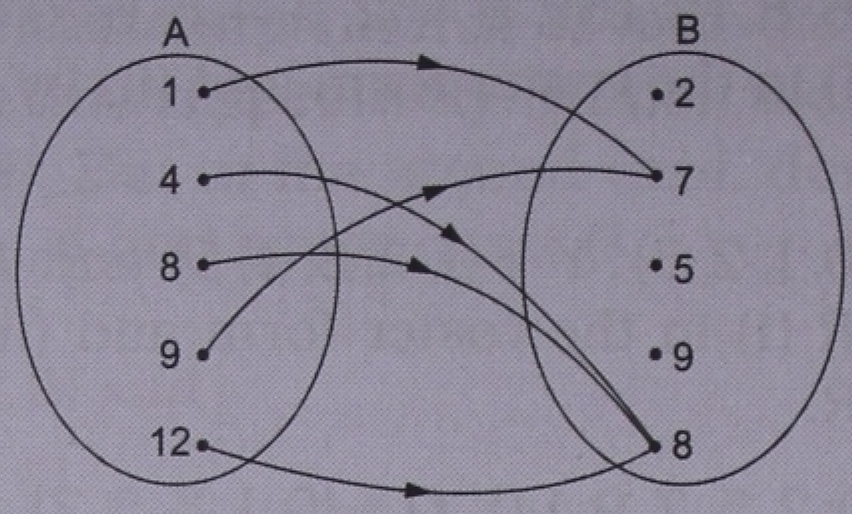


(iii) Domain = $\{-2, -1, 0, 1, 2\}$, range = $\{-2, 0, 2\}$

Downloaded from https:// www.studiestoday.com

Downloaded from https:// www.studiestoday.com

7. (i) $R = \{(1, 7), (4, 8), (8, 8), (9, 7), (12, 8)\}$ (ii)



(iii) Domain = $\{1, 4, 8, 9, 12\}$, range = $\{7, 8\}$

Mappings

Let A and B be two non-empty sets. Then a relation which associates each member of A with exactly one member of B is called a mapping from set A to set B. The unique element b of set B that is linked to an element a of set A is called the image of a in B. A mapping is also known as a function.

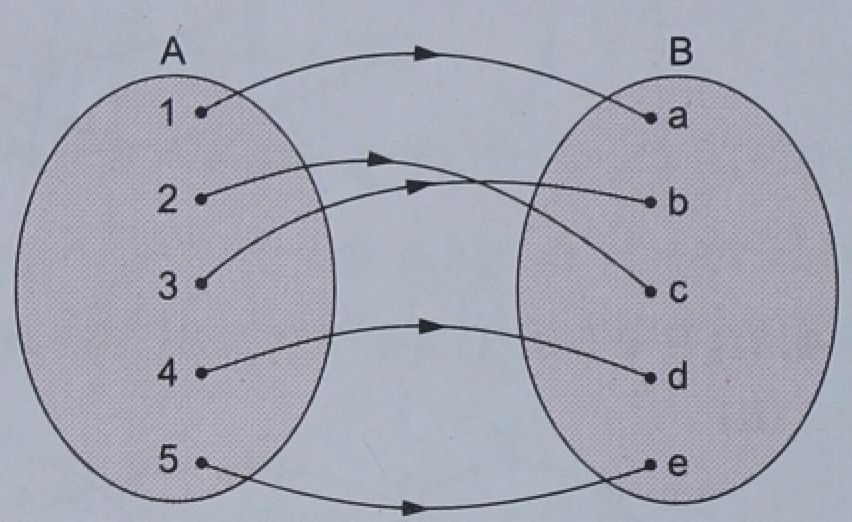
Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{a, b, c, d, e\}$. Consider the following relations from set A to set B.

(i) $R = \{(1, a), (2, c), (3, b), (4, d), (5, e)\}$

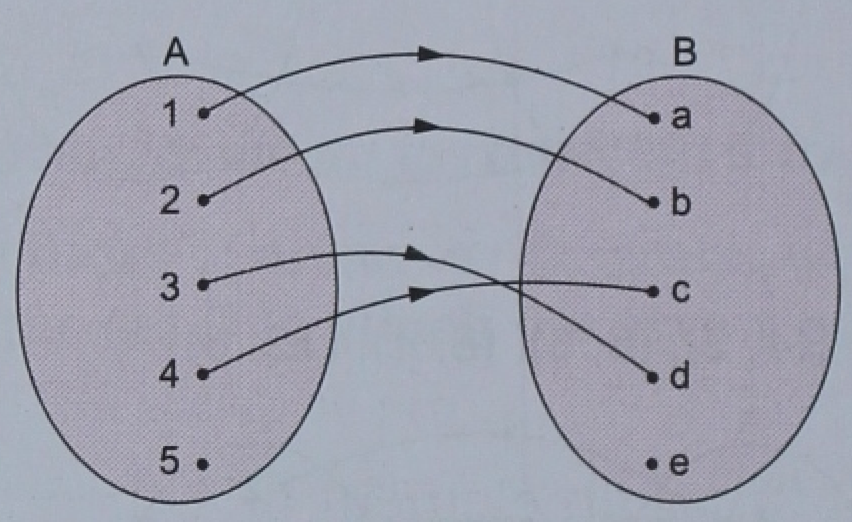
(ii) $R = \{(1, a), (2, b), (3, d), (4, c)\}$

(iii) $R = \{(1, a), (2, a), (3, b), (4, c), (5, d)\}$

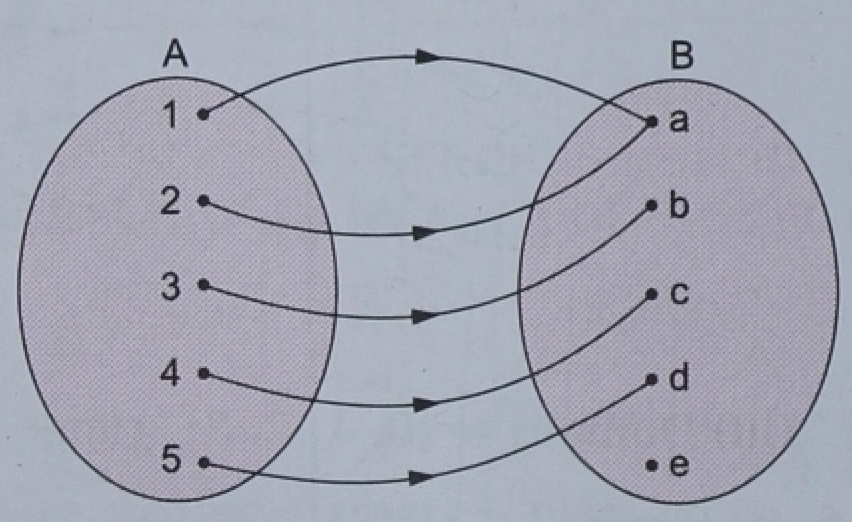
(iv) $R = \{(1, a), (2, b), (2, c), (3, d), (4, e), (5, e)\}$



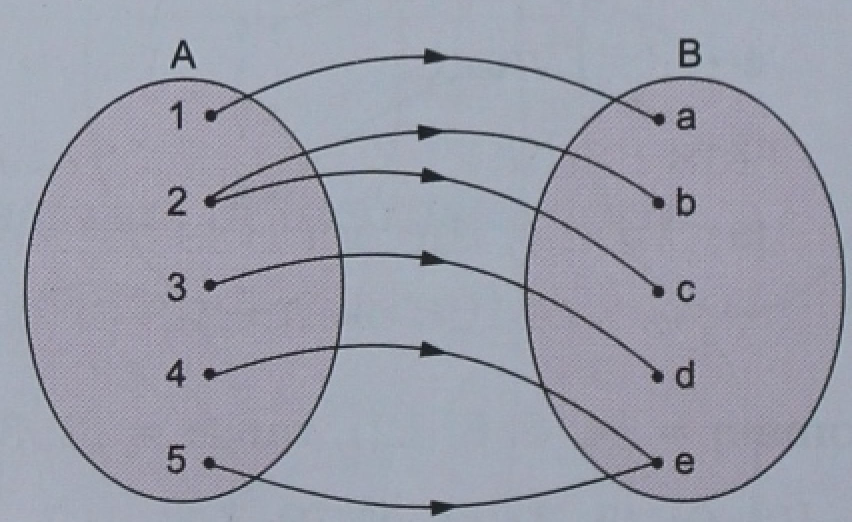
Relation (i)



Relation (ii)



Relation (iii)



Relation (iv)

In (i), each member of set A is associated with only one member of B. So, relation (i) is a mapping or function.

In (ii), the member '5' of set A is not associated with any member of set B. So, relation (ii) is not a mapping or function.

In (iii), each member of set A is associated with only one member of set B. Thus, even though two members of A, (1 and 2) are associated with the same member of B and no member of A is associated with the member 'e' of B, the relation (iii) is a mapping or function.

Downloaded from https:// www.studiestoday.com

Downloaded from https:// www.studiestoday.com

In (iv), each member of set A is associated with members of set B but the member '2' of A is associated with two members, 'b' and 'c', of set B. So, this relation is not a mapping or function.

All functons are relations but all relations are not functions.

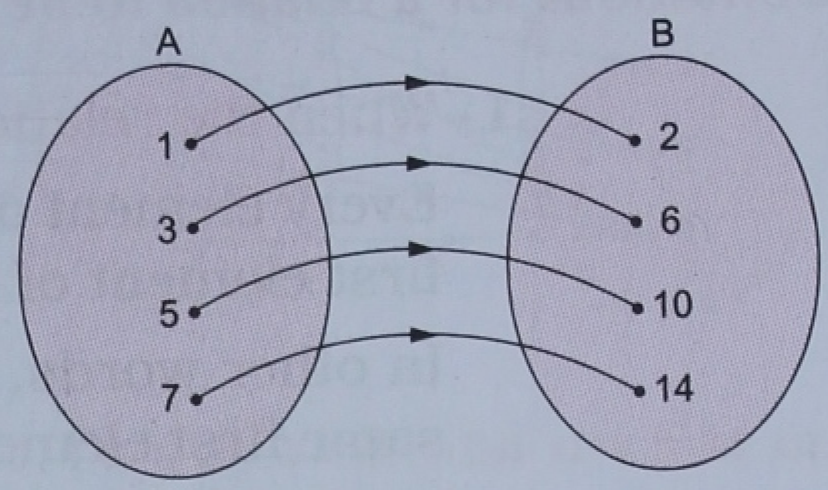
The domain of relation (i) = {1, 2, 3, 4, 5} = set A and the domain of relation (iii) = {1, 2, 3, 4, 5} = set A.

The domain of a mapping or function from set A to set B = set A.

The range of relation (i) = $\{a, b, c, d, e\}$ = set B, while the range of relation (iii) = $\{a, b, c, d\} \subset$ set B.

The set of images, i.e., the range of a mapping or function from set A to set B may be equal to set B or a subset of B.

Next, consider the relation "is half of" from set $A = \{1, 3, 5, 7\}$ to set $B = \{2, 6, 10, 14\}$, illustrated by the arrow diagram alongside.



It is clear that $1 R 2, 3 R 6, 5 R 10$ and $7 R 14$. The relation is a function because each element of A is associated with only one member of B.

If we denote this relation by f then we represent the function f from set A to set B as $f: A \to B$ or $A \xrightarrow{f} B$. We write $f(1) = 2$. Similarly, $f(3) = 6$, $f(5) = 10$ and $f(7) = 14$. In other words the images of 1, 3, 5 and 7 are 2, 6, 10 and 14 respectively. We can represent f in the roster form as

$f = \{(1, 2), (3, 6), (5, 10), (7, 14)\}$ where 1, 3, 5, 7 belong to set A and 2, 6, 10, 14 belong to set B.

or $f = \{(x, y) : x \in A, y \in B \text{ and } y = 2x\}$.

If x is an element of A then the image of x is $2x$, i.e., $f(x) = 2x$.

$\therefore \quad f = \{(x, f(x)) \text{ such that } x \in A\} = \{(x, f(x)) : x \in A\}$.

We also write $f: A \to B$ such that $f(x) = 2x, x \in A$.

This is the equation form of the functions. The set A is called the domain of the function f, the set B is called the co-domain of the function f and the set of all the images of the elements of A is called the range of the function f.

EXAMPLE **Let A = {Chandigarh, Shillong, Patna, Lucknow} and B = {Punjab, Haryana, Meghalaya, Bihar, Uttar Pradesh}.**

(i) Represent the relation "is the capital of" from set A to set B (a) in the roster form and (b) by an arrow diagram.

(ii) Find the domain and range of R.

(iii) Is R a mapping? Give reasons.

Solution Since Chandigarh is the capital of Punjab as well as Haryana, (Chandigarh, Punjab) and (Chandigarh, Haryana) are two matching pairs. Shillong is the capital of Meghalaya, Patna is the capital of Bihar and Lucknow is the capital of Uttar

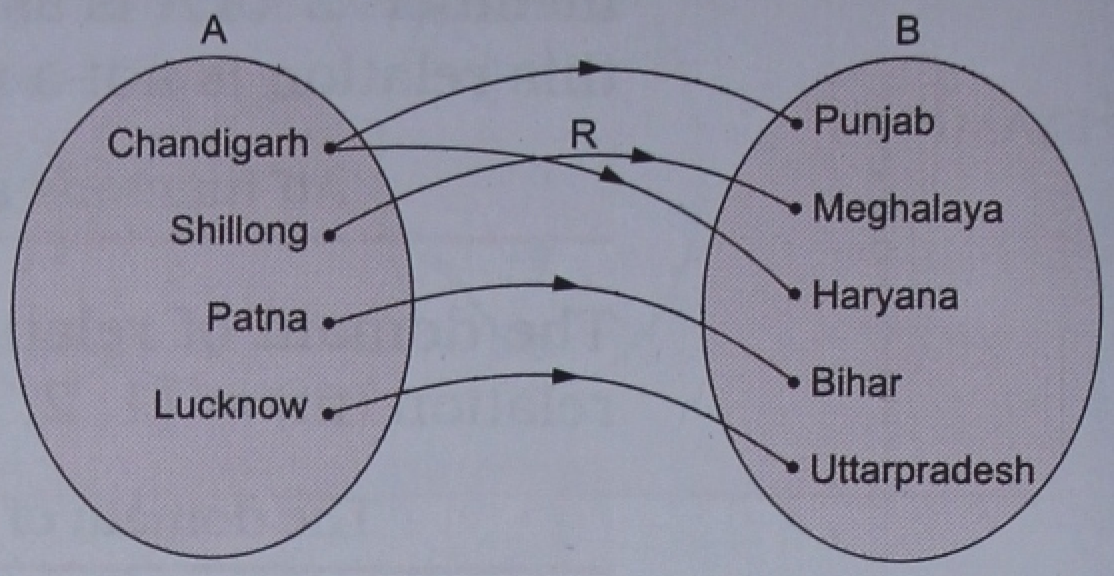
Downloaded from https:// www.studiestoday.com

Downloaded from https:// www.studiestoday.com

Pradesh. So, (Shillong, Meghalaya), (Patna, Bihar) and (Lucknow, Uttar Pradesh) are the other matching pairs.

(i) The relation R in the roster form is R = {(Chandigarh, Punjab), (Chandigarh, Haryana), (Shillong, Meghalaya), (Patna, Bihar), (Lucknow, Uttar Pradesh)}

The arrow diagram is shown alongside.



(ii) The domain of R = {Chandigarh, Shillong, Patna, Lucknow} and the range of R = {Punjab, Meghalaya, Haryana, Bihar, Uttar Pradesh}.

(iii) The relation R is not a mapping because the member "Chandigarh" of set A is matched with more than one member of set B.

Conditions for a relation to be a function

1. When the relation is expressed in the roster form

   Every element of A must appear as first element of an ordered pair. Also, the first element of the ordered pairs must be different.

   In other words, two or more ordered pairs of the relation must not have the same first element. However, the second element of the ordered pairs may be repeated.

2. When the relation is represented by an arrow diagram:

   Every element of A must have one and only one image in B. However, two or more elements of A may have the same image in B.

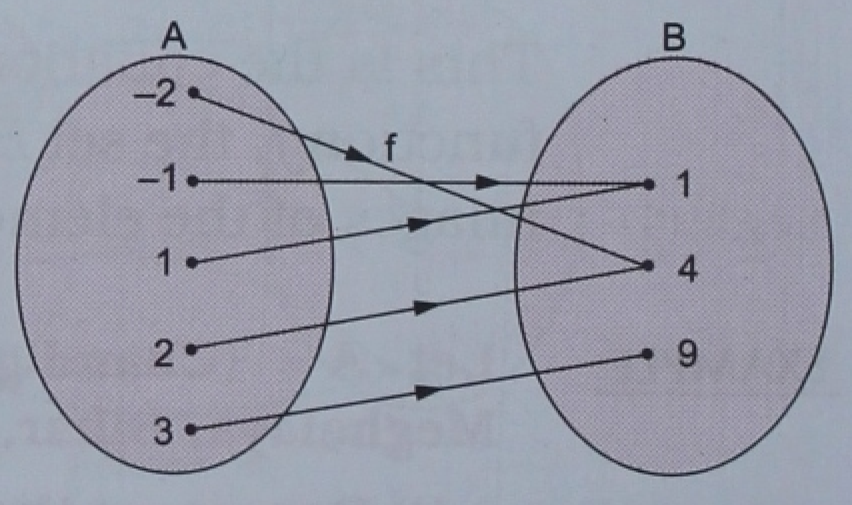
EXAMPLE **Determine whether the given relation can be a function.**

(i) $R = \{(5, 8), (6, 8), (7, 8)\}$ **(ii) $R = \{(7, 2), (0, 0), (7, -2)\}$**

Solution (i) The first elements of the ordered pairs of R are different. So, this relation can be a function.

(ii) This relation can not be a function because the number '7' is the first element in more than one ordered pair.

EXAMPLE **Does the arrow diagram represent a function? If so, express the function in the roster form and in the equation form. Also, find the domain and range of the function.**



Solution The relation is a function from set A to set B because each element of A has a unique image in B.

The function f in the roster form is

$f = \{(-2, 4), (-1, 1), (1, 1), (2, 4), (3, 9)\}$.

Since $f(-2) = 4 = (-2)^2$, $f(-1) = 1 = (-1)^2$, $f(1) = 1 = 1^2$, $f(2) = 4 = 2^2$

and $f(3) = 9 = 3^2$, we have $f(x) = x^2, x \in A$.

$\therefore$ the equation form of function f is

$f(x) = x^2$ or $f : x \to x^2, x \in A$.

The domain of $f = \{-2, -1, 1, 2, 3\}$ and the range of $f = \{1, 4, 9\}$.

Downloaded from https:// www.studiestoday.com

Downloaded from https:// www.studiestoday.com

Solved Examples

EXAMPLE 1 **Let $A = \{3, 4, 5, 6\}$, $B = \{2, 3, 4, 6\}$ and R be the relation "is less than or equal to" from A to B.**

(i) Write R in the roster form. **(ii) Represent R by an arrow diagram.**

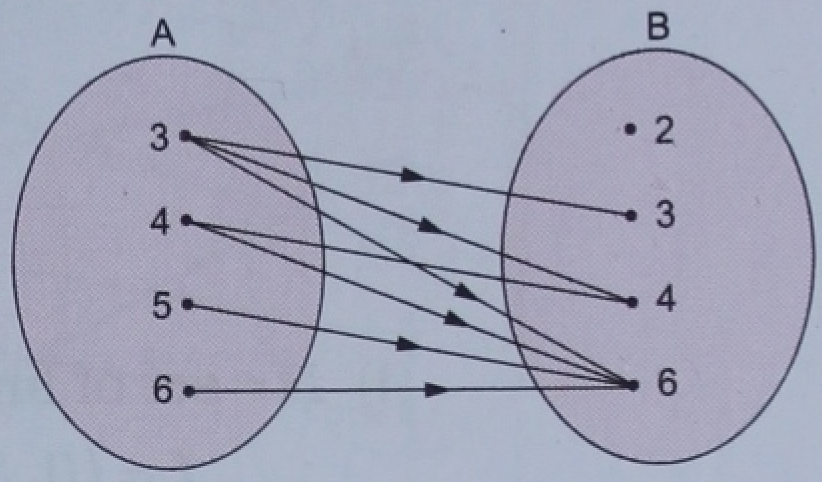
(iii) Find the domain and the range of R. **(iv) Is R a mapping? Give reasons.**

Solution $3 = 3, 3 < 4, 3 < 6; 4 = 4, 4 < 6; 5 < 6; 6 = 6.$

So, (3, 3), (3, 4), (3, 6), (4, 4), (4, 6), (5, 6), (6, 6) are the ordered pairs of R.

(i) The relation in the roster form is
$R = \{(3, 3), (3, 4), (3, 6), (4, 4), (4, 6), (5, 6), (6, 6)\}$.

(ii) The adjoining arrow diagram represents R.



(iii) The domain of $R = \{3, 4, 5, 6\}$.
The range of $R = \{3, 4, 6\}$.

(iv) This relation is not a mapping because the members 3 and 4 of the set A are matched with more than one member of the set B.

EXAMPLE 2 **Let $A = \{0, 1, 2, 3, 4\}$, $B = \{1, 2, 3, 4, 5\}$ and $R = \{(a, b) : a \in A, b \in B$ and $a + b$ is an even number$\}$.**

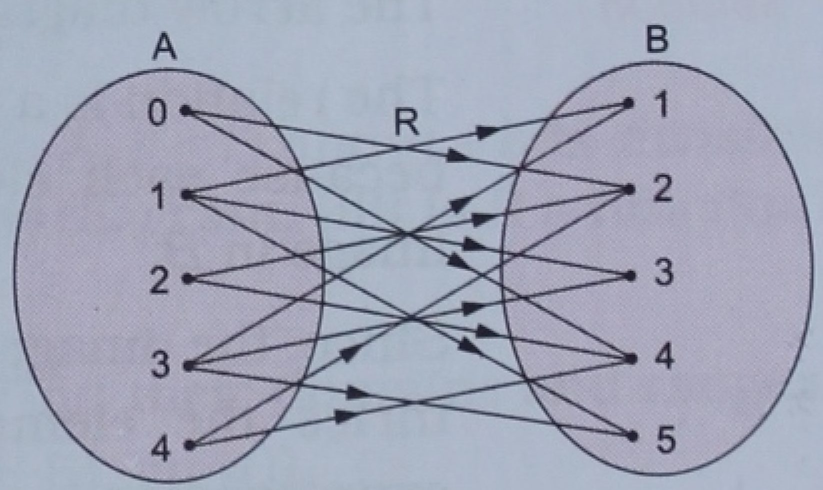
(i) Write R in the roster form **(ii) Represent R by an arrow diagram.**

(iii) Find the domain and range of R. **(iv) Is R a mapping? Give reasons.**

Solution (0, 2), (0, 4), (1, 1), (1, 3), (1, 5), (2, 2), (2, 4), (3, 1), (3, 3), (3, 5), (4, 2), (4, 4) are the ordered pairs (a, b) such that $a + b$ is an even number.

(i) $\therefore$ $R = \{(0, 2), (0, 4), (1, 1), (1, 3), (1, 5), (2, 2), (2, 4), (3, 1), (3, 3), (3, 5), (4, 2), (4, 4)\}$.

(ii) The adjoining figure represents R.



(iii) The domain of $R = \{0, 1, 2, 3, 4\}$.
The range of $R = \{1, 2, 3, 4, 5\}$.

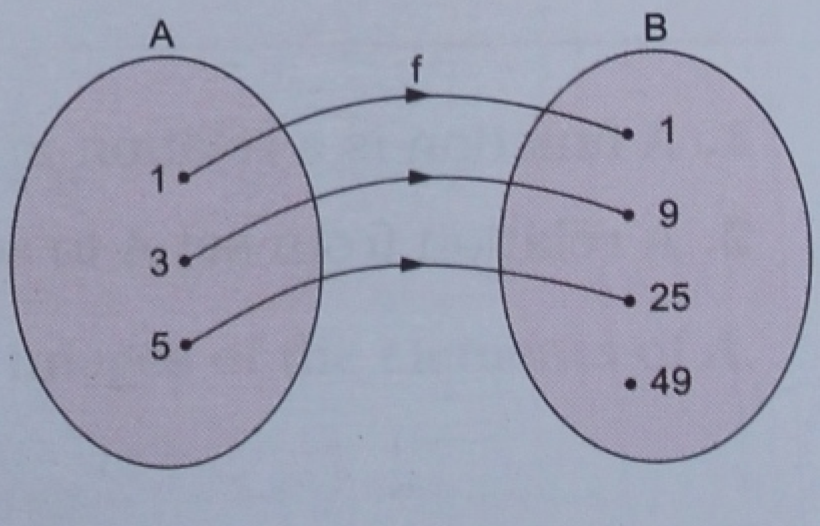
(iv) The relation R is not a mapping because the members of set A are matched with more than one member of set B.

EXAMPLE 3 **Let $A = \{1, 3, 5\}$, $B = \{1, 9, 25, 49\}$ and $f : A \to B$ where $f(x) = x^2$, $x \in A$.**

(i) Draw an arrow diagram for this mapping.
(ii) Represent f in the roster form.
(iii) Find the domain and range of f.

Solution Here, $f(x) = x^2$. So, $f(1) = 1^2 = 1$; $f(3) = 3^2 = 9$ and $f(5) = 5^2 = 25$.

(i) The arrow diagram is shown alongside.



(ii) The ordered pairs of $f = (1, 1), (3, 9), (5, 25)$.
$\therefore$ in the roster form $f = \{(1, 1), (3, 9), (5, 25)\}$.

(iii) The domain of $f = \{1, 3, 5\}$.
The range of $f = \{1, 9, 25\}$.

Downloaded from https:// www.studiestoday.com

Downloaded from https:// www.studiestoday.com

EXAMPLE 4 **Let $f : x \to 2x + 1$, where $A = \{1, 2, 3, 4\}$ and B is the set of images of the elements of A.**

(i) Find B and f. **(ii) Draw an arrow diagram to represent f.**

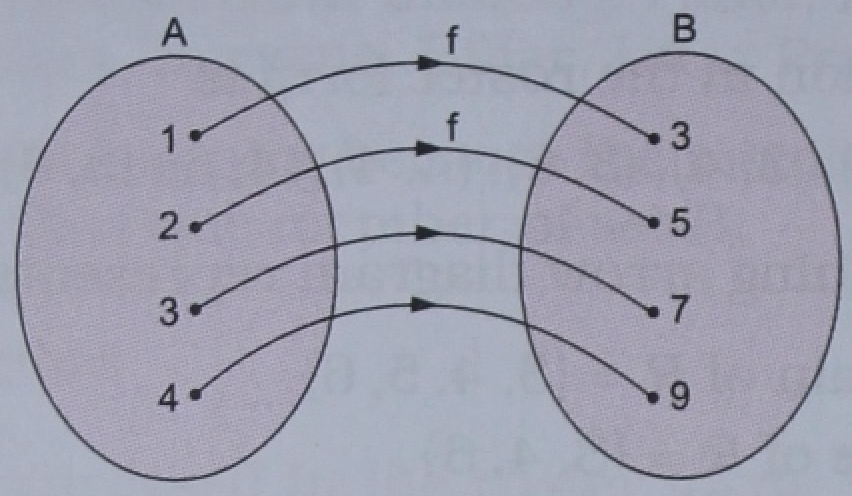
(iii) Find the domain and range of f.

Solution Here, $f(x) = 2x + 1$. ... (1)

$\because$ $A = \{1, 2, 3, 4\}$, substituting $x = 1, 2, 3, 4$ respectively in (1), we get

$f(1) = 2 \times 1 + 1 = 3,\ f(2) = 2 \times 2 + 1 = 5,\ f(3) = 2 \times 3 + 1 = 7,\ f(4) = 2 \times 4 + 1 = 9.$

So, the images of the elements 1, 2, 3 and 4 of A are 3, 5, 7 and 9 respectively.



(i) B = set of images of elements of $A = \{3, 5, 7, 9\}$.

$\therefore$ $f = \{(1, 3), (2, 5), (3, 7), (4, 9)\}$.

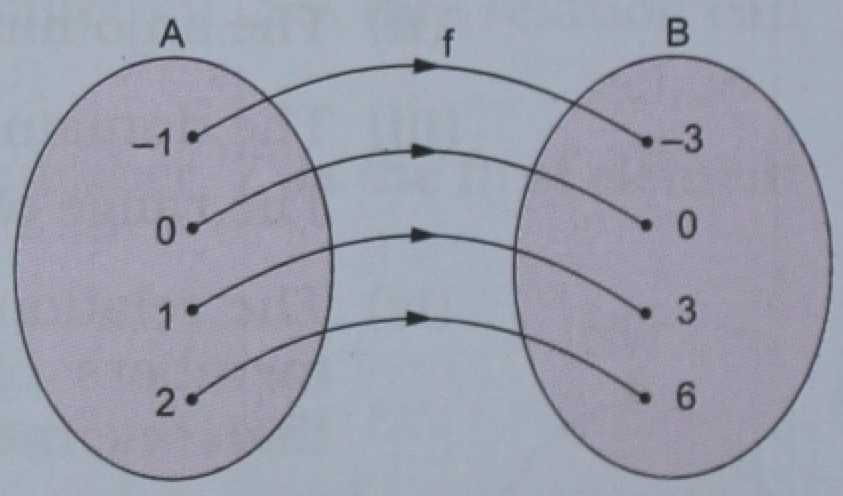
(ii) The arrow diagram is shown above.

(iii) The domain of $f = \{1, 2, 3, 4\}$.

The range of $f = \{3, 5, 7, 9\}$.

EXAMPLE 5 **Draw an arrow diagram for the relation $R = \{(-1, -3), (0, 0), (1, 3), (2, 6)\}$. Is this relation a function? If so, express it in the form of an equation. Also, find the domain and the range of the function.**

Solution The arrow diagram is shown alongside.



The relation is a functon from set A to set B because each element of A has a unique image in B.

Since the image of each element of A is thrice the element, the function can be expressed as

$f(x) = 3x, x \in A$ or $f : x \to 3x, x \in A$.

The domain of $f = \{-1, 0, 1, 2\}$ and the range of $f = \{-3, 0, 3, 6\}$.

Remember These

1. A function is a relation in which no two ordered pairs have the same first element.

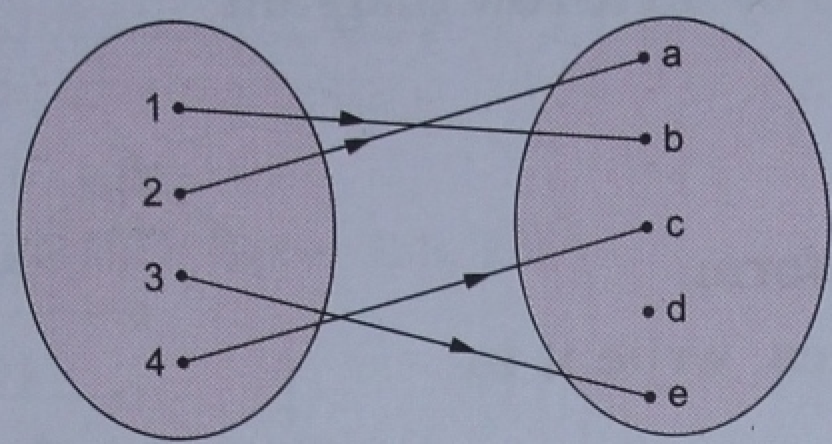
2. A relation from set A to set B is a function if each element of A has a unique image in B.

Downloaded from https:// www.studiestoday.com

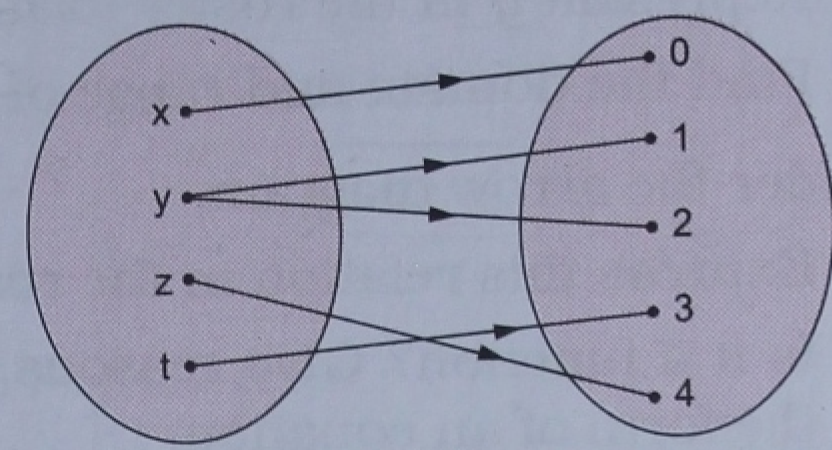
Downloaded from https:// www.studiestoday.com

EXERCISE 12B

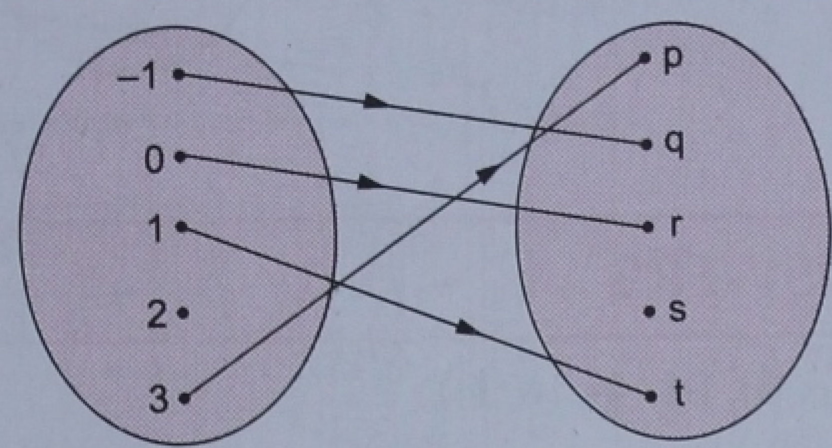
1. Which of the following arrow diagrams represent mappings? Represent the mappings in the roster form.



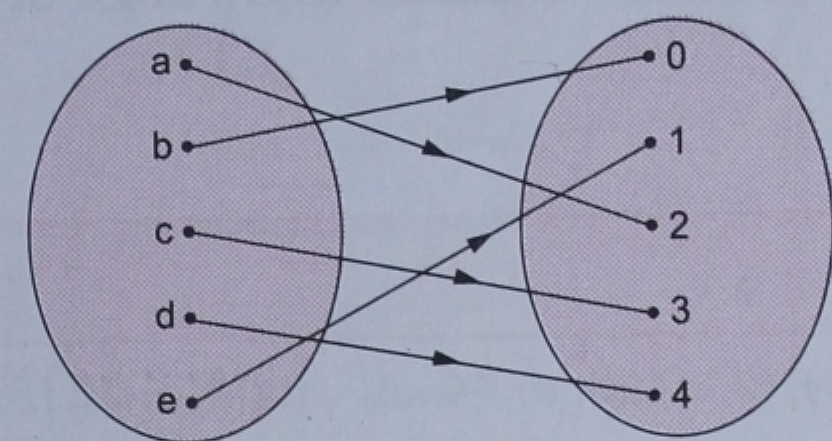
(i)



(ii)



(iii)



(iv)

2. Let $A = \{4, 9, 12, 16\}$ and $B = \{11, 18, 5, 14, 6\}$. Represent the mapping "is 2 less than" from the set A to the set B (i) in the roster form and (ii) by an arrow diagram. Find the domain and range of the mapping.

3. Let $A = \{2, 3, 4, 5\}$, $B = \{2, 4, 6, 8, 9\}$ and R be a relation from the set A to the set B defined by "divides". Represent R (i) in the roster form, (ii) R by an arrow diagram. Is R a mapping? Give reasons.

4. Let $A = \{2, 3, 7, 5\}$, $B = \{2, 3, 4\}$ and $R = \{(a, b) : a \in A, b \in B$ and $a + b$ is divisible by $3\}$. Represent the relation R (i) in the roster form and (ii) by an arrow diagram. (iii) Is R a function? Give reasons. Find the domain and the range of R.

5. Let $A = \{1, 4, 8, 9, 12\}$, $B = \{2, 5, 7, 8, 10\}$ and $R = \{(a, b) : a \in A, b \in B$ and $a + b$ is divisible by $4\}$. Represent R (i) in the roster form and (ii) by an arrow diagram. (iii) Is R a function? Find the domain and the range of R.

6. Which of the following set of ordered pairs are functions? Find their domains and ranges.
   (i) $\{(1, 1), (2, 3), (3, 4)\}$
   (ii) $\{(1, 1), (2, 5), (1, 4)\}$
   (iii) $\{(2, -2), (5, -2), (-5, 4), (-2, 3), (-2, 5)\}$
   (iv) $\{(0, 3), (1, 5), (2, -3), (-2, 5), (-1, 4)\}$.

7. Which of the following relations can be functions? Draw arrow diagrams for them and express them in the equation form. Find the domain and the range of each function.
   (i) $f = \{(1, 1), (2, 4), (3, 9), (-3, 9)\}$
   (ii) $f = \{(1, 1), (2, 3), (3, 5), (4, 7), (5, 9)\}$
   (iii) $R = \{(1, 3), (2, 5), (3, 7), (2, -3)$
   (iv) $g = \{(-1, 0), (1, 2), (2, 3), (0, 1)\}$

8. Let $f : x \to 2x - 1$ where $x \in A = \{1, 2, 3, 5, 7\}$.
   (i) Find the set of images of the elements of A.
   (ii) Represent f in the roster form and by an arrow diagram.
   (iii) Find the domain and the range of f.

9. Let $f : x \to 3x + 1$ where $A = \{0, -1, 1, 3\}$ and B is the set of images of the elements of A.
   (i) Find B.

Downloaded from https:// www.studiestoday.com

Downloaded from https:// www.studiestoday.com

(ii) Represent f in the roster form and by an arrow diagram.

(iii) Find the domain and range of f.

10. Let $g : x \to 2x^2$, where $x \in X = \{-1, 0, 1, 2, 3\}$ and Y is the set of images of the element of X.

(i) Find Y.

(ii) Represent g in the roster form and by an arrow diagram.

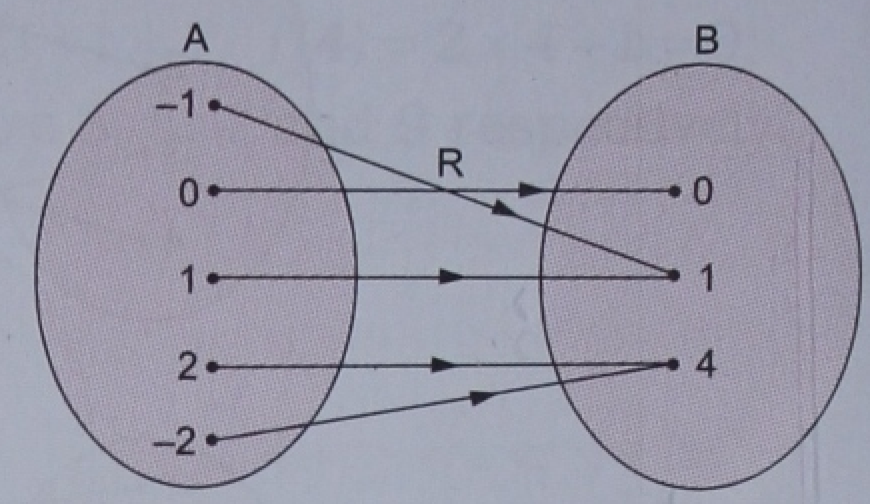
(iii) Find the domain and range of g.

11. Consider the arrow diagram.

(i) Express this relation in the roster form.

(ii) Is it a function? Give reasons. If yes, write it in the form of an equation.

(iii) Find the domain and range of R.

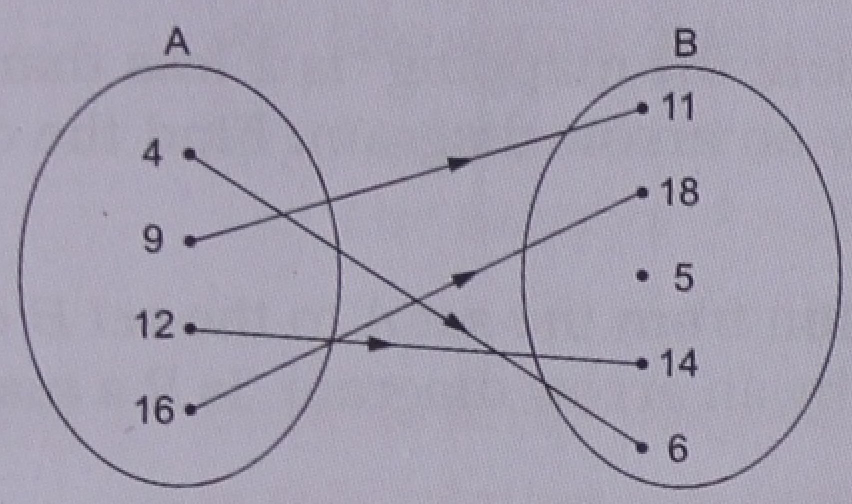


ANSWERS

1. (i) $\{(1, b), (2, a), (3, e), (4, c)\}$ (iv) $\{(a, 2), (b, 0), (c, 3), (d, 4), (e, 1)\}$

2. (i) $\{(4, 6), (9, 11), (12, 14), (16, 18)\}$

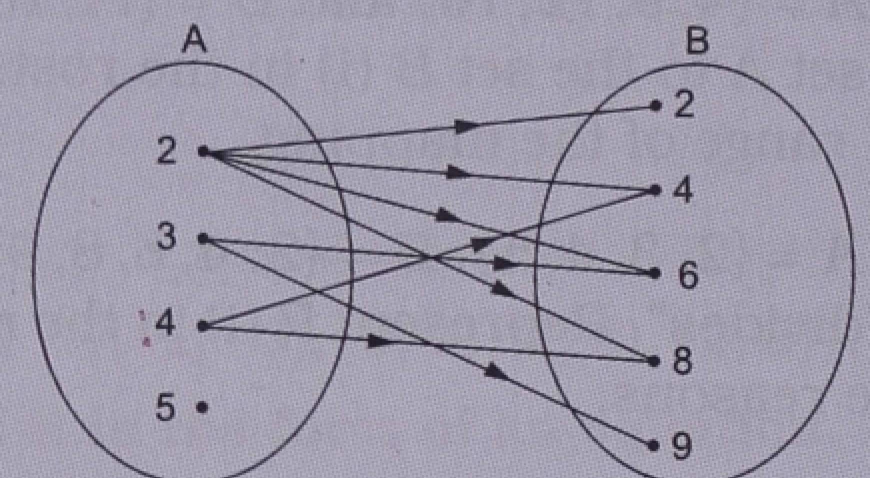
(ii)



Domain = $\{4, 9, 12, 16\}$,
range = $\{11, 18, 14, 6\}$

3. (i) $\{(2, 2), (2, 4), (2, 6), (2, 8), (3, 6), (3, 9), (4, 4), (4, 8)\}$

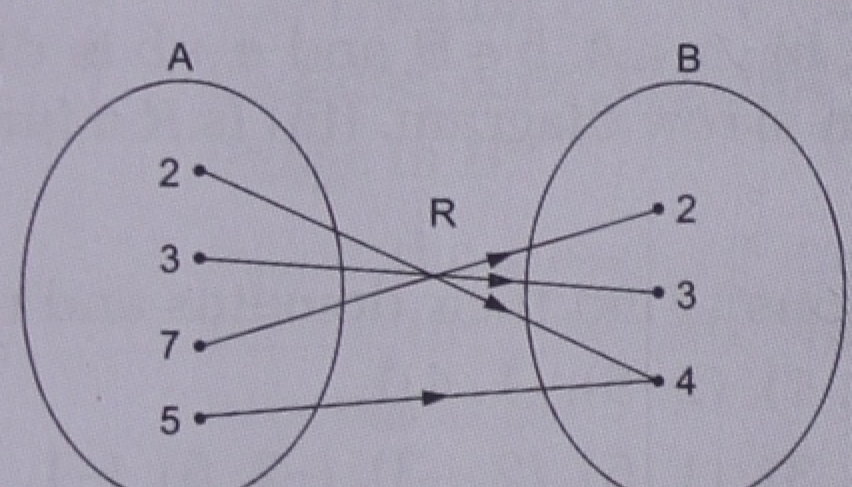
(ii)



(iii) No; 2, 3 and 4 have more than one image in B and 5 has no image in B.

4. (i) $\{(2, 4), (3, 3), (7, 2), (5, 4)\}$

(ii)

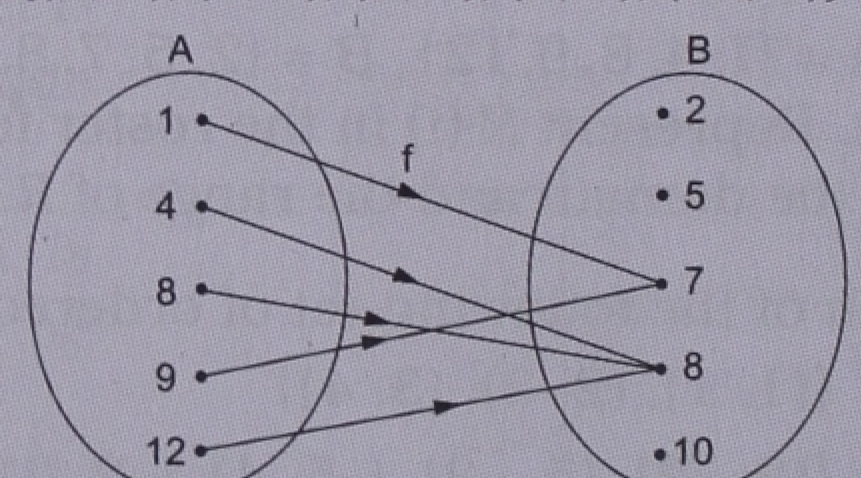


(iii) R is a function because each element of A has a unique image in B.

Domain = $\{2, 3, 7, 5\}$, range = $\{2, 3, 4\}$

5. (i) $\{(1, 7), (4, 8), (8, 8), (9, 7), (12, 8)\}$

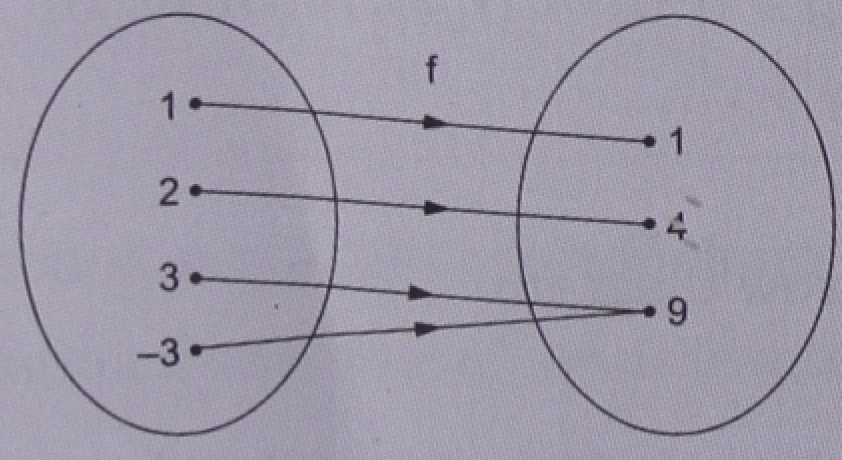
(ii)



(iii) Yes; domain = $\{1, 4, 8, 9, 12\}$, range = $\{7, 8\}$

6. (i) domain = $\{1, 2, 3\}$, range = $\{1, 3, 4\}$ (iv) domain = $\{0, 1, 2, -2, -1\}$, range = $\{3, 5, -3, 4\}$

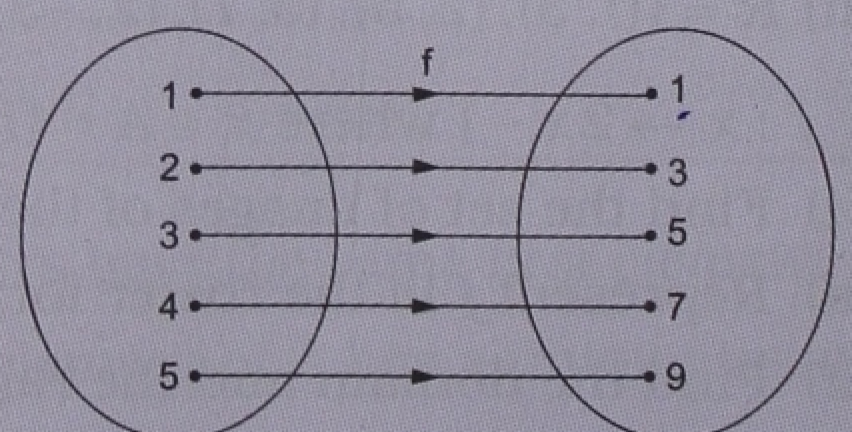
7. (i) Yes,



$f(x) = x^2$ or $f : x \to x^2$,

Domain = $\{1, 2, 3, -3\}$, range = $\{1, 4, 9\}$.

(ii) Yes,



$f(x) = 2x - 1$ or $f : x \to 2x - 1$

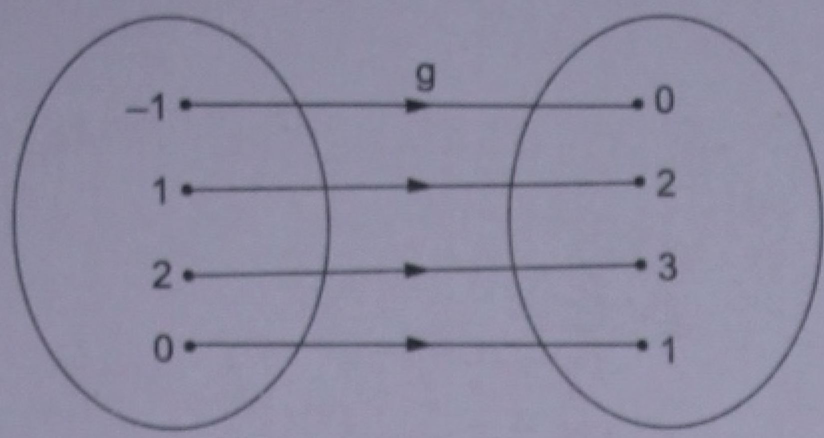
Domain = $\{1, 2, 3, 4, 5\}$, range = $\{1, 3, 5, 7, 9\}$.

Downloaded from https:// www.studiestoday.com

Downloaded from https:// www.studiestoday.com

(iii) No

(iv) Yes,



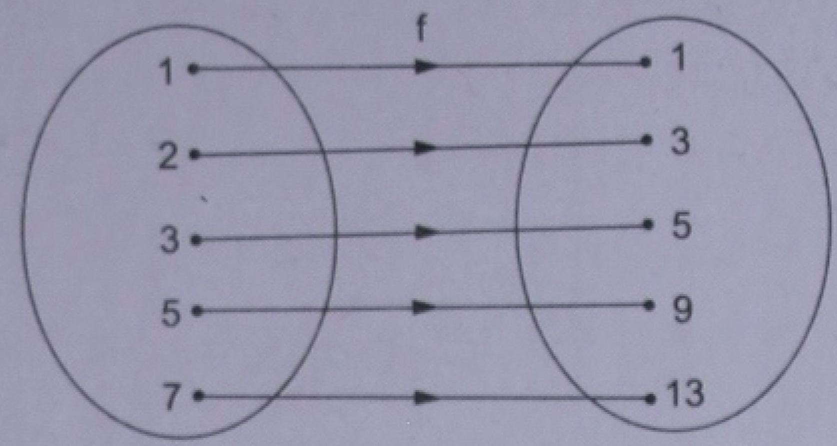
$g(x) = x + 1$ or $g : x \to x + 1$

Domain = $\{-1, 1, 2, 0\}$, range = $\{0, 2, 3, 1\}$

8. (i) The set of images = $\{1, 3, 5, 9, 13\}$

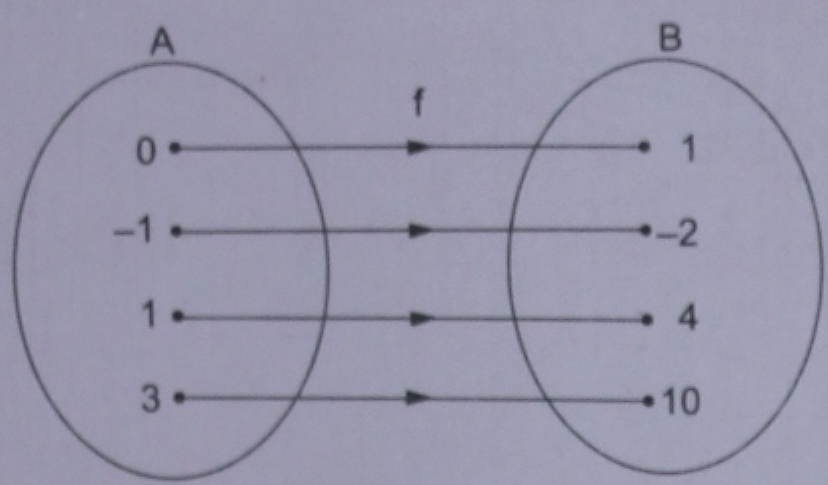
(ii) $f = \{(1, 1), (2, 3), (3, 5), (5, 9), (7, 13)\}$

(iii) Domain = $\{1, 2, 3, 5, 7\}$, range = $\{1, 3, 5, 9, 13\}$



9. (i) $B = \{1, -2, 4, 10\}$

(ii) $f = \{(0, 1), (-1, -2), (1, 4), (3, 10)\}$

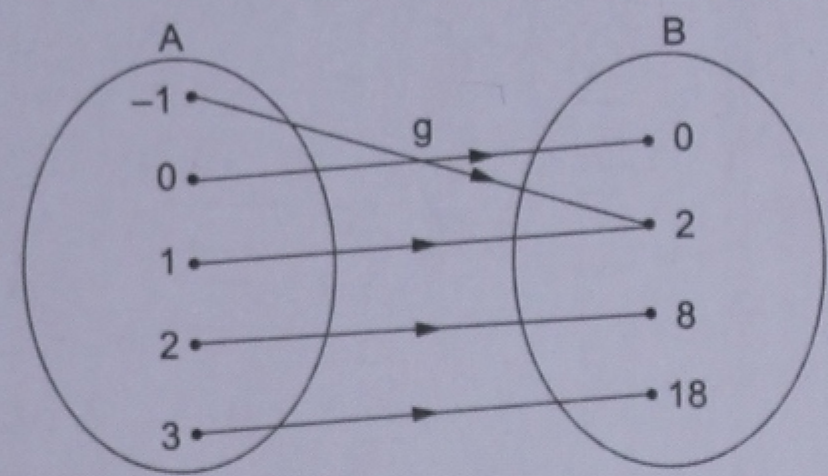


(iii) domain = $\{0, -1, 1, 3\}$,

range = $\{1, -2, 4, 10\}$

10. (i) $Y = \{0, 2, 8, 18\}$

(ii) $g = \{(-1, 2), (0, 0), (1, 2), (2, 8), (3, 18)\}$



(iii) Domain = $\{-1, 0, 1, 2, 3\}$,

range = $\{0, 2, 8, 18\}$

11. (i) $R = \{(-1, 1), (0, 0), (1, 1), (2, 4), (-2, 4)\}$

(ii) Yes, each element of A has a unique image in B. The equation form of R is $R : x \to x^2, x \in A$ or $R(x) = x^2$.

(iii) Domain = $\{-1, 0, 1, 2, -2\}$, range = $\{0, 1, 4\}$

❖

Downloaded from https:// www.studiestoday.com