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5
Factorization

If a given algebraic expression can be expressed as a product of two or more expressions then those expressions are called factors of the given expression.

Examples (i) $6+2x=2(3+x)$, so 2 and $3+x$ are factors of $6+2x$.

(ii) $a^2-b^2=(a+b)(a-b)$, so $a+b$ and $a-b$ are factors of a^2-b^2.

(iii) $(x+1)(x+2)=x^2+3x+2$, so $x+1$ and $x+2$ are factors of x^2+3x+2.

Methods of Factorization

To resolve an expression into its factors is called factorization. The method we use to factorize an expression depends on the type of expression. In this chapter, we will take a look at a few methods of factorization used commonly.

Taking out common factors

Follow these steps when all the terms of a polynomial have common factors.

Steps
1. Find the highest common factor (HCF) of the terms by (i) finding the HCF of the numerical coefficients of the terms, (ii) finding the highest power of each variable common to the terms, and (iii) multiplying all the common factors.
2. Divide each term of the polynomial by this common factor.
3. Write the quotient within brackets and the common factor outside the brackets.

Solved Examples

EXAMPLE 1 **Factorize $4xy-8y^2$.**

Solution $4xy=4\times x\times y$, $8y^2=4\times 2\times y\times y$.

The HCF of the numerical coefficients of the terms = 4.

The highest power of y common to all the terms = y.

$\therefore$ the HCF of the terms = $4y$.

Dividing the expression by the common factor, $\frac{4xy}{4y}-\frac{8y^2}{4y}=x-2y$.

$\therefore$ $4xy-8y^2=4y(x-2y)$.

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EXAMPLE 2 **Factorize $3a^2b^2 - 9ab^3 + 15ab^4$.**

Solution $3a^2b^2 = 3 \times a \times a \times b \times b,\ 9ab^3 = 3 \times 3 \times a \times b \times b \times b,\ 15ab^4 = 3 \times 5 \times a \times b \times b \times b \times b$

The HCF of the numerical coefficients = 3.

The highest common power of $a = a$.

The highest common power of $b = b^2$.

$\therefore$ the HCF of the terms $= 3ab^2$.

$$\therefore\quad 3a^2b^2 - 9ab^3 + 15ab^2 = 3ab^2\left(\frac{3a^2b^2}{3ab^2} - \frac{9ab^3}{3ab^2} + \frac{15ab^4}{3ab^2}\right) = 3ab^2(a - 3b + 5b^2).$$

EXAMPLE 3 **Factorize $2(x+y) + 3(x+y)^2$.**

Solution The HCF of the numerical coefficients = 1.

The highest power of the binomial $(x+y)$ common to both the terms $= x+y$.

$\therefore$ the HCF of the terms $= 1 \times (x+y) = x+y$.

$$\therefore\quad 2(x+y) + 3(x+y)^2 = (x+y)\left\{\frac{2(x+y)}{x+y} + \frac{3(x+y)^2}{x+y}\right\}$$

$$= (x+y)\{2 + 3(x+y)\} = (x+y)(3x+3y+2).$$

EXAMPLE 4 **Factorize $5(3a+2b)^2 - 20(3a+2b)^4 + 60(3a+2b)^5$.**

Solution The HCF of 5, 20 and 60 = 5.

The highest power of the binomial $(3a+2b)$ common to all the terms $= (3a+2b)^2$.

$\therefore$ the HCF of the terms $= 5 \times (3a+2b)^2 = 5(3a+2b)^2$.

$$\therefore\quad 5(3a+2b)^2 - 20(3a+2b)^4 + 60(3a+2b)^5$$

$$= 5(3a+2b)^2\left\{\frac{5(3a+2b)^2 - 20(3a+2b)^4 + 60(3a+2b)^5}{5(3a+2b)^2}\right\}$$

$$= 5(3a+2b)^2\{1 - 4(3a+2b)^2 + 12(3a+2b)^3\}.$$

EXERCISE 5A

Factorize the following.

1. (i) $2a - 4b$ (ii) $8x^2 + 16y^2$
2. (i) $5x^2y - 10xy^2$ (ii) $12a^2b^3 - 16a^3b^2$
3. (i) $39a^2b^3c^4 - 26a^3b^2c$ (ii) $12x^2y^3z^4 + 16xz^5$
4. (i) $3a + 3b + 3c$ (ii) $2x^2 - 10xy + 18x$
   (iii) $5x^3 - 10x^2y + 25xy^2$ (iv) $15x^{10}y^8 + 24x^9y^9 - 27x^{11}y^7$
5. (i) $5x^3y^2z + 10x^2y^3 + 25xy^3$ (ii) $6x^3yz - 8xy^3z + 10xyz^3$
   (iii) $12abc - 6a^2b^2c^2 + 3a^3b^3c^3$
6. (i) $a(5a+6b) - b(5a+6b)$ (ii) $4(x-3y) - 2y(x-3y)$
7. (i) $a(x-y) - b(x-y) + c(x-y)$ (ii) $2x(2a+3b) + 3y(2a+3b) - 4z(2a+3b)$
   (iii) $(a+b)(2a+b) + (3a+2b)(a+b) - (a+b)(a+4b)$
   (iv) $6ab(x^2+y^2+z^2) + 8cd(x^2+y^2+z^2) - 9bc(x^2+y^2+z^2)$

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ANSWERS

1. (i) $2(a-2b)$ (ii) $8(x^2+2y^2)$ **2.** (i) $5xy(x-2y)$ (ii) $4a^2b^2(3b-4a)$

3. (i) $13a^2b^2c(3bc^3-2a)$ (ii) $4xz^4(3xy^3+4z)$

4. (i) $3(a+b+c)$ (ii) $2x(x-5y+9)$ (iii) $5x(x^2-2xy+5y^2)$ (iv) $3x^9y^7(5xy+8y^2-9x^2)$

5. (i) $5xy^2(x^2z+2xy+5y)$ (ii) $2xyz(3x^2-4y^2+5z^2)$ (iii) $3abc(4-2abc+a^2b^2c^2)$

6. (i) $(5a+6b)(a-b)$ (ii) $2(x-3y)(2-y)$

7. (i) $(x-y)(a-b+c)$ (ii) $(2a+3b)(2x+3y-4z)$ (iii) $(a+b)(4a-b)$ (iv) $(x^2+y^2+z^2)(6ab+8cd-9bc)$

Grouping terms

Some algebraic expressions can be factorized by rearranging the terms suitably in pairs such that

(i) the terms in each pair have a common factor and

(ii) when this common factor is taken out, the same expression is left in each pair.

Example We can regroup $ac+bd+ad+bc$ as $(ac+ad)+(bc+bd)$.

Then, taking out the common factor in each pair,

$$ac+bd+ad+bc=a(c+d)+b(c+d).$$

So, $c+d$ is a factor common to both parts of the expression.

Taking $(c+d)$ out, $ac+bd+ad+bc=(c+d)(a+b)$.

Another possible way of grouping the terms of the given expression in order to carry out factorization is as follows.

$$ac+bd+ad+bc=(ac+bc)+(ad+bd)=(a+b)c+(a+b)d=(a+b)(c+d)$$

Note There is more than one possible way of grouping the terms of an expression.

Solved Examples

EXAMPLE 1 **Factorize $a^2+3b+3a+ab$.**

Solution $a^2+3b+3a+ab=(a^2+3a)+(ab+3b)=a(a+3)+b(a+3)=(a+3)(a+b)$.

Alternative way

$a^2+3b+3a+ab=(a^2+ab)+(3a+3b)=a(a+b)+3(a+b)=(a+b)(a+3)$.

EXAMPLE 2 **Factorize $xy-y-x+1$.**

Solution $xy-y-x+1=(xy-y)-(x-1)=y(x-1)-1(x-1)=(x-1)(y-1)$.

Alternative way

$xy-y-x+1=(xy-x)-(y-1)=x(y-1)-1(y-1)=(y-1)(x-1)$.

EXAMPLE 3 **Factorize $8x^2y-4x^2+6y-3$.**

Solution $8x^2y-4x^2+6y-3=(8x^2y-4x^2)+(6y-3)$

$=4x^2(2y-1)+3(2y-1)=(2y-1)(4x^2+3)$.

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Alternative way

$$8x^2y-4x^2+6y-3=(8x^2y+6y)-(4x^2+3)$$
$$=2y(4x^2+3)-1(4x^2+3)=(4x^2+3)(2y-1).$$

EXAMPLE 4 **Factorize $14a+2b^2-14b-2ab$.**

Solution $14a+2b^2-14b-2ab=2(7a+b^2-7b-ab)=2\{(7a-7b)+(b^2-ab)\}$
$=2\{7(a-b)+b(b-a)\}=2\{7(a-b)-b(a-b)\}$
$=2(a-b)(7-b).$

EXAMPLE 5 **Factorize $x+y+z+ax+ay+az$.**

Solution $x+y+z+ax+ay+az=(x+y+z)+(ax+ay+az)$
$=1(x+y+z)+a(x+y+z)=(x+y+z)(1+a).$

EXAMPLE 6 **Factorize $2mn^2+2n^2p+m+4np+4mn+p$.**

Solution $2mn^2+2n^2p+m+4np+4mn+p=(2mn^2+2n^2p)+(m+p)+(4np+4mn)$
$=2n^2(m+p)+1(m+p)+4n(p+m)$
$=(m+p)(2n^2+1+4n)=(m+p)(2n^2+4n+1).$

EXERCISE 5B

Factorize by grouping terms.

1. (i) $ax+ay-bx-by$ (ii) $4p+4q-ap-aq$ (iii) $1+x-y-xy$
   (iv) $ab+5a-b-5$ (v) $3xy+3y+2ax+2a$
2. (i) $a^2-4ax+ab-4bx$ (ii) $x^2-ax-bx+ab$ (iii) $ax-x^2-bx+ab$
   (iv) $m^2+mn+mp+np$
3. (i) $m^2n^2+m^2+n^2+1$ (ii) $5xy^2-5y^2-3a+3ax$ (iii) $-x^2y-x+3xy+3$
   (iv) $3xy^2-6y^2+4x-8$ (v) $2a^2-a^2b-bc^2+2c^2$
4. (i) $6y^3-24y^2+2y-8$ (ii) $2ab+6b-4a-12$ (iii) $3a^2b+12a^2+9b+36$
5. (i) $ax+bx+cx-a-b-c$ (ii) $ax+bx+ay+by+az+bz$
   (iii) $a^2c+c+a+a^3-2a^2b-2b$ (iv) $2mn^2p-2m+2n^3p-2n-n^2p^2+p$

ANSWERS

1. (i) $(x+y)(a-b)$ (ii) $(p+q)(4-a)$ (iii) $(1+x)(1-y)$ (iv) $(b+5)(a-1)$ (v) $(x+1)(3y+2a)$
2. (i) $(a+b)(a-4x)$ (ii) $(x-a)(x-b)$ (iii) $(a-x)(x+b)$ (iv) $(m+n)(m+p)$
3. (i) $(n^2+1)(m^2+1)$ (ii) $(x-1)(5y^2+3a)$ (iii) $(xy+1)(3-x)$ (iv) $(x-2)(3y^2+4)$ (v) $(2-b)(a^2+c^2)$
4. (i) $2(y-4)(3y^2+1)$ (ii) $2(a+3)(b-2)$ (iii) $3(b+4)(a^2+3)$
5. (i) $(a+b+c)(x-1)$ (ii) $(x+y+z)(a+b)$ (iii) $(a^2+1)(c+a-2b)$ (iv) $(n^2p-1)(2m+2n-p)$

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Difference of two squares

The difference of two squares can always be factorized by the formula $a^2 - b^2 = (a+b)(a-b)$.

Solved Examples

EXAMPLE 1 **Factorize $9x^2 - 16$.**

Solution $9x^2 = (3x)^2$ and $16 = 4^2$.

$\therefore \quad 9x^2 - 16 = (3x)^2 - 4^2 = (3x+4)(3x-4)$.

EXAMPLE 2 **Factorize $1 - \frac{36a^2}{49}$.**

Solution $1 - \frac{36}{49}a^2 = 1^2 - \left(\frac{6}{7}a\right)^2 = \left(1 + \frac{6}{7}a\right)\left(1 - \frac{6}{7}a\right)$.

EXAMPLE 3 **Factorize $9x^2 - 16y^2$.**

Solution $9x^2 - 16y^2 = (3x)^2 - (4y)^2 = (3x+4y)(3x-4y)$.

EXAMPLE 4 **Factorize $3x^2 - 75y^2$.**

Solution The HCF of the numerical coefficients in the two terms = 3. So, we take 3 out as the common factor.

$\therefore \quad 3x^2 - 75y^2 = 3(x^2 - 25y^2) = 3\{x^2 - (5y)^2\} = 3(x+5y)(x-5y)$.

EXAMPLE 5 **Factorize $x^2 - (y+z)^2$.**

Solution $x^2 - (y+z)^2 = \{x + (y+z)\}\{x - (y+z)\} = (x+y+z)(x-y-z)$.

EXAMPLE 6 **Factorize $9(a+b)^2 - 25c^2$.**

Solution $9(a+b)^2 - 25c^2 = \{3(a+b)\}^2 - (5c)^2 = \{3(a+b)+5c\}\{3(a+b)-5c\}$

$= (3a+3b+5c)(3a+3b-5c)$.

EXAMPLE 7 **Factorize $9(a+b)^3 - 16(a+b)$.**

Solution $9(a+b)^3 - 16(a+b) = (a+b)\{9(a+b)^2 - 16\} = (a+b)[\{3(a+b)\}^2 - 4^2]$

$= (a+b)[\{3(a+b)+4\}\{3(a+b)-4\}]$

$= (a+b)(3a+3b+4)(3a+3b-4)$.

EXAMPLE 8 **Factorize $16a^2 - b^2 + 4b - 4$.**

Solution $16a^2 - b^2 + 4b - 4 = 16a^2 - (b^2 - 4b + 4) = 16a^2 - (b^2 - 2 \times b \times 2 + 2^2)$

$= (4a)^2 - (b-2)^2 = \{4a + (b-2)\}\{4a - (b-2)\}$

$= (4a+b-2)(4a-b+2)$.

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EXAMPLE 9 **Factorize $x^4 - \frac{16}{81}$.**

Solution $x^4 - \frac{16}{81} = (x^2)^2 - \left(\frac{4}{9}\right)^2 = \left(x^2 + \frac{4}{9}\right)\left(x^2 - \frac{4}{9}\right)$

$= \left(x^2 + \frac{4}{9}\right)\left\{x^2 - \left(\frac{2}{3}\right)^2\right\} = \left(x^2 + \frac{4}{9}\right)\left(x + \frac{2}{3}\right)\left(x - \frac{2}{3}\right)$.

EXAMPLE 10 **Factorize $2a^4 - 32b^4$.**

Solution $2a^4 - 32b^4 = 2(a^4 - 16b^4) = 2\{(a^2)^2 - (4b^2)^2\} = 2(a^2 + 4b^2)(a^2 - 4b^2)$

$= 2(a^2 + 4b^2)\{a^2 - (2b)^2\} = 2(a^2 + 4b^2)(a + 2b)(a - 2b)$.

EXAMPLE 11 **Find the value of $(168)^2 - (132)^2$.**

Solution $(168)^2 - (132)^2 = (168 + 132)(168 - 132) = 300 \times 36 = 10800$.

EXAMPLE 12 **Find the value of $\frac{6.4 \times 6.4 - 3.6 \times 3.6}{6.4 - 3.6}$.**

Solution $\frac{6.4 \times 6.4 - 3.6 \times 3.6}{6.4 - 3.6} = \frac{(6.4)^2 - (3.6)^2}{6.4 - 3.6} = \frac{(6.4 + 3.6)(6.4 - 3.6)}{(6.4 - 3.6)} = 6.4 + 3.6 = 10.$

EXERCISE 5C

Factorize the following.

1. (i) $4 - 9a^2$ (ii) $\frac{25}{36}x^2 - 49$ (iii) $\frac{1}{4}m^2 - \frac{9}{49}$
2. (i) $16a^2 - 81b^2$ (ii) $144m^2 - 169n^2$ (iii) $\frac{1}{9}x^2 - \frac{1}{16}y^2$ (iv) $\frac{9}{25}a^2 - \frac{4}{9}b^2$
3. (i) $2x^2 - 32$ (ii) $48 - \frac{27}{49}y^2$ (iii) $6a^2 - 54b^2$
4. (i) $64x^3y^2 - 81xz^2$ (ii) $6a^3 - 54ab^2$ (iii) $28x^3 - 63x$
5. (i) $4x^2 - 81(y + z)^2$ (ii) $(a - 5)^2 - 36$ (iii) $(x + 2y)^2 - 16z^2$ (iv) $(a + b)^2 - (a - b)^2$ (v) $\frac{1}{4}(m + n)^2 - \frac{25}{49}(x + y)^2$
6. (i) $50(x + y)^2 - 32(x - y)^2$ (ii) $4(x + y)^3 - 25(x + y)$
7. (i) $a^2 - b^2 - 2b - 1$ (ii) $m^2 + 4m + 4 - n^2$
8. (i) $a^4 - b^4$ (ii) $x^4 - 81$ (iii) $16m^4 - 81n^4$
9. (i) $48m^4 - 3$ (ii) $32xy^4 - 2x^5$

Evaluate the following.

10. $(59)^2 - (49)^2$
11. $(7.9)^2 - (2.1)^2$
12. $\left(6\frac{2}{5}\right)^2 - \left(3\frac{3}{5}\right)^2$

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13. $(1.892)^2 - (1.108)^2$ **14.** $\dfrac{7.8 \times 7.8 - 2.2 \times 2.2}{7.8 - 2.2}$ **15.** $\dfrac{2.5 \times 2.5 - 1.5 \times 1.5}{2.5 - 1.5}$

ANSWERS

1. (i) $(2+3a)(2-3a)$ (ii) $\left(\frac{5}{6}x+7\right)\left(\frac{5}{6}x-7\right)$ (iii) $\left(\frac{1}{2}m+\frac{3}{7}\right)\left(\frac{1}{2}m-\frac{3}{7}\right)$

2. (i) $(4a+9b)(4a-9b)$ (ii) $(12m+13n)(12m-13n)$ (iii) $\left(\frac{1}{3}x+\frac{1}{4}y\right)\left(\frac{1}{3}x-\frac{1}{4}y\right)$

(iv) $\left(\frac{3}{5}a+\frac{2}{3}b\right)\left(\frac{3}{5}a-\frac{2}{3}b\right)$

3. (i) $2(x+4)(x-4)$ (ii) $3\left(4+\frac{3}{7}y\right)\left(4-\frac{3}{7}y\right)$ (iii) $6(a+3b)(a-3b)$

4. (i) $x(8xy+9z)(8xy-9z)$ (ii) $6a(a+3b)(a-3b)$ (iii) $7x(2x+3)(2x-3)$

5. (i) $(2x+9y+9z)(2x-9y-9z)$ (ii) $(a+1)(a-11)$ (iii) $(x+2y+4z)(x+2y-4z)$ (iv) $4ab$

(v) $\left\{\frac{1}{2}(m+n)+\frac{5}{7}(x+y)\right\}\left\{\frac{1}{2}(m+n)-\frac{5}{7}(x+y)\right\}$

6. (i) $2(9x+y)(x+9y)$ (ii) $(x+y)(2x+2y+5)(2x+2y-5)$

7. (i) $(a+b+1)(a-b-1)$ (ii) $(m+n+2)(m-n+2)$

8. (i) $(a^2+b^2)(a+b)(a-b)$ (ii) $(x^2+9)(x+3)(x-3)$ (iii) $(4m^2+9n^2)(2m+3n)(2m-3n)$

9. (i) $3(4m^2+1)(2m+1)(2m-1)$ (ii) $2x(4y^2+x^2)(2y+x)(2y-x)$

10. 1080 **11.** 58 **12.** 28 **13.** 2.352 **14.** 10 **15.** 4

Trinomials of the type ax^2+bx+c

There can be two cases:

1. The coefficient of x^2 is unity, that is, the expression is of the form x^2+bx+c.
2. The coefficient of x^2 is not unity.

Case I To factorize an expression of the type x^2+bx+c, we find two factors p and q of c such that

$$pq = c \text{ and } p+q = b.$$

$$\text{Then } x^2+bx+c = x^2+(p+q)x+pq = x^2+px+qx+pq$$
$$= x(x+p)+q(x+p) = (x+p)(x+q).$$

EXAMPLE **Factorize $x^2+7x+12$.**

Solution We have to find a pair of factors of 12 such that their product is 12 and their sum is 7.

The possible pairs of factors of 12 = (1, 12), (2, 6), (3, 4).

By inspection we find that the product and the sum of the factors 3 and 4 are equal to 12 and 7 respectively.

$\therefore \quad x^2+7x+12 = x^2+3x+4x+12 = x(x+3)+4(x+3) = (x+3)(x+4).$

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Case II To factorize an expression of the form $ax^2 + bx + c$, we find two factors p and q of the product ac (the product of the coefficient of x^2 and the constant term) such that

$$pq = ac \text{ and } p + q = b.$$

EXAMPLE **Factorize $3x^2 + 11x + 10$.**

Solution The coefficient of $x^2 = 3$ and the constant term = 10.

Their product $= 3 \times 10 = 30$.

The possible factors of $30 = (1, 30), (2, 15), (3, 10), (5, 6)$.

By inspection, (5, 6) is the pair of factors such that $5 \times 6 = 30$ and $5 + 6 = 11$.

$\therefore \quad 3x^2 + 11x + 10 = 3x^2 + 5x + 6x + 10 = x(3x + 5) + 2(3x + 5) = (3x + 5)(x + 2)$.

Alternatively

$3x^2 + 11x + 10 = 3x^2 + 6x + 5x + 10 = 3x(x + 2) + 5(x + 2) = (x + 2)(3x + 5)$.

Solved Examples

EXAMPLE 1 **Factorize $x^2 - 18x + 45$.**

Solution We have to find two numbers such that their product is 45 and sum is –18.

By inspection, $(-3) \times (-15) = 45$ and $(-3) + (-15) = -18$.

$\therefore \quad x^2 - 18x + 45 = x^2 - 3x - 15x + 45 = x(x - 3) - 15(x - 3) = (x - 3)(x - 15)$.

EXAMPLE 2 **Resolve into factors the expression $x^2 + 2x - 24$.**

Solution We have to find two numbers such that their product is –24 and sum is 2.

The numbers are 6 and –4 since $6 \times (-4) = -24$ and $6 + (-4) = 2$.

$\therefore \quad x^2 + 2x - 24 = x^2 + 6x - 4x - 24 = x(x + 6) - 4(x + 6) = (x + 6)(x - 4)$.

EXAMPLE 3 **Factorize $m^2 - 21m - 100$.**

Solution We have to find two numbers such that their product is –100 and sum equals –21.

By trial, we find that $(-25) \times 4 = -100$ and $-25 + 4 = -21$.

$$\therefore \quad m^2 - 21m - 100 = m^2 - 25m + 4m - 100$$
$$= m(m - 25) + 4(m - 25) = (m - 25)(m + 4).$$

EXAMPLE 4 **Factorize the trinomial $4x^2 - 20x + 9$.**

Solution The coefficient of $x^2 = 4$ and the constant term = 9.

The product of the two $= 4 \times 9 = 36$.

We have to find two numbers such that their product = 36 and sum = –20.

We find that $(-18) \times (-2) = 36$ and $(-18) + (-2) = -20$.

$\therefore \quad 4x^2 - 20x + 9 = 4x^2 - 18x - 2x + 9 = 2x(2x - 9) - 1(2x - 9) = (2x - 9)(2x - 1)$.

EXAMPLE 5 **Factorize $5x^2 + 7x - 6$.**

Solution The coefficient of $x^2 = 5$ and the constant term = –6.

The product of the two $= 5 \times (-6) = -30$.

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We have to find two numbers such that their product = –30 and sum = 7.
We find that $10 \times (-3) = -30$ and $10 + (-3) = 7$.
$\therefore \quad 5x^2 + 7x - 6 = 5x^2 + 10x - 3x - 6 = 5x(x+2) - 3(x+2) = (x+2)(5x-3)$.

EXAMPLE 6 **Resolve into factors the expression $3x^2 - 7x - 6$.**

Solution The product of the coefficient of x^2 and the constant term $= 3 \times (-6) = -18$.
We have to find two numbers such that their product = –18 and sum = –7.
We observe that $(-9) \times 2 = -18$ and $(-9) + 2 = -7$.
$\therefore \quad 3x^2 - 7x - 6 = 3x^2 - 9x + 2x - 6 = 3x(x-3) + 2(x-3) = (x-3)(3x+2)$.

EXAMPLE 7 **Factorize the trinomial $2x^2 + 15xy + 27y^2$.**

Solution Here, $2 \times 27 = 54$.
We have to find two numbers such that their product = 54 and sum = 15.
We find that $9 \times 6 = 54$ and $9 + 6 = 15$.
$\therefore \quad 2x^2 + 15xy + 27y^2 = 2x^2 + 9xy + 6xy + 27y^2$
$= x(2x+9y) + 3y(2x+9y) = (2x+9y)(x+3y)$.

EXAMPLE 8 **Factorize $8(a+b)^2 + 14(a+b) + 3$.**

Solution Let $a + b = x$.
Then $8(a+b)^2 + 14(a+b) + 3 = 8x^2 + 14x + 3$.
We have to find two numbers such that their product $= 8 \times 3 = 24$, and their sum = 14.
We find that $12 \times 2 = 24$ and $12 + 2 = 14$.
$\therefore \quad 8x^2 + 14x + 3 = 8x^2 + 12x + 2x + 3 = 4x(2x+3) + 1(2x+3)$
$= (2x+3)(4x+1) = \{2(a+b)+3\}\{4(a+b)+1\} \quad [\because x = a+b]$
$= (2a+2b+3)(4a+4b+1)$.

EXERCISE 5D

Find the factors of the following trinomials.

1. (i) $x^2 + 5x + 6$ (ii) $x^2 + 3x + 2$ (iii) $x^2 + 16x + 39$ (iv) $a^2 + 13a + 40$
2. (i) $x^2 - 5x + 4$ (ii) $x^2 - 22x + 40$
3. (i) $x^2 + 3x - 28$ (ii) $x^2 + 13x - 48$
4. (i) $x^2 - 6x - 27$ (ii) $m^2 - 2m - 63$
5. (i) $5x^2 + 17x + 6$ (ii) $2x^2 + 11x + 15$ (iii) $4x^2 + 5x + 1$ (iv) $6a^2 + 19a + 10$
6. (i) $12x^2 - 11x + 2$ (ii) $3x^2 - 10x + 8$
7. (i) $8m^2 + 10m - 3$ (ii) $5t^2 + 7t - 6$
8. (i) $6x^2 - x - 15$ (ii) $12x^2 - 19x - 10$
9. (i) $x^2 + 5xy + 4y^2$ (ii) $12a^2 - 19ab + 7b^2$
10. (i) $5(x+2)^2 + 6(x+2) + 1$ (ii) $2(m+2n)^2 + 5(m+2n) + 2$

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ANSWERS

1. (i) $(x+3)(x+2)$ (ii) $(x+2)(x+1)$ (iii) $(x+13)(x+3)$ (iv) $(a+8)(a+5)$

2. (i) $(x-4)(x-1)$ (ii) $(x-20)(x-2)$

3. (i) $(x+7)(x-4)$ (ii) $(x+16)(x-3)$

4. (i) $(x-9)(x+3)$ (ii) $(m-9)(m+7)$

5. (i) $(5x+2)(x+3)$ (ii) $(x+3)(2x+5)$ (iii) $(4x+1)(x+1)$ (iv) $(3a+2)(2a+5)$

6. (i) $(3x-2)(4x-1)$ (ii) $(3x-4)(x-2)$

7. (i) $(2m+3)(4m-1)$ (ii) $(t+2)(5t-3)$

8. (i) $(2x+3)(3x-5)$ (ii) $(x-2)(12x+5)$

9. (i) $(x+4y)(x+y)$ (ii) $(12a-7b)(a-b)$

10. (i) $(5x+11)(x+3)$ (ii) $(m+2n+2)(2m+4n+1)$

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