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Simultaneous Linear Equations

Linear equation in two variables

A linear equation in two variables (say, x and y) contains the variables in the first degree and in separate terms. The general form of such an equation is $ax + by + c = 0$, where a, b and c are real numbers and a and b are nonzero numbers.

Examples $2x - 5y = 7$ and $5x + \frac{y}{2} + 3 = 0$ are linear equations in x and y. However, $5xy = 9$ is not a linear equation because x and y are not contained in separate terms.

Simultaneous linear equations

If two linear equations in x and y are satisfied by the same values of x and y then the equations are called simultaneous linear equations. The general form of such equations is $ax + by + c = 0$ and $px + qy + r = 0$.

Example The equations $x + 2y = 5$ and $2x + y = 4$ are satisfied by the values $x = 1, y = 2$. Therefore, $x + 2y = 5$ and $2x + y = 4$ are simultaneous linear equations and their solution is $x = 1, y = 2$.

To find a solution to simultaneous linear equations, we must find a pair of values of the variables that satisfy both the equations. There are two ways of doing this.

(i) By substitution (ii) By elimination

Substitution method

This method involves taking the following steps.

Steps
1. Using one of the equations write y in terms of x (or x in terms of y) and the constant.
2. Substitute the expression for y (or x) in the second equation.
3. Solve the resulting linear equation in x (or y).
4. Substitute the value of x (or y) in either of the equations.
5. Solve the resulting linear equation in y (or x).
6. Verify the correctness of the solution by substituting the values of x and y in the given equations.

EXAMPLE **Solve the equations $5x + y = 10$ and $14x + 3y = 18$.**

Solution Given, $5x + y = 10$ ... (1)

$14x + 3y = 18$ ... (2)

From the equation (1), $5x + y = 10$ or $y = 10 - 5x$.

Substituting the expression $10 - 5x$ for y in equation (2),

$14x + 3(10 - 5x) = 18$ or $14x + 30 - 15x = 18$ or $-x + 12 = 0$ or $x = 12$.

Substituting $x = 12$ in the equation (2),

$14 \times 12 + 3y = 18$ or $3y = 18 - 168 = -150$ or $y = -50$

Hence, the solution is $x = 12, y = -50$.

Elimination method

This method is also called the addition-subtraction method.

Steps

1. Decide which variable will be easier to eliminate. Try to avoid fractions.
2. Multiply one or both the equations by suitable numbers to ensure that the coefficients of the variable to be eliminated are the same in both the equations.
3. Add or subtract the resulting equations to eliminate the variable.
4. Solve the resulting equation in one variable.
5. Substitute the value of the variable obtained in Step 4 in either of the given equations.
6. Solve the resulting equation.
7. Verify the correctness of the solution by substituting the values of the variables in the given equations.

EXAMPLE **Solve the equations $5x + 3y = 7$ and $2x + 5y = 1$.**

Solution Given, $5x + 3y = 7$ ... (1)

$2x + 5y = 1$ ... (2)

To eliminate x, let us multiply the equation (1) by 2 and the equation (2) by 5.

Thus, $10x + 6y = 14$ ... (3)

$10x + 25y = 5$ ... (4)

Subtracting the equation (4) from the equation (3),

$$-19y = 9 \quad \text{or} \quad y = \frac{9}{-19} = -\frac{9}{19}.$$

Substituting $y = \frac{-9}{19}$ in the equation (1),

$$5x + 3 \times \frac{-9}{19} = 7 \quad \text{or} \quad 5x = 7 + \frac{27}{19} = \frac{133 + 27}{19} = \frac{160}{19}.$$

$$\therefore \quad x = \frac{1}{5} \times \frac{160}{19} = \frac{32}{19}.$$

$\therefore$ the solution is $x = \frac{32}{19}, y = \frac{-9}{19}$.

Verification

Substitute $x = \frac{32}{19}$, $y = \frac{-9}{19}$ in the equation (2),

$$2 \times \frac{32}{19} + 5 \times \frac{-9}{19} = 1 \quad \text{or} \quad \frac{64}{19} - \frac{45}{19} = 1 \quad \text{or} \quad \frac{19}{19} = 1$$

which is true. Hence, the solution is correct.

Solved Examples

EXAMPLE 1 **Solve $3y - 2x = 1$, $3x + 4y = 24$ by the substitution method.**

Solution Given, $3y - 2x = 1$ ... (1)

$3x + 4y = 24$ ... (2)

Let us solve the equation (1) for x.

$$3y - 2x = 1 \quad \text{or} \quad 2x = 3y - 1 \quad \text{or} \quad x = \frac{3y-1}{2}.$$

Substituting this value of x in the equation (2),

$$3 \times \frac{3y-1}{2} + 4y = 24.$$

Multiplying both sides by 2,

$$3(3y-1) + 8y = 48 \quad \text{or} \quad 9y - 3 + 8y = 48 \quad \text{or} \quad 17y = 48 + 3 = 51.$$

$$\therefore \quad y = \frac{51}{17} = 3.$$

Substituting the value of y in the equation (1),

$$3 \times 3 - 2x = 1 \quad \text{or} \quad 9 - 2x = 1 \quad \text{or} \quad 2x = 9 - 1 = 8 \quad \text{or} \quad x = 4.$$

Hence, the solution is $x = 4$, $y = 3$.

EXAMPLE 2 **Solve $8a + 5b = 9$ and $3a + 2b = 4$.**

Solution Given, $8a + 5b = 9$ ... (1)

$3a + 2b = 4$ ... (2)

To eliminate a, let us multiply the equation (1) by 3 and the equation (2) by 8.

Thus, $24a + 15b = 27$ ... (3)

$24a + 16b = 32$ ... (4)

Subtracting the equation (3) from the equation (4),

$$b = 32 - 27 = 5.$$

Substituting the value of b in the equation (1),

$$8a + 5 \times 5 = 9 \quad \text{or} \quad 8a = 9 - 25 = -16 \quad \text{or} \quad a = \frac{-16}{8} = -2.$$

Hence, the solution is $a = -2$, $b = 5$.

EXAMPLE 3 **Solve $7x + 3(y - 3) = 5(x + y)$ and $7(x - 1) - 6y = 5(x - y)$.**

Solution Given, $7x + 3(y-3) = 5(x+y)$ ... (1)

$7(x-1) - 6y = 5(x-y)$ ... (2)

Simplifying the two equations,

$7x + 3y - 9 = 5x + 5y$ or $2x - 2y = 9$ ... (3)

$7x - 7 - 6y = 5x - 5y$ or $2x - y = 7$ ... (4)

Subtracting (3) from (4), $y = 7 - 9 = -2$.

Substituting $y = -2$ in (4), $2x - y = 7$ or $2x - (-2) = 7$ or $2x = 5$ or $x = \frac{5}{2}$.

Hence, the solution is $x = \frac{5}{2}$ and $y = -2$.

EXAMPLE 4 **Solve $\frac{2}{x} - y = 2$ and $\frac{3}{x} + 2y = 10$.**

Solution Given, $\frac{2}{x} - y = 2$ and $\frac{3}{x} + 2y = 10$.

Substituting $\frac{1}{x} = z$, the given equations become

$2z - y = 2$ ... (1)

and $3z + 2y = 10$ .. (2)

Multiplying (1) by 2, $4z - 2y = 4$ ... (3)

Adding (2) and (3), $3z + 4z = 10 + 4$ or $7z = 14$.

$\therefore$ $z = \frac{14}{7} = 2$, that is, $\frac{1}{x} = 2$ or $x = \frac{1}{2}$.

Substituting the value of z in (1), $2 \times 2 - y = 2$ or $y = 4 - 2 = 2$.

Hence, the solution is $x = \frac{1}{2}$, $y = 2$.

EXAMPLE 5 **Solve $\frac{2}{x} + \frac{3}{y} = -1$ and $\frac{3}{x} + \frac{5}{y} = -2$.**

Solution Let $\frac{1}{x} = p$ and $\frac{1}{y} = q$.

Then $2p + 3q = -1$ ... (1)

and $3p + 5q = -2$ ... (2)

Multiplying (1) by 3 and (2) by 2,

$6p + 9q = -3$ ... (3)

$6p + 10q = -4$ ... (4)

Subtracting (3) from (4), $q = -4 + 3 = -1$ or $\frac{1}{y} = -1$ or $y = -1$.

Substituting the value of q in (1),

$2p + 3 \times (-1) = -1$ or $2p - 3 = -1$ or $2p = 3 - 1 = 2$

or $p = 1$ or $\frac{1}{x} = 1$ or $x = 1$.

Hence, the solution is $x = 1$, $y = -1$.

Remember These

1. An equation of the form $ax + by + c = 0$, where a, b and c are real numbers and a and b are nonzero numbers, is called a linear equation in two variables.
2. When two linear equations are satisfied by the same values of the two variables, the equations are called simultaneous equations.
3. There are two ways of solving simultaneous equations—by substitution and by elimination.

4. The substitution method essentially consists of using one of the equations to express one variable in terms of the other and then substituting this expression for the variable in the second equation.

5. The elimination method essentially consists of multiplying one or both the equations by suitable numbers so that one of the variables can be eliminated by adding or subtracting the resulting equations.

EXERCISE 9A

1. Solve the following by the substitution method.

(i) $2x+y=3,\quad x+2y=0$

(ii) $3x+4y=11,\quad 2x+3y=8$

(iii) $2x-y=3,\quad -3x+5y=13$

(iv) $x-y=2,\quad 4x+5y=-1$

(v) $4x+3y=17,\quad 3x-4y+6=0$

(vi) $2x-3y=0,\quad 3x+5y=\frac{19}{6}$

(vii) $\frac{x}{4}+\frac{y}{5}=1,\quad \frac{2x}{9}-\frac{y}{9}=-2$

2. Use the elimination method to solve the following.

(i) $6x+7y-11=0,\quad 5x+2y=13$

(ii) $2x+3y=1,\quad 3x-2y=-5$

(iii) $5x+4y+14=0,\quad 3x=-10-4y$

(iv) $x+y=3,\quad -2x+7y=-6$

(v) $2x+y-8=0,\quad x-\frac{1}{3}y=-1$

(vi) $3x-2y=7,\quad 4x-3y=9$

3. Solve:

(i) $\frac{x+1}{y+1}=2,\quad \frac{2x+1}{2y+1}=\frac{1}{3}$ [or $x+1=2(y+1), 3(2x+1)=2y+1$]

(ii) $3(x-y)-\frac{1}{3}(x+y)=30,\quad x+y+\frac{5}{3}(x-y)=22$

(iii) $\frac{x-1}{3}+\frac{y+2}{2}=3,\quad \frac{1-x}{6}-\frac{y-4}{2}=\frac{1}{2}$

(iv) $\frac{x}{3}-\frac{y}{2}=1,\quad \frac{2x+3}{5}-\frac{5y+1}{11}=2$

4. Solve:

(i) $\frac{7}{x}+4y=1,\quad \frac{5}{x}+2y=-1$

(ii) $2x+\frac{5}{y}=16,\quad x-\frac{1}{y}=1$

(iii) $4x+\frac{3}{y}=9,\quad 3x+\frac{6}{y}=8$

(iv) $\frac{5}{x}-2y=-19,\quad \frac{2}{x}+3y=0$

5. Solve the following simultaneous equations.

(i) $\frac{3}{x}-\frac{4}{y}=10,\quad \frac{4}{x}+\frac{3}{y}=5$

(ii) $\frac{5}{x}-\frac{3}{y}=1,\quad \frac{2}{x}+\frac{5}{y}=19$

(iii) $\frac{2}{a}+\frac{3}{b}=8,\quad \frac{3}{a}-\frac{4}{b}=-5$

(iv) $\frac{2}{x}-\frac{1}{y}=10,\quad \frac{3}{x}+\frac{2}{y}=29$

(v) $\frac{-2}{p}+\frac{3}{q}=10,\quad \frac{1}{p}+\frac{2}{q}=2$

ANSWERS

1. (i) $x = 2, y = -1$ (ii) $x = 1, y = 2$ (iii) $x = 4, y = 5$ (iv) $x = 1, y = -1$ (v) $x = 2, y = 3$ (vi) $x = \frac{1}{2}, y = \frac{1}{3}$ (vii) $x = -4, y = 10$

2. (i) $x = 3, y = -1$ (ii) $x = -1, y = 1$ (iii) $x = -2, y = -1$ (iv) $x = 3, y = 0$ (v) $x = 1, y = 6$ (vi) $x = 3, y = 1$

3. (i) $x = \frac{-3}{5}, y = \frac{-4}{5}$ (ii) $x = \frac{15}{2}, y = -3$ (iii) $x = 4, y = 2$ (iv) $x = 6, y = 2$

4. (i) $x = -1, y = 2$ (ii) $x = 3, y = \frac{1}{2}$ (iii) $x = 2, y = 3$ (iv) $x = \frac{-1}{3}, y = 2$

5. (i) $x = \frac{1}{2}, y = -1$ (ii) $x = \frac{1}{2}, y = \frac{1}{3}$ (iii) $a = 1, b = \frac{1}{2}$ (iv) $x = \frac{1}{7}, y = \frac{1}{4}$ (v) $p = \frac{-1}{2}, q = \frac{1}{2}$

Problems leading to simultaneous equations

Many word problems involving two unknown quantities can be solved by framing simultaneous equations for the unknown quantities. The method is to represent each unknown quantity by a variable and then to write two linear equations using the conditions mentioned in the problem. Solving the simultaneous equations thus framed gives the required values of the quantities.

Solved Examples

EXAMPLE 1 **The sum of two numbers is 11. Twice the first number plus three times the second number equals 25. Find the numbers.**

Solution Let the numbers be x and y.

From question, $x + y = 11$ ... (1)

and $2x + 3y = 25$ ... (2)

Multiplying (1) by 2, $2x + 2y = 22$ ... (3)

Subtracting (3) from (2), $y = 25 - 22 = 3$.

Substituting $y = 3$ in (1), $x + 3 = 11$ or $x = 11 - 3 = 8$.

Hence, the numbers are 8 and 3.

EXAMPLE 2 **In a two-digit number, the digit in the units place is one more than twice the digit in the tens place. When the digits are reversed, the number obtained is 45 more than the original number. Find the original number.**

Solution Let the digit at the tens place $= x$ and the digit at the units place $= y$.

Then the number $= 10x + y$.

Given, $y = 2x + 1$ or $y - 2x = 1$ ... (1)

The number obtained after reversing the digits $= 10y + x$.

Given, $10y + x = 10x + y + 45$ or $9y - 9x = 45$

or $9(y - x) = 45$ or $y - x = 5$ ... (2)

Subtracting (1) from (2), $2x - x = 5 - 1$ or $x = 4$.

Substituting the value of x in (2), $y - 4 = 5$ or $y = 9$.

$\therefore$ the original number $= 10x + y = 10 \times 4 + 9 = 49$.

EXAMPLE 3 **If 5 is added to the numerator of a fraction and 3 is subtracted from its denominator, the fraction equals 2. If 3 is subtracted from the numerator and 3 is added to the denominator, the fraction become $\frac{1}{3}$. Find the fraction.**

Solution Let the fraction be $\frac{x}{y}$.

Given, $\frac{x+5}{y-3}=2$ or $x+5=2y-6$ or $x-2y=-11$ ... (1)

Also, $\frac{x-3}{y+3}=\frac{1}{3}$ or $3x-9=y+3$ or $3x-y=12$ ... (2)

Multiplying (2) by 2,

$6x-2y=24$ ... (3)

Subtracting (3) from (1), $x-6x=-11-24$ or $-5x=-35$ or $x=7$.

Substituting the value of x in (1), $7-2y=-11$ or $7+11=2y$

or $2y=18$ or $y=\frac{18}{2}=9$.

Hence, the required fraction $=\frac{7}{9}$.

EXAMPLE 4 **Eight years ago a man's age was thrice that of his son. Two years later, the man will be twice as old as his son. How old are the man and his son at present?**

Solution Let the man's present age $= x$ years and his son's present age $= y$ years.

From the question, $x-8=3(y-8)$ ... (1)

and $x+2=2(y+2)$ ... (2)

From (1), $x-8=3y-24$ or $x-3y=-16$ ... (3)

From (2), $x+2=2y+4$ or $x-2y=2$ ... (4)

Subtracting (3) from (4), $3y-2y=2+16$ or $y=18$.

Substituting the value of y in (3),

$x-3\times 18=-16$ or $x=54-16=38$.

Hence, the present age of the man is 38 years and that of his son is 18 years.

EXAMPLE 5 **The cost of 3 L of diesel and 5 L of petrol is Rs 346 whereas the cost of 7 L of diesel and 2 L of petrol is Rs 324. Find the cost of diesel and petrol per litre.**

Solution Let the cost of 1 L of diesel = Rs x and the cost of 1 L petrol = Rs y.

From the question, $3x+5y=346$ ... (1)

and $7x+2y=324$ ... (2)

To eliminate y, we multiply (1) by 2 and (2) by 5.

Thus, $6x+10y=692$ ... (3)

and $35x+10y=1620$ ... (4)

Subtracting (3) from (4),

$29x=1620-692=928$ or $x=\frac{928}{29}=32$.

Substituting the value of x in (1),

$3\times 32+5y=346$ or $5y=346-96=250$ or $y=\frac{250}{5}=50$.

Hence, the cost of diesel = Rs 32/L and the cost of petrol = Rs 50/L.

EXAMPLE 6 **26 tickets were sold for a total of ₹ 330 at a fair. If an adult ticket cost Rs 15 and a child's ticket cost ₹ 5, how many of each kind of ticket were sold.**

Solution Let x = the number of adult tickets and y = the number of children's tickets.

From the question, $x+y=26$ ... (3)

Since an adult ticket costs ₹ 15 and a child's ticket costs ₹ 5,

$15x+5y=330$ or $5(3x+y)=330$ or $3x+y=66$ ... (4)

Subtracting (3) from (4), $2x=66-26=40$ or $x=\frac{40}{2}=20$.

Substituting the value of x in (3), $20+y=26$ or $y=26-20=6$.

Hence, 20 adult tickets and 6 children's tickets were sold.

EXAMPLE 7 **A collection has 36 one-rupee and two-rupee coins. How many coins of each kind are there in the collection if the total value of the coins is Rs 47?**

Solution Let the number of one-rupee coins = x and the number of two-rupee coins = y.

$\therefore$ the value of the one-rupee coins = Rs x

and the value of the two-rupee coins = Rs $2y$.

From the question, the total number of coins = 36 or $x+y=36$ ... (1)

Also, the total value of coins = Rs 47 or $x+2y=47$ ... (2)

Subtracting (1) from (2), $y=47-36=11$.

Substituting this value in (1), $x+11=36$ or $x=36-11=25$.

Hence, the number of one-rupee coins = 25

and the number of two-rupee coins = 11.

EXAMPLE 8 **A plane flying with the wind takes 2 hours to make a 900-km trip from one city to another. On the return trip against the wind, it takes 2 hours and 15 minutes. Find the speed of the plane in still air and the speed of the wind.**

Solution Let the speed of the plane in still air = x km/h and the speed of the wind = y km/h.

Then the speed of the plane with the wind = $(x+y)$ km/h

and the speed of the plane against the wind = $(x-y)$ km/h.

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

$$\therefore \quad (x+y)\text{ km/h} = \frac{900\text{ km}}{2\text{ hours}} \text{ and } (x-y)\text{ km/h} = \frac{900\text{ km}}{2\text{ hours 15 min}}$$

$$\text{or} \quad x+y=\frac{900}{2}=450 \text{ and } x-y=\frac{900}{2\frac{1}{4}}=900\times\frac{4}{9}=400.$$

Thus, $x+y=450$ ... (1)

and $x-y=400$ ... (2)

Adding (1) and (2), $2x=850$ or $x=\frac{850}{2}=425$.

Substituting the value of x in (1),

$425+y=450$ or $y=450-425=25$.

Thus, the speed of the plane in still air = 425 km/h

and the speed of the wind = 25 km/h.

EXERCISE 9B

1. The sum of two numbers is 64 and their difference is 24. Find the numbers.
2. The sum of two numbers is 41 and their difference is 31. Find the numbers.
3. The difference of two numbers is 13. If the smaller number is 2 more than one fourth the larger, what are the numbers?
4. The difference of two numbers is 4. Three times the smaller number is 4 more than twice the larger number. Find the numbers.
5. The sum of the digits of a two-digit number is 5. If 9 is added to the number, the digits are interchanged. Find the number.
6. In a two-digit number, the digit in the units place is 6 less than twice the digit in the tens place. When the digits are reversed, the number increases by 9. What is the number?
7. The digit in the tens place of a two-digit number is four times the digit in the units place. When the digits are reversed, the number obtained is 27 less than the original number. Find the original number.
8. The sum of the digits of a two-digit number is equal to $\frac{1}{8}$ of the number. If the digits of the number are reversed, the new number is 45 less than the original number. Find the original number.
9. When 9 is added to the numerator and 10 is subtracted from the denominator of a fraction, the fraction becomes $\frac{4}{3}$. When 5 is added to the denominator of the fraction and 1 is subtracted from its numerator, the fraction equals $\frac{1}{3}$. Find the fraction.
10. If 3 is added to the numerator and 2 is subtracted from the denominator of a fraction, the fraction equals 1. If 1 is subtracted from its numerator and 2 is added to its denominator, the fraction equals $\frac{1}{9}$. Find the fraction.
11. The sum of the ages of a man and his son is 52 years. Eight years ago, the man was eight times as old as his son. Find the present ages of the man and his son.
12. A year ago, an adult was five times as old as a child. A year hence, the adult will be four times as old as the child. Find the present ages of the adult and the child.
13. The cost of 3 kg of rice and 4 kg of wheat is Rs 93. The cost of 5 kg of rice and 7 kg of wheat is Rs 159. Find the prices of rice and wheat per kg.
14. A pencil and two pens cost ₹ 22. Three pencils and five pens cost ₹ 56. Find the cost of one pencil and one pen.
15. Pinky bought a toy train and a doll for ₹ 760. If the price of the train was ₹ 20 less than twice that of the doll, what was the price of each?
16. The front seats for a concert cost Rs 40 each while the back seats cost Rs 30 each. If 500 tickets were sold for the concert and the total receipts were Rs 17,000. How many front seats and how many back seats were sold?

17. The tickets for a show were priced at Rs 10 for adults and Rs 5 for children. How many of each kind of ticket were sold if a total of 200 tickets were sold for Rs 1750?

18. Ravi has 20 coins worth Rs 85. If he has only 5-rupee and 2-rupee coins, find the number of each type of coin.

19. Ray carries only 10-rupee and 20-rupee notes in his purse. The total amount in his purse is ₹ 500. If the total number of notes is 35, how many of each kind does he have?

20. Bob went to the post office to buy 2-rupee and 5-rupee stamps. He bought 20 stamps in all and paid Rs 55. How many of each kind of stamp did he buy?

21. A motor boat travelling with the current covers 100 km in 4 hours. When it travels against the current, it takes 5 hours to cover the same distance. Find the speed of the motor boat in calm water and the speed of the current.

22. A plane took 4 hours to cover 2400 km while flying with the wind. Against the wind, it took 6 hours to fly the same distance. Find the speed of the plane in still air and the speed of the wind.

ANSWERS

1. 44, 20 **2.** 36, 5 **3.** 20, 7 **4.** 16, 12 **5.** 23 **6.** 78

7. 41 **8.** 72 **9.** $\frac{11}{25}$ **10.** $\frac{2}{7}$

11. Man's age = 40 years, son's age = 12 years

12. Adult's age = 31 years, child's age = 7 years

13. Rice : Rs 15/kg, wheat: Rs 12/kg

14. Pencil: ₹ 2, pen: ₹ 10

15. Train: ₹ 500, doll: ₹ 260

16. 200 front seats, 300 back seats

17. 150 adults, 50 children

18. 15 five-rupee coins, 5 two-rupee coins

19. 20 ten-rupee notes, 15 twenty-rupee notes

20. 15 two-rupee stamps, 5 five-rupee stamps

21. Speed of motor boat = 22.5 km/h, speed of current = 2.5 km/h

22. Speed of plane in still air = 500 km/h, speed of wind = 100 km/h

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