

Algebraic Fractions

Fractions in which the numerator or denominator or both are algebraic expressions are called **algebraic fractions**.

Example $\frac{x}{2}, \frac{2}{x^2+3}$ and $\frac{a-6}{a^2+3a+2}$ are algebraic fractions.

Algebraic fractions can be reduced, added, subtracted, multiplied and divided according to the same rules that govern arithmetic fractions.

Reduction to lowest terms

When the numerator and denominator of an algebraic fraction do not have any common factor except 1, the algebraic fraction is said to be expressed in the lowest terms.

To reduce an algebraic fraction to the lowest terms, take the following steps.

Steps 1. Factorize the numerator and the denominator.

2. Cancel the factors that are common to the numerator and the denominator.

Examples (i)
$$\frac{15x^2}{25x^3} = \frac{3 \times 5 \times x \times x}{5 \times 5 \times x \times x \times x} = \frac{3}{5 \times x} = \frac{3}{5x}.$$

Alternatively,
$$\begin{aligned} \frac{15x^2}{25x^3} &= \frac{15}{25} \times \frac{x^2}{x^3} \\ &= \frac{3}{5} \times \frac{1}{x^{3-2}}, \quad \left\{ \because \frac{x^m}{x^n} = \frac{1}{x^{n-m}}, n > m \right\} \\ &= \frac{3}{5x}. \end{aligned}$$

(ii)
$$\frac{2a^2b}{6ab^4} = \frac{2 \times a \times a \times b}{2 \times 3 \times a \times b \times b \times b \times b} = \frac{a}{3 \times b \times b \times b} = \frac{a}{3b^3}.$$

Alternatively,
$$\frac{2a^2b}{6ab^4} = \frac{2}{6} \times \frac{a^2}{a} \times \frac{b}{b^4} = \frac{1}{3} \times \frac{a^{2-1}}{b^{4-1}} = \frac{a}{3b^3}.$$

(iii)
$$\frac{2a^2 - 6a}{9b - 3ab} = \frac{2a(a-3)}{3b(3-a)} = \frac{2a(a-3)}{-3b(a-3)} = -\frac{2a}{3b}.$$

(iv)
$$\begin{aligned} \frac{x^2 - 5x + 6}{x^2 - 6x + 9} &= \frac{x^2 - 3x - 2x + 6}{x^2 - 2 \times x \times 3 + 3^2} = \frac{x(x-3) - 2(x-3)}{(x-3)^2} \\ &= \frac{(x-3)(x-2)}{(x-3)^2} = \frac{x-2}{x-3}. \end{aligned}$$

Addition and subtraction

If algebraic fractions have the same denominator then add or subtract the fractions as follows.

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$$

$$\frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$$

To add or subtract algebraic fractions that do not have the same denominator, first find the LCM of their denominators and then proceed as in arithmetic.

$$\frac{a}{b} + \frac{c}{d} = \frac{a(bd \div b) + c \times (bd \div d)}{bd} = \frac{(a \times d) + (c \times b)}{bd}.$$

Thus,

$$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$$

$$\frac{a}{b} - \frac{c}{d} = \frac{ad-bc}{bd}$$

Examples (i) $\frac{4}{5a} + \frac{6}{5a} = \frac{4+6}{5a} = \frac{10}{5a} = \frac{2}{a}.$

(ii) $\frac{5}{x-1} + \frac{x-6}{x-1} = \frac{5+x-6}{x-1} = \frac{x-1}{x-1} = 1.$

(iii) $\frac{7}{3m^2} - \frac{1}{3m^2} = \frac{7-1}{3m^2} = \frac{6}{3m^2} = \frac{2}{m^2}.$

(iv) $\frac{2x}{x^2-4} - \frac{4}{x^2-4} = \frac{2x-4}{x^2-4} = \frac{2(x-2)}{(x+2)(x-2)} = \frac{2}{x+2}.$

(v) $\frac{13}{x} + \frac{24}{y} = \frac{13 \times y + 24 \times x}{xy} = \frac{13y+24x}{xy}.$

(vi)
$$\begin{aligned} \frac{3a}{a+5} - \frac{8a}{a-5} &= \frac{3a(a-5) - 8a(a+5)}{(a+5)(a-5)} = \frac{3a^2 - 15a - 8a^2 - 40a}{(a+5)(a-5)} \\ &= \frac{-5a^2 - 55a}{(a+5)(a-5)} = \frac{-5a(a+11)}{a^2 - 5^2} = \frac{-5a(a+11)}{a^2 - 25}. \end{aligned}$$

Multiplication

To find the product of two algebraic fractions, multiply the numerator of one by the numerator of the other and the denominator of one by the denominator of the other. Then reduce the fraction to the lowest terms.

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d} = \frac{ac}{bd}, \text{ where } b \neq 0, d \neq 0$$

Examples (i)
$$\begin{aligned} 2a^2 \times \frac{3b^2}{5a^3} \times \frac{5}{4ab} &= \frac{2a^2}{1} \times \frac{3b^2}{5a^3} \times \frac{5}{4ab} = \frac{2a^2 \times 3b^2 \times 5}{1 \times 5a^3 \times 4ab} \\ &= \frac{2 \times a \times a \times 3 \times b \times b \times 5}{5 \times a \times a \times a \times 2 \times 2 \times a \times b} \\ &= \frac{3 \times b}{2 \times a \times a} = \frac{3b}{2a^2}. \end{aligned}$$

$$\text{(ii)} \frac{3a+12}{a^4} \times \frac{a^2}{a^2-16} = \frac{3(a+4) \times a \times a}{a \times a \times a \times a \times (a+4) \times (a-4)}$$

$$= \frac{3}{a \times a \times (a-4)} = \frac{3}{a^2(a-4)}.$$

Division

To divide one algebraic fraction by another, multiply the first by the reciprocal of the second.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}, \text{ where } b \neq 0, c \neq 0, d \neq 0$$

Examples (i) $\frac{12}{a^2} \div \frac{4}{a^3} = \frac{12}{a^2} \times \frac{a^3}{4} = \frac{2 \times 2 \times 3}{a \times a} \times \frac{a \times a \times a}{2 \times 2} = 3a.$

$$\text{(ii)} \frac{2x^2-8}{x^2-9} \div \frac{x-2}{3x+9} = \frac{2x^2-8}{x^2-9} \times \frac{3x+9}{x-2} = \frac{2(x^2-4)}{(x+3)(x-3)} \times \frac{3x+9}{x-2}$$

$$= \frac{2(x+2)(x-2)}{(x+3)(x-3)} \times \frac{3(x+3)}{x-2} = \frac{6(x+2)}{(x-3)}.$$

Solved Examples

EXAMPLE 1 Simplify $\frac{a+3}{2a+8} - \frac{4}{a^2-16}.$

Solution $2a+8 = 2(a+4)$ and $a^2-16 = (a+4)(a-4).$

\therefore the LCM of the denominators = $2(a+4)(a-4).$

$$\begin{aligned} \therefore \frac{a+3}{2a+8} - \frac{4}{a^2-16} &= \frac{(a+3)(a-4) - 4 \times 2}{2(a+4)(a-4)} = \frac{a^2 - a - 12 - 8}{2(a+4)(a-4)} \\ &= \frac{a^2 - a - 20}{2(a+4)(a-4)} = \frac{a^2 + 4a - 5a - 20}{2(a+4)(a-4)} \\ &= \frac{a(a+4) - 5(a+4)}{2(a+4)(a-4)} = \frac{(a+4)(a-5)}{2(a+4)(a-4)} = \frac{a-5}{2(a-4)}. \end{aligned}$$

EXAMPLE 2 Simplify $\frac{x+1}{4} - \frac{2(x+5)}{9} + \frac{5x+6}{12}.$

Solution The LCM of 4, 9 and 12 = 36.

$$\begin{aligned} \therefore \frac{x+1}{4} - \frac{2(x+5)}{9} + \frac{5x+6}{12} &= \frac{9(x+1) - 4 \times 2(x+5) + 3(5x+6)}{36} \\ &= \frac{9x + 9 - 8x - 40 + 15x + 18}{36} = \frac{16x - 13}{36}. \end{aligned}$$

EXAMPLE 3 Simplify $\frac{x}{x+2} + \frac{1}{2x+4} - \frac{3}{x^2+2x}.$

Solution $2x+4 = 2(x+2)$ and $x^2+2x = x(x+2).$

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\therefore the LCM of the denominators = $2x(x + 2)$.

$$\begin{aligned}\therefore \frac{x}{x+2} + \frac{1}{2x+4} - \frac{3}{x^2+2x} &= \frac{x}{x+2} + \frac{1}{2(x+2)} - \frac{3}{x(x+2)} \\&= \frac{x \times 2x + x - 3 \times 2}{2x(x+2)} = \frac{2x^2 + x - 6}{2x(x+2)} \\&= \frac{2x^2 + 4x - 3x - 6}{2x(x+2)} = \frac{2x(x+2) - 3(x+2)}{2x(x+2)} \\&= \frac{(x+2)(2x-3)}{2x(x+2)} = \frac{2x-3}{2x}.\end{aligned}$$

EXAMPLE 4 Simplify $\frac{x+4}{x^2+5x+6} - \frac{x+5}{x^2+4x+3}$.

$$\text{Solution } x^2 + 5x + 6 = x^2 + 3x + 2x + 6 = x(x+3) + 2(x+3) = (x+3)(x+2).$$

$$x^2 + 4x + 3 = x^2 + 3x + x + 3 = x(x+3) + 1(x+3) = (x+3)(x+1).$$

\therefore the LCM of the denominators = $(x+1)(x+2)(x+3)$.

$$\begin{aligned}\therefore \frac{x+4}{x^2+5x+6} - \frac{x+5}{x^2+4x+3} &= \frac{x+4}{(x+3)(x+2)} - \frac{x+5}{(x+3)(x+1)} \\&= \frac{(x+4)(x+1) - (x+5)(x+2)}{(x+1)(x+2)(x+3)} \\&= \frac{(x^2 + 5x + 4) - (x^2 + 7x + 10)}{(x+1)(x+2)(x+3)} \\&= \frac{x^2 + 5x + 4 - x^2 - 7x - 10}{(x+1)(x+2)(x+3)} = \frac{-2x - 6}{(x+1)(x+2)(x+3)} \\&= \frac{-2(x+3)}{(x+1)(x+2)(x+3)} = \frac{-2}{(x+1)(x+2)}.\end{aligned}$$

EXAMPLE 5 Simplify $\frac{a^2 + 6a + 9}{2a - 2} \times \frac{a^2 - 1}{a^2 + 4a + 3}$.

$$\begin{aligned}\text{Solution } \frac{a^2 + 6a + 9}{2a - 2} \times \frac{a^2 - 1}{a^2 + 4a + 3} &= \frac{a^2 + 2 \times a \times 3 + 3^2}{2(a-1)} \times \frac{(a+1)(a-1)}{a^2 + 3a + a + 3} \\&= \frac{(a+3)^2}{2(a-1)} \times \frac{(a+1)(a-1)}{a(a+3) + 1(a+3)} \\&= \frac{(a+3)^2}{2(a-1)} \times \frac{(a+1)(a-1)}{(a+3)(a+1)} = \frac{a+3}{2}.\end{aligned}$$

EXAMPLE 6 Simplify $\frac{x^2 + 7x + 12}{x^2 - 25} \div \frac{x^2 + 9x + 18}{x^2 - 7x + 10}$.

$$\begin{aligned}\text{Solution } \frac{x^2 + 7x + 12}{x^2 - 25} \div \frac{x^2 + 9x + 18}{x^2 - 7x + 10} &= \frac{x^2 + 7x + 12}{x^2 - 25} \times \frac{x^2 - 7x + 10}{x^2 + 9x + 18} \\&= \frac{x^2 + 4x + 3x + 12}{x^2 - 5^2} \times \frac{x^2 - 5x - 2x + 10}{x^2 + 3x + 6x + 18} \\&= \frac{x(x+4) + 3(x+4)}{(x+5)(x-5)} \times \frac{x(x-5) - 2(x-5)}{x(x+3) + 6(x+3)} \\&= \frac{(x+4)(x+3)}{(x+5)(x-5)} \times \frac{(x-5)(x-2)}{(x+3)(x+6)} = \frac{(x+4)(x-2)}{(x+5)(x+6)}.\end{aligned}$$

EXAMPLE 7 Simplify $\left(\frac{4x}{x^2 + 6x + 5} - \frac{3x}{x^2 + 5x + 4} \right)$ of $\frac{x^2 - 16}{x^3 - 7x^2}$.

$$\text{Solution} \quad x^2 + 6x + 5 = x^2 + 5x + x + 5 = x(x+5) + 1(x+5) = (x+5)(x+1).$$

$$x^2 + 5x + 4 = x^2 + 4x + x + 4 = x(x+4) + 1(x+4) = (x+4)(x+1).$$

∴ the LCM of $x^2 + 6x + 5$ and $x^2 + 5x + 4 = (x+1)(x+4)(x+5)$.

$$\therefore \frac{4x}{x^2 + 6x + 5} - \frac{3x}{x^2 + 5x + 4} = \frac{4x}{(x+5)(x+1)} - \frac{3x}{(x+4)(x+1)} = \frac{4x(x+4) - 3x(x+5)}{(x+1)(x+4)(x+5)}$$

$$= \frac{4x^2 + 16x - 3x^2 - 15x}{(x+1)(x+4)(x+5)} = \frac{x^2 + x}{(x+1)(x+4)(x+5)}$$

$$= \frac{x(x+1)}{(x+1)(x+4)(x+5)} = \frac{x}{(x+4)(x+5)}.$$

$$\therefore \left(\frac{4x}{x^2 + 6x + 5} - \frac{3x}{x^2 + 5x + 4} \right) \text{ of } \frac{x^2 - 16}{x^3 - 7x^2} = \frac{x}{(x+4)(x+5)} \text{ of } \frac{(x+4)(x-4)}{x^2(x-7)}$$

$$= \frac{x}{(x+4)(x+5)} \times \frac{(x+4)(x-4)}{x^2(x-7)}$$

$$= \frac{x-4}{x(x+5)(x-7)}.$$

Remember These

1. To reduce an algebraic fraction to the lowest terms, factorize the numerator and the denominator and cancel the factors which are common to both.
2. $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$ and $\frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$.
3. $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$ and $\frac{a}{b} - \frac{c}{d} = \frac{ad-bc}{bd}$.
4. $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d} = \frac{ac}{bd}$, where $b \neq 0, d \neq 0$.
5. $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$, where $b \neq 0, c \neq 0, d \neq 0$.

EXERCISE

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1. Reduce the following to the lowest terms.

$$(i) \frac{24x^4}{4x^2}$$

$$(ii) \frac{20a^3b^2}{-25a^4b}$$

$$(iii) \frac{2xyz}{8x^2z^3}$$

$$(iv) \frac{-25a^2cd^3}{-30ac^2d}$$

$$(v) \frac{x+y}{x^2-y^2}$$

$$(vi) \frac{12a^2(a-b)}{15a(a-b)^2}$$

$$(vii) \frac{6-3x}{x^2-4x+4}$$

$$(viii) \frac{x^2-2x}{x^2-5x+6}$$

$$(ix) \frac{a^2+7a+6}{a^2+10a+24}$$

$$(x) \frac{2m^2+5m+3}{4m^2-4m-15}$$

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2. Express each of the following as a single fraction.

(i) $\frac{3}{x} + \frac{5}{x}$

(ii) $\frac{15}{2m} - \frac{3}{-2m}$

(iii) $\frac{a}{a-6} + \frac{6}{6-a}$

(iv) $\frac{3}{5x} + \frac{1}{10}$

(v) $\frac{4x}{9} - \frac{1}{2x}$

(vi) $\frac{5}{x+2} - \frac{2}{x+1}$

(vii) $\frac{1}{x+3} + \frac{6}{x^2-9}$

(viii) $\frac{x}{3x+9} - \frac{3}{x^2+3x}$

3. Simplify:

(i) $\frac{3x-8}{x^2-16} - \frac{1}{x+4}$

(ii) $\frac{4m}{4m^2-9} - \frac{1}{2m-3}$

(iii) $\frac{x+7}{x-3} - \frac{x-5}{x+2}$

(iv) $\frac{2}{2x^2+5x-3} - \frac{1}{x^2+5x+6}$

(v) $\frac{x+4}{x^2+5x+6} - \frac{x+2}{x^2+7x+12}$

(vi) $\frac{1}{x} - \frac{5}{x^2} - \frac{3}{x^3}$

(vii) $\frac{3}{x+1} - \frac{2}{x-1} + \frac{x+3}{x^2-1}$

(viii) $\frac{m-2}{m^2-3m} + \frac{2m-1}{m^2+3m} - \frac{2}{m^2-9}$

(ix) $\frac{2}{x^2+7x+12} + \frac{3}{x^2+5x+6}$

(x) $\frac{2m+13}{m^2+11m+30} - \frac{m+2}{m^2+9m+20}$

4. Multiply and reduce to the lowest terms.

(i) $\frac{10x^2}{25y^3} \times \frac{15y^2}{2x^4} \times 3x^2$

(ii) $\frac{x^2-16}{x^2-9} \times \frac{x+3}{x-4}$

(iii) $\frac{a^2-b^2}{a^2+2ab+b^2} \times \frac{ab+b^2}{a^2-ab}$

(iv) $\frac{a-3}{a^2+a-6} \times \frac{a^2+5a+6}{a^2+2a-15}$

5. Divide and reduce to the lowest terms.

(i) $\frac{36}{y^6} \div \frac{15}{y^2}$

(ii) $\frac{a-3}{8a} \div \frac{9-a^2}{6a^2}$

(iii) $\frac{m^2+5m+6}{m^2+2m-3} \div \frac{2m^2-8}{m^2-3m+2}$

6. Simplify:

(i) $\left(\frac{2}{3m} + \frac{3}{4m} \right) \div \frac{5}{3m}$

(ii) $\left(\frac{1}{2-x} - \frac{2}{4-x} \right) \text{ of } \frac{x^2-4x}{3}$

ANSWERS

1. (i) $6x^2$

(ii) $-\frac{4b}{5a}$

(iii) $\frac{y}{4xz^2}$

(iv) $\frac{5ad^2}{6c}$

(v) $\frac{1}{x-y}$

(vi) $\frac{4a}{5(a-b)}$

(vii) $\frac{-3}{x-2}$

(viii) $\frac{x}{x-3}$

(ix) $\frac{a+1}{a+4}$

(x) $\frac{m+1}{2m-5}$

2. (i) $\frac{8}{x}$

(ii) $\frac{9}{m}$

(iii) 1

(iv) $\frac{6+x}{10x}$

(v) $\frac{8x^2-9}{18x}$

(vi) $\frac{3x+1}{(x+1)(x+2)}$

(vii) $\frac{1}{x-3}$

(viii) $\frac{x-3}{3x}$

3. (i) $\frac{2(x-2)}{(x+4)(x-4)}$

(ii) $\frac{1}{2m+3}$

(iii) $\frac{17x-1}{(x-3)(x+2)}$

(iv) $\frac{5}{(x+2)(x+3)(2x-1)}$

(v) $\frac{4}{(x+2)(x+4)}$

(vi) $\frac{x^2-5x-3}{x^3}$

(vii) $\frac{2}{x+1}$

(viii) $\frac{3m+1}{m(m+3)}$

(ix) $\frac{5x+16}{(x+2)(x+3)(x+4)}$

(x) $\frac{m+8}{(m+4)(m+6)}$

4. (i) $\frac{9}{y}$

(ii) $\frac{x+4}{x-3}$

(iii) $\frac{b}{a}$

(iv) $\frac{a+2}{(a-2)(a+5)}$

5. (i) $\frac{12}{5y^4}$

(ii) $-\frac{3a}{4(a+3)}$

(iii) $\frac{1}{2}$

6. (i) $\frac{17}{20}$ (ii) $\frac{x^2}{3(x-2)}$

Revision Exercise 5

1. Find the following products.

(i) $(x+y)(x+2)$

(ii) $(x+4)(x-1)$

(iii) $(a+3)(a-7)$

(iv) $(m-2)(m-n)$

(v) $(3x-4y)^2$

(vi) $(2a-3b+4c)^2$

2. Factorize:

(i) $4(x-1)+x(x-1)$ (ii) $9x^2 + 42xy + 49y^2$ (iii) $16a^4 - 81$ (iv) $9x^2 + 37x + 4$

(v) $12a^2 - 7a - 12$ (vi) $12a^2 + 7a - 12$ (vii) $3x^2 - 11x + 6$ (viii) $5 - 14x - 3x^2$

3. Evaluate:

(i) $\frac{(6732)^2 - (6720)^2}{13452}$ (ii) $\frac{15.9 \times 15.9 - 4.1 \times 4.1}{15.9 - 4.1}$ (iii) $\frac{(7.88)^2 - (4.38)^2}{12.26}$ of Rs 2.40

4. (i) If $a + \frac{1}{a} = 4$, prove that $a^2 + \frac{1}{a^2} = 14$ and $a^4 + \frac{1}{a^4} = 194$.

(ii) If $z - \frac{1}{2z} = 7$, prove that $z^2 + \frac{1}{4z^2} = 50$.

5. (i) If $a + b + c = 3\sqrt{3}$ and $a^2 + b^2 + c^2 = 27$, prove that $ab + bc + ca = 0$.

(ii) If $a + b + c = 10$ and $ab + bc + ca = 25$, prove that $a^2 + b^2 + c^2 = 50$.

6. Find the HCF and LCM of the following.

(i) $40a^2b^2x^3y^3$, $50a^5b^5x^4y^4$ and $60a^3b^3x^5y^5$

(ii) $8(2x-1)$, $12(4x^2-1)$ and $20(4x^2-4x+1)$

7. Reduce the following to the lowest terms.

(i) $\frac{81a^2b^2(a^2 - 6ab + 9b^2)}{27a^3b^2 - 81a^2b^3}$ (ii) $\frac{(a^2 - 4ab + 4b^2)(a - 3b)}{a^2 - 5ab + 6b^2}$

8. Simplify the following.

(i) $\frac{x}{x+y} + \frac{y}{x-y}$

(ii) $\frac{a-b}{a+b} + \frac{a+b}{a-b}$

(iii) $\frac{2a+3b}{2a-3b} - \frac{2a-3b}{2a+3b}$

(iv) $\frac{x}{x-y} + \frac{y}{x+y} + \frac{x^2 + y^2}{y^2 - x^2}$

(v) $\frac{x^2 - 24x + 144}{x^2 - 15x + 36} \times \frac{x-3}{x-12}$

(vi) $\frac{a^2 - 81b^2}{a^3 - 9a^2b} \times \frac{a^2}{a+9b}$

(vii) $\left(\frac{x}{x+y} + \frac{y}{x-y} \right) \div \left(\frac{1}{x-y} - \frac{1}{x+y} \right)$

ANSWERS

1. (i) $x^2 + x(y+2) + 2y$ (ii) $x^2 + 3x - 4$ (iii) $a^2 - 4a - 21$ (iv) $m^2 - (2+n)m + 2n$ (v) $9x^2 - 24xy + 16y^2$

(vi) $4a^2 + 9b^2 + 16c^2 - 12ab - 24bc + 16ca$

2. (i) $(x-1)(4+x)$ (ii) $(3x+7y)^2$ (iii) $(4a^2+9)(2a+3)(2a-3)$ (iv) $(9x+1)(x+4)$ (v) $(3a-4)(4a+3)$

(vi) $(4a-3)(3a+4)$ (vii) $(3x-2)(x-3)$ (viii) $(5+x)(1-3x)$

3. (i) 12 (ii) 20 (iii) Rs 8.40

6. (i) HCF = $10a^2b^2x^3y^3$ and LCM = $600a^5b^5x^5y^5$ (ii) HCF = $4(2x-1)$ and LCM = $120(2x-1)^2(2x+1)$

7. (i) $3(a-3b)$ (ii) $a-2b$

8. (i) $\frac{x^2 + y^2}{x^2 - y^2}$ (ii) $\frac{2(a^2 + b^2)}{a^2 - b^2}$ (iii) $\frac{24ab}{4a^2 - 9b^2}$ (iv) $\frac{2y}{x+y}$ (v) 1 (vi) 1 (vii) $\frac{x^2 + y^2}{2y}$

