2

Formulae

A formula is a relation between certain quantities, expressed with the help of variables and mathematical symbols.

Examples (i) "When a number is multiplied by 7 and 20 is deducted from the product, the result is 5 more than twice the number." If we denote the number by x, we can represent this statement by the formula, $7x - 20 = 2x + 5$.

(ii) The volume (V) of a cuboid is the product of its length (l), breadth (b) and height (h). This can be expressed by the formula $V = lbh$.

(iii) The area (A) of a rectangle is the product of its length (l) and its breadth (b). This can be expressed as $A = l \times b$.

Framing a formula

Steps 1. Use variables such as a, b, x, y, A, R and T for the quantities for which you want to frame the formula. Certain symbols are traditionally used to denote certain variables.

Examples (i) V is used for volume, A for area, l for length, b for breadth, h for height and S for surface area in mensuration.

(ii) v and u are used for speed, s for distance, and t for time in physics.

(iii) P is used for principal, A for amount, R for rate, I for interest and T for time in arithmetic.

2. Use the rules or conditions relevant to the context to establish a relationship between the variables.

Solved Examples

EXAMPLE 1 **Ram is 22 years older than his son Rohit. After 8 years his age will be 5 years more than twice the age of Rohit. Frame a formula for these statements.**

Solution Let Rohit's present age = x years.

Then Ram's age = $(x + 22)$ years.

After 8 years, Rohit's age = $(x + 8)$ years

and Ram's age = $[(x + 22) + 8]$ years = $(x + 30)$ years.

From the question, $x + 30 = 2(x + 8) + 5$.

This is the required formula.

EXAMPLE 2 **In a two-digit number, the digit in the tens place exceeds the digit in the units place by 5. Write a formula for the number.**

Solution Let the digit in the units place be x.

Then the digit in the tens place = $x + 5$.

$\therefore$ the number = $10 \times$ digit in tens place + digit in units place

$$= 10(x+5) + x = 11x + 50.$$

Remember These

1. A formula is a relation of equality or inequality between two or more quantities (or variables).
2. To frame a formula for a statement, we use literals (or variables) to represent the quantities concerned and express the relation between the quantities by an equality (or inequality).

EXERCISE 2A

1. Frame a formula for each of the following statements.
   (i) If 4 is subtracted from twice a certain integer n, the result is greater than the integer by 6.
   (ii) In a two-digit number the digit in the tens place is 2 more than the digit in the units place. The number is seven times the sum of its digits.
   (iii) The sum of three consecutive even numbers equals four times the smallest of the numbers (n).
   (iv) A man's age is 20 years more than that of his son and the sum of their ages is 80 years.
   (v) Sam is x years old. In 3 years, he will be thrice as old as he is now.
   (vi) Annie is x years old. Aru is twice as old as her and Kris is 3 years younger than her. The sum of their ages is 20 years.
2. Express the following as formulae.
   (i) The area A of a triangle is half of the product of its base b and altitude h.
   (ii) The larger of two supplementary angles measures 30° more than the smaller angle, which measures $x°$.
   (iii) The length of a rectangle is 2 m less than three times its width and its perimeter is six times its width.
3. Sachin has an average score of 62 runs in x innings and an average of 58 runs in y innings. Find his average score A for x and y innings.
4. Ravi earns Rs x per day on weekdays and Rs 50 more than the normal rate when he works on Sundays. Frame a formula for his earnings E in a 30-day month of 4 Sundays when he works on 2 Sundays in addition to weekdays.
5. Ramesh earns a profit of Rs 200 by selling 16 toys at the rate of Rs x per toy. If the cost price of each toy is Rs 50, frame a formula for the profit.
6. Samir bought a pencil and a sharpener for Rs 10. The pencil cost Rs 2 less than half the price of the sharpener. Frame a formulae to express this, taking the cost of the sharpener to be Rs x.

7. In all, 200 tickets were sold for a charity show. Adult tickets cost Rs 50 each and student tickets cost Rs 20 each. If the number of adult tickets sold was x, construct a formula for the income I (in rupees) from the show.

8. A man weighing 85 kg steps on a weighing scale while carrying a briefcase. If the scale reads x kg, frame a formula for the weight W (in kg) of the briefcase.

ANSWERS

1. (i) $2n - 4 = n + 6$ (ii) $10(a+2) + a = 7\{(a+2) + a\}$ (iii) $n + (n+2) + (n+4) = 4n$ (iv) $(x + 20) + x = 80$
(v) $x + 3 = 3x$ (vi) $2x + x + (x - 3) = 20$

2. (i) $A = \frac{1}{2}bh$ (ii) $(x + 30°) + x = 180°$ (iii) $2\{(3x - 2) + x\} = 6x$

3. $A = \dfrac{62x + 58y}{x + y}$ runs **4.** $E =$ Rs $[26x + 2(x + 50)]$ **5.** $16x - 50 \times 16 = 200$

6. $\left(\frac{x}{2} - 2\right) + x = 10$ **7.** $I = 50x + 20(200 - x)$ **8.** $W = x - 85$

Subject of a formula

When one quantity (or variable) is expressed in terms of other quantities (or variables), the quantity (or variable) thus expressed is called the subject of the formula.

Examples (i) The area (A) of a square of side a is $A = a^2$.
Here, A is the subject of the formula.

(ii) If P, l and b denote the perimeter, length and breadth of a rectangle respectively then $P = 2(l + b)$.
Here, P is the subject of the formula.

Changing the subject of a formula

In the formula $V = l \times b \times h$ for the volume of a cuboid, the subject is V. To express l in terms of the volume V, breadth b and height h, we write the formula as

$$l = \frac{V}{b \times h}.$$

The subject of the formula is now l.

This process of transforming a formula is called changing the subject of the formula.

EXAMPLE **Change the subject of the formula $v = u + ft$ to (i) f and (ii) t.**

Solution (i) Given, $v = u + ft$.

$\therefore \quad v - u = ft$ or $f = \dfrac{v - u}{t}$.

Now, f is the subject of the formula.

(ii) Again, $ft = v - u$ or $t = \frac{v-u}{f}$.

Here, t is the subject of the formula.

Substitution

The process of replacing the variables in an expression by numbers is called substitution. The value thus obtained is called the value of the expression for those values of the variables.

EXAMPLE **If $x = 1$, $y = -2$, $z = 3$ then find the value of $x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$.**

Solution Substituting the given values of x, y and z in the given expression,

$$x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$$
$$= 1^2 + (-2)^2 + 3^2 + 2 \times 1 \times (-2) + 2 \times (-2) \times 3 + 2 \times 3 \times 1$$
$$= 1 + 4 + 9 - 4 - 12 + 6 = 4.$$

Substitution in a formula

To find the value of an unknown variable from a formula when the values of the other variables are given, take the following steps.

Steps
1. Make the unknown variable the subject of the formula.
2. Substitute the values of the known variables in the formula and find the value of the unknown variable.

Solved Examples

EXAMPLE 1 **(i) The interest I is equal to the product of the principal P, rate of interest R and time T divided by 100. Frame a formula for this statement.**

(ii) Change the subject of the formula to R.

(iii) Find the value of I when P = Rs 6000, R = 6% and T = 3 years.

Solution (i) The given statement can be expressed by the formula $I = \frac{P \times R \times T}{100}$.

(ii) $I = \frac{P \times R \times T}{100}$

$\therefore$ $100I = P \times R \times T$ or $R = \frac{100I}{P \times T}$.

(iii) Substituting P = Rs 6000, R = 6%, T = 3 years in the original formula,

$$I = \frac{P \times R \times T}{100} = \text{Rs } \frac{6000 \times 6 \times 3}{100} = \text{Rs } 1080.$$

EXAMPLE 2 **(i) The area A of a circular ring is π times the difference between the squares of its outer radius R and inner radius r. Write a formula to express this statement.**

(ii) Change the subject of the formula to R.

(iii) Find the area of a ring of R = 5 m and r = 2 m.

Solution (i) The given statement can be expressed as $A = \pi(R^2 - r^2)$.

(ii) Given, $A = \pi(R^2 - r^2)$.

$\therefore \quad \frac{A}{\pi} = R^2 - r^2$ or $R^2 = r^2 + \frac{A}{\pi}$.

$\therefore \quad R = \sqrt{r^2 + \frac{A}{\pi}}$.

Here, R is the subject.

(iii) Given that $R = 5$ m, $r = 2$ m.

$\therefore \quad A = \pi(R^2 - r^2) = \frac{22}{7}(5^2 - 2^2)\text{ m}^2 = \frac{22}{7} \times 21\text{ m}^2 = 66\text{ m}^2$.

EXAMPLE 3 **In the formula $s = ut + \frac{1}{2}ft^2$, make u the subject. Also, find u when $t = 2$, $s = 68$ and $f = 32$.**

Solution Given, $s = ut + \frac{1}{2}ft^2$.

$\therefore \quad ut = s - \frac{1}{2}ft^2$ or $u = \frac{1}{t}\left(s - \frac{1}{2}ft^2\right)$.

Thus, $u = \frac{s}{t} - \frac{1}{2}ft$. This is the required formula.

Given that $t = 2$, $s = 68$, $f = 32$.

$\therefore \quad u = \frac{s}{t} - \frac{1}{2}ft = \frac{68}{2} - \frac{1}{2} \times 32 \times 2 = 34 - 32 = 2$.

EXAMPLE 4 **(i) The average A of three numbers, m_1, m_2 and m_3 is given by the sum of the three numbers divided by 3. Construct a formula for this statement.**

(ii) Change the subject to m_1.

(iii) Find m_1 if $A = 44$, $m_2 = 42$, $m_3 = 50$.

Solution (i) The given statement can be expressed by the formula $A = \frac{m_1 + m_2 + m_3}{3}$.

(ii) $A = \frac{m_1 + m_2 + m_3}{3}$.

$\therefore \quad 3A = m_1 + m_2 + m_3$.

So, $m_1 = 3A - m_2 - m_3$.

(iii) Given that $A = 44$, $m_2 = 42$, $m_3 = 50$.

$\therefore \quad m_1 = 3A - m_2 - m_3 = 3 \times 44 - 42 - 50 = 132 - 92 = 40$.

EXAMPLE 5 **(i) Make m the subject of the formula $\frac{m}{a} = \frac{b}{m}$.**

(ii) If $a = 9$ and $b = 4$, find the positive value of m.

Solution (i) Given that $\frac{m}{a} = \frac{b}{m}$.

Multiplying both sides by am,

$am \times \frac{m}{a} = am \times \frac{b}{m}$ or $m^2 = ab$.

$\therefore \quad m = \sqrt{ab}$.

(ii) Given that $a = 9$, $b = 4$.

$\therefore \quad m = \sqrt{ab} = \sqrt{9 \times 4} = 6.$

EXAMPLE 6 **If $x = \dfrac{3l + m}{5l + 2m}$, find m when $x = 3$ and $l = 10$.**

Solution To find m, we have to change the subject of the given formula to m.

Given, $x = \dfrac{3l + m}{5l + 2m}$.

$\therefore \quad (5l + 2m)x = 3l + m$ [multiplying both sides by $(5l + 2m)$]

or $5lx + 2mx = 3l + m$ or $2mx - m = 3l - 5lx$

or $m(2x - 1) = l(3 - 5x)$ or $m = \dfrac{l(3 - 5x)}{2x - 1}$.

Substituting the values of x and l,

$$m = \frac{10 \times (3 - 5 \times 3)}{2 \times 3 - 1} = \frac{10 \times (3 - 15)}{5} = 2 \times (-12) = -24.$$

Remember These

1. To change the subject of a formula, use the laws of equality that you learnt in the previous class.
2. To find the value of an expression, replace the variables by their given values and simplify.
3. To find the value of the subject of a formula for given values of the other variables, substitute the given values in the formula and simplify.

EXERCISE 2B

1. Change the subject of each formula to the corresponding variable given against it.

(i) $A = P + I$; P (ii) $A = 2a + 3b$; a (iii) $v^2 = u^2 + 2fs$; f (iv) $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$; u

(v) $A = \pi r^2$; r (vi) $T = 2\pi\sqrt{\dfrac{l}{g}}$; l (vii) $s = u + \dfrac{1}{2}f(2n - 1)$; n

(viii) $m = \dfrac{2c - 3d}{c + 2d}$; c (ix) $a = \sqrt{2b}$; b

2. Find the value of each of the following algebraic expressions (by substitution).

(i) $2x^2 - 3xy + y^2$, when $x = 2$, $y = -3$

(ii) $2a^2 - 3bc + 4ca - b^2$, when $a = 4$, $b = 5$, $c = -6$

(iii) $-5x^2 + 6y^2 - 3z^2 - 24xyz$, when $x = 2$, $y = -3$, $z = 4$

(iv) $7abc - 2ab + 4bc - 3ac + 2a^2$, when $a = 1$, $b = 2$, $c = 3$

(v) $3y^2 - 5x^2 - 7z^2 - 2xz + 2yz + 6xy$, when $x = -2$, $y = -1$, $z = 3$

3. Evaluate:

(i) $\dfrac{a-3b}{a^2+b^2+1}-\dfrac{2a+b-c}{a+b+c}$, if $a=-2$, $b=4$ and $c=-5$

(ii) $\dfrac{3pq-7qr+7rp}{-15p+7pr-8qr}$, if $p=15$, $q=-5$ and $r=0$

4. From the formula $S=\dfrac{a+b+c}{2}$, find a when $S=25$, $b=12$ and $c=10$.

5. Make R the subject of the formula $A=P+\dfrac{PRT}{100}\cdot$ Also, find R when $A=1600$, $P=1000$ and $T=5$.

6. Make x the subject of $z=\dfrac{2xy}{x+y}$, and find x when $y=5$ and $z=8$.

7. Change the subject of the formula $R=\sqrt{\dfrac{3V}{\pi h}}$ to h, and find h when $R=3$, $V=66$ and $\pi=\dfrac{22}{7}\cdot$

8. Change the subject of the formula $A=2\pi r(r+h)$ to h, and find h when $A=88$, $r=2$ and $\pi=\dfrac{22}{7}\cdot$

9. Change the subject of $s=u+\dfrac{1}{2}f(2x-1)$ to f, and find f when $s=49$, $u=1$ and $x=2$.

10. Use the formula $E=V+IR$ to find I when $E=25$, $V=10$ and $R=6$.

11. Change the subject of $2a=\dfrac{x+y}{x-y}$ to x, and find x when $y=15$ and $a=3$.

12. Change the subject of $x=\dfrac{a-b}{a-5}$ to a, and find a if $x=4$ and $b=2$.

ANSWERS

1. (i) $P=A-I$ (ii) $a=\dfrac{1}{2}(A-3b)$ (iii) $f=\dfrac{1}{2s}(v^2-u^2)$ (iv) $u=\dfrac{vf}{f-v}$ (v) $r=\sqrt{\dfrac{A}{\pi}}$ (vi) $l=\dfrac{T^2g}{4\pi^2}$ (vii) $n=\dfrac{1}{2f}[f+2(s-u)]$ (viii) $c=\dfrac{d(3+2m)}{2-m}$ (ix) $b=\dfrac{1}{2}a^2$

2. (i) 35 (ii) 1 (iii) 562 (iv) 55 (v) −62

3. (i) 1 (ii) 1

4. 28

5. $R=\dfrac{100\times(A-P)}{P\times T}$; 12

6. $x=\dfrac{yz}{2y-z}$; 20

7. $h=\dfrac{3V}{\pi R^2}$; 7

8. $h=\dfrac{A}{2\pi r}-r$; 5

9. $f=\dfrac{2(s-u)}{2x-1}$; 32

10. 2.5

11. $x=\dfrac{(2a+1)y}{2a-1}$; 21

12. $a=\dfrac{5x-b}{x-1}$; 6

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Revision Exercise 2

1. Identify the polynomials and specify their degrees.

   (i) $6m^3 - 5m + \frac{3}{m}$ (ii) $p^2 + \sqrt{2}p - 7$

   (iii) $9x^2y - 19xyz + \frac{6}{\sqrt{7}}xy^2z^2 - 13x^2y^2z^7$ (iv) $\frac{2}{\sqrt{3}}abc - \frac{6}{5}a^2b + 7ab^2 - 8\sqrt{3}b^5c^2 + \sqrt{11}$

2. By how much does $11x^6 - 7x^5 + 4x^3 - 5$ exceed $-2x^6 + 6x^4 - 3x^3 + 2x^2 + 7x$?
3. The perimeter of a rectangle is $2a - 4b + 6c$ and its length is $2a - 3b + 5c$. Find its breadth.
4. Multiply $2m^3 - 6m^2 + 7m - 9$ by $3 - 2m + 4m^2$.
5. The length and the breadth of a rectangle are $(2x+1)$ units and $(x+y+1)$ units respectively. Find its area.
6. Divide $8x^3 - 12x^2 + 6x - 1$ by $4x^2 - 4x + 1$.
7. Divide $10x^4 - 19x^3 + 17x^2 + 12x - 36$ by $5x^2 - 2x - 7$.
8. The area of a rectangle is $2x^4 + x^3 - 31x^2 + 7x + 30$ and its length is $x^2 + 3x - 5$. Find its breadth.
9. Simplify $6x - 2[5(2z+3x) - 3\{4x - 7(2y - \overline{3x + 7z - 8y})\}]$.
10. A man buys a car for Rs P and sells it for Rs Q. Find a formula for his loss percentage.
11. The average age of x boys in a class is m years. If a boy of age y years joins the class, find the formula for the average age of the class.
12. Make l the subject of the formula $T = 2\pi\sqrt{\frac{l}{g}}$. Find l when $g = 980$ cm/s^2, $\pi = \frac{22}{7}$ and $T = \frac{33}{14}$ s.
13. Make h the subject of the formula $A = \frac{1}{2}bh$. Find h when $A = 357$ cm^2 and $b = 21$ cm.

ANSWERS

1. (i) not a polynomial (ii) polynomial of degree 2 (iii) polynomial of degree 11 (iv) polynomial of degree 7
2. $13x^6 - 7x^5 - 6x^4 + 7x^3 - 2x^2 - 7x - 5$
3. $-a + b - 2c$
4. $8m^5 - 28m^4 + 46m^3 - 68m^2 + 39m - 27$
5. $2x^2 + 2xy + 3x + y + 1$
6. quotient $= 2x - 1$, remainder $= 0$
7. quotient $= 2x^2 - 3x + 5$, remainder $= x - 1$
8. $2x^2 - 5x - 6$
9. $126x - 420y + 274z$
10. loss% $= \frac{P - Q}{P} \times 100$
11. average age $= \left(\frac{mx + y}{x + 1}\right)$ years
12. $l = \frac{T^2 g}{4\pi^2}$, $l = 137.81$ cm
13. $h = \frac{2A}{b}$, $h = 34$ cm

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