1

Fundamental Concepts and Operations

Fundamental Concepts

You are already familiar with the basic terms and concepts of algebra. Let us recall what you have learnt in previous classes.

Literals and constants

A symbol, such as a, b, c, x, y and z, representing an unspecified number or member of a class of objects is called a literal or a variable.

A symbol which has a fixed value is known as a constant. Numbers such as $36, 0, \frac{2}{3}, 5.6$ and -2 have a specific value, so they are constants.

Operations on literals

Since literals represent numbers, we can add, subtract, multiply and divide literals and numbers.

Addition (i) $x+y$ is the sum (or addition) of two literals x and y.

(ii) $x+x=2, x+x+x=3x$, etc.

(iii) $x+0=x, 0+x=x$.

Subtraction (i) $a-b$ means b subtracted from a.

(ii) $a-a=0$.

(iii) 5 subtracted from a is $a-5$.

Multiplication (i) ab means $a \times b$ or a multiplied by b.

(ii) $5x$ means $5 \times x$ or 5 multiplied by x.

(iii) $1 \cdot x = x$.

Division (i) $a \div b$ or $\frac{a}{b}$ means a divided by b.

(ii) $a \div 3$ or $\frac{a}{3}$ means a divided by 3.

(iii) $6 \div a$ or $\frac{6}{a}$ means 6 divided by a.

(iv) $\frac{x}{x}=1$.

(v) $\frac{x}{1}=x$.

Laws of operations

All the laws of addition and multiplication of numbers are valid for operations on literals.

1. $a+b=b+a$ (commutative law of addition)
2. $ab=ba$ (commutative law of multiplication)
3. $a+(b+c)=(a+b)+c$ (associative law of addition)
4. $a(bc)=(ab)c$ (associative law of multiplication)
5. $a(b+c)=ab+ac$ (distributive law)

Powers of a literal

Product of a literal multiplied by itself is called a power of the literal, for example, $x \times x = x^2$ and $x \times x \times x = x^3$. The literal is called the base of the power and the number of times it is multiplied is called the index or exponent of the power.

Examples (i) In x^2, x is the base and 2 is the index of the power.

(ii) In y^3, base $= y$ and index of power = 3.

(iii) In $p \times p \times p \times p \times p = p^5$, base $= p$ and index of power = 5.

Terms

A combination of constants and variables connected only by the operations of multiplication and division is called a term. Some examples are $4a, -3x^2, 2xy$ and $\frac{16}{3}yz$.

Algebraic expressions

A collection of numbers and variables connected by operations is called an algebraic expression or simply an expression. Parts of an expression connected by '+' or '–' signs are called terms of the expression. If a part has a negative sign before it, the '–' sign is included in the term.

Examples (i) $3x-5y+6z$ has three terms $3x, -5y$ and $6z$.

(ii) $\frac{5}{2}ab+\frac{6}{5}bc-\frac{8}{9}cd+7f$ has four terms $\frac{5}{2}ab, \frac{6}{5}bc, -\frac{8}{9}cd$ and $7f$.

If an algebraic expression has one term, it is called a monomial and if it has more than one term, it is called a multinomial. Multinomials with two and three terms are called binomials and trinomials respectively.

Factors and coefficients

The constants and variables that are multiplied together to form a term are called factors of the term. Each factor is a coefficient of the product of the remaining factors of the term. In particular, the number or constant which appears in a term is called the numerical coefficient of the term.

Example Consider the term $5x^2yz$.

5 is the numerical coefficient of x^2yz.

$5x^2$ is the coefficient of yz.
$5y$ is the coefficient of x^2z.
$5x^2y$ is the coefficient of z.
$5yz$ is the coefficient of x^2.
$5xy$ is the coefficient of xz.

Like and unlike terms

Terms which are alike in their variables are called like terms. Terms which are not like are called unlike terms.

Examples (i) $-\frac{2}{3}ab, 5ab$ are like terms.

(ii) $\frac{5}{6}x, \frac{5}{6}xy$ are unlike terms.

(iii) $3x^2, -4x^3$ are unlike terms.

Degree of a term

The highest index of power or the sum of the indices of power of the variable(s) in a term is called the degree of the term.

Examples (i) $-6x^2$ is of the second degree.

(ii) The degree of xy^2 is $3(=1+2)$.

(iii) $12x^8y^9z^5$ is a term of degree $22\ (=8+9+5)$.

Polynomial in one variable

An algebraic expression is called a polynomial if it is a finite sum of terms which contain only non-negative integral exponents of a variable. A polynomial in one variable (say x) contains only such terms as can be expressed in the form ax^n, where a is a constant and n is a non-negative integer. In other words, a polynomial in x cannot have terms, such as $\frac{1}{x}, \frac{1}{x^2}$ and $\frac{1}{x^3}$ because they are not of the form x^n, where n is a non-negative integer.

Examples (i) $-3x^2+4x+2$ is a polynomial in x with three terms $-3x^2$, $4x$ and 2. The term 2 is called a constant term.

(ii) $x^2+\frac{2}{x}$ is not a polynomial because the term $\frac{2}{x}$ is not of the form ax^n, where n is a non-negative integer.

Polynomial in two or more variables

A polynomial in two or more variables is a sum of terms that contain only non-negative integral exponents of those variables. For example, a polynomial in x, y and z is a sum of one or more terms of the form $ax^my^nz^p$, where a is any constant and m, n and p are non-negative integers.

Examples (i) $5xy-2yz+3zx$ is a polynomial in three variables x, y and z.

(ii) $6a^2bc+8ab^2cd+9abc^2d^2+7$ is a polynomial in four variables a, b, c and d.

Degree of a polynomial

The degree of a polynomial in one variable is the highest index of powers of the variable in any term of the polynomial.

Examples (i) The degree of the polynomial $x^5 - 6x^2 + 2$ in the variable x is 5 since this is the highest index of the powers of x in the terms.

(ii) Similarly, the degree of the polynomial $5y^6 + y^3 - 2y$ is 6 since the highest of the indices of power of y is 6.

The degree of a polynomial in more than one variable is the highest of the sums of the indices of power of the variables in any of the terms of the polynomial.

Examples (i) The expression $5x^2yz + 7xy$ is a polynomial in x, y and z. The degrees of its terms are 4 and 2 respectively. So the degree of the polynomial is 4.

(ii) If we treat this expression as a polynomial in x and y, we get $(5z)x^2y + 7xy$. The degree of the polynomial then becomes 3 since the highest of the sums of indices of the powers of x and y in any of the terms is 3 (in the first term).

(iii) Similarly, treating the expression as a polynomial in x, we get $(5yz)x^2 + (7y)x$. The degree of the polynomial then becomes 2.

(iv) If we treat the expression as a polynomial in y, we get $(5x^2z)y + (7x)y$, which has the degree 1.

(v) If we treat it as a polynomial in z, we get $(5x^2y)z + 7xy$, of degree 1.

Polynomials are called by particular names in accordance with their degrees.

1. A polynomial of degree 1 is called a linear polynomial.

Examples (i) $x + 5$ is a linear polynomial in one variable (x).

(ii) $-3a + 5b$ is a linear polynomial in two variables (a and b).

(iii) $7x - 8y + 9z$ is a linear polynomial in three variables (x, y and z).

2. A polynomial of degree 2 is called a quadratic polynomial.

Examples (i) x^2, $4a^2 + 3$, $6x^2 - 5x$ and $7x^2 + 5x + 6$ are quadratic polynomials in one variable.

(ii) $x^2 + 5xy - 9y^2$, $7a^2 + 5ab + 9$, $x^2 + xy - 4$ are quadratic polynomials in two variables.

(iii) $4x^2 + y^2 - z^2 + 3xy - 4yz + 2zx$ is a quadratic polynomial in three variables.

3. A polynomial of degree 3 is called a cubic polynomial.

Examples (i) $4x^3 + 3x^2 + 2x + 1$, $-6x^3 + 4x^2 + 5$, $7a^3 - 8a + 9$ and $8a^3 - 3$ are cubic polynomials in one variable.

(ii) $2x^3 + 3y^2 + xy$ and $12x^2y + 9y^3 - 5xy^2 + 7$ are cubic polynomials in two variables.

(iii) $3x^2y - 5yz^2$ and $8xyz + 6yz^2 + 9xz^2 - xy + 5$ are cubic polynomials in three variables.

4. A polynomial of degree 4 is called a quartic polynomial.

Examples (i) $-6a^4 + 3a^2 + \frac{7}{5}$ and $8x^4 - 9$ are quartic polynomials in one variable.

(ii) $3x^4 + 7x^2y^2 - 8xy$ and $7a^4 - 8a^3b + 9ab^2 - 4$ are quartic polynomials in two variables.

(iii) $7x^2y^2 + 9xy^2z - 3xyz^2$ and $x^4 + 3x^3y - yz^2 - 4$ are quartic polynomials in three variables.

(iv) $a^2bc + ab^2d + 6bc^2d - 9$ and $a^4 + 6ab^2c - 9abd^2 + 15abcd$ are quartic polynomials in four variables.

EXERCISE 1A

1. Identify the literals and constants.

(i) x (ii) 2 (iii) $2 \times x$ (iv) y^2

(v) $\frac{x}{2}$

2. Find the number of terms in the following expressions.

(i) 5 (ii) $x - 11$ (iii) $11y$ (iv) $\frac{y}{11}$

(v) $-3 + 2x^2 + \frac{z}{5}$ (vi) $x - \frac{2y}{3} + \frac{z}{3x}$ (vii) $2 + \frac{3x^2}{4} + 5x - 9y$

3. Identify the monomials, binomials and trinomials.

(i) $2a$ (ii) $5 - x$ (iii) $\frac{m^2}{n}$ (iv) $m + 2n - 3$

(v) $m^2 - 2n^2$ (vi) $\frac{m}{3} + \frac{x}{5} - \frac{z}{6}$

4. In the term $\frac{-6}{7}a^4b^5c^3$, find the coefficient of

(i) a^4 (ii) $-3b^5$ (iii) $\frac{-3c^3}{7}$ (iv) a^3

(v) a^2b^3 (vi) abc (vii) $-b^3c^2$ (viii) $2bc^3$

(ix) $-\frac{3}{7}a^2b^2c$ (x) $\frac{6}{7}a^2b^5$

5. Identify the like terms.

(i) $2ab, -2\frac{a}{b}, -\frac{2}{3}ab, \frac{6}{5}bc, \frac{3b}{a}$ (ii) $5xy^2, 0.2xy^2, x^2y, (xy)^2$

6. Identify the polynomials. Also, write the degrees of the polynomials.

(i) $8x^2 - x^3 + \frac{2}{3}x$ (ii) $7y^3 + 5y - \frac{1}{2y} - 6$ (iii) $10x^2y + \frac{5x^2}{y} - 9x^4 + 6x^2$

(iv) $x^7y^{12} - 6x^3y^5 + \frac{3x^7}{7}y^2 - x^{10}y^{20}$

ANSWERS

1. (i) Literal (ii) Constant (iii) Literal (iv) Literal (v) Literal

2. (i) 1 (ii) 2 (iii) 1 (iv) 1 (v) 3 (vi) 3 (vii) 4

3. (i) Monomial (ii) Binomial (iii) Monomial (iv) Trinomial (v) Binomial (vi) Trinomial

4. (i) $-\frac{6}{7}b^5c^3$ (ii) $\frac{2}{7}a^4c^3$ (iii) $2a^4b^5$ (iv) $-\frac{6}{7}ab^5c^3$ (v) $-\frac{6}{7}a^2b^2c^3$ (vi) $-\frac{6}{7}a^3b^4c^2$ (vii) $\frac{6}{7}a^4b^2c$ (viii) $\frac{-3}{7}a^4b^4$ (ix) $2a^2b^3c^2$ (x) $-a^2c^3$

5. (i) $2ab, -\frac{2}{3}ab$ (ii) $5xy^2, 0.2xy^2$

6. (i) Polynomial of degree 3 (ii) Not a polynomial (iii) Not a polynomial (iv) Polynomial of degree 30

Addition and Subtraction of Algebraic Expressions

You are familiar with the addition and subtraction of algebraic expressions. Let us review what you have learnt already.

Addition of like terms

To add like terms, add their numerical coefficients.

Examples (i) $3xy + 7xy + \left(-\frac{5}{2}xy\right) = \left\{3 + 7 + \left(-\frac{5}{2}\right)\right\}xy = \frac{15}{2}xy.$

(ii) $11x^2y^3 + 16x^2y^3 + 24x^2y^3 + (-48x^2y^3)$

$= \{11 + 16 + 24 + (-48)\}x^2y^3 = 3x^2y^3.$

(iii) $ax^2 + bx^2 + (-cx^2) + dx^2 = (a + b - c + d)x^2.$

Addition of unlike terms

To add unlike terms, write an algebraic expression with all the terms along with their signs.

Examples (i) The sum of $6x$ and $-9y = 6x - 9y$.

(ii) The sum of $3xy$, $-5x^2y$ and $6xyz = 3xy - 5x^2y + 6xyz$.

Addition of algebraic expressions

To add two or more algebraic expressions, group the like terms of the expressions and add each group of like terms separately. This method is known as the horizontal method of addition.

EXAMPLE **Add $6x + 5y - 2z + 9$, $15y - 3x + 6z$ and $6z - 3y + 5$.**

Solution The sum $= (6x + 5y - 2z + 9) + (15y - 3x + 6z) + (6z - 3y + 5)$

$= (6x - 3x) + (5y + 15y - 3y) + (-2z + 6z + 6z) + (9 + 5)$ [grouping the like terms]

$= (6 - 3)x + (5 + 15 - 3)y + (-2 + 6 + 6)z + 14$ [adding the like terms]

$= 3x + 17y + 10z + 14.$

Alternative method

Write the expressions one below the other such that the like terms are in the same column. Then add each column of like terms. This is known as the column method of addition.

$$\begin{array}{r} 6x + 5y - 2z + 9 \\ -3x + 15y + 6z \\ - 3y + 6z + 5 \\ \hline 3x + 17y + 10z + 14 \end{array}$$

$[\because\ 6x - 3x = 3x, 5y + 15y - 3y = 17y$
$-2z + 6z + 6z = 10z, 9 + 5 = 14]$

Negative of an algebraic expression

1. To find the negative of a term, change the sign of the term.
2. To find the negative of an algebraic expression, change the sign of each term of the expression.

Examples (i) The negative of $3x$ is $-3x$.

(ii) The negative of $-\frac{3}{2}xy$ is $\frac{3}{2}xy$.

(iii) The negative of $5x - 6y$ is $-5x + 6y$, that is, $-(5x - 6y) = -5x + 6y$.

Subtraction of algebraic expressions

To subtract an algebraic expression from another, add the negative of the first expression to the second expression.

Examples (i) To subtract $5x^2y$ from $9x^2y$, we add $-5x^2y$ to $9x^2y$.

$\therefore\quad 9x^2y - 5x^2y = 9x^2y + (-5x^2y) = [9 + (-5)]x^2y = 4x^2y.$

(ii) To subtract $-\frac{3}{2}abc$ from $-\frac{7}{2}abc$, we add $-\left(-\frac{3}{2}abc\right)$ or $\frac{3}{2}abc$ to $-\frac{7}{2}abc$.

$\therefore\quad -\frac{7}{2}abc - \left(-\frac{3}{2}abc\right) = -\frac{7}{2}abc + \frac{3}{2}abc = \left(-\frac{7}{2} + \frac{3}{2}\right)abc = -2abc.$

(iii) To subtract $7x - 5y$ from $12x + 6y$, we add $-(7x - 5y)$ or $-7x + 5y$ to $12x + 6y$.

$$\begin{aligned} \therefore\quad (12x + 6y) - (7x - 5y) &= (12x + 6y) + \{-(7x - 5y)\} \\ &= (12x + 6y) + (-7x + 5y) \\ &= \{12x + (-7x)\} + (6y + 5y) \\ &= \{12 + (-7)\}x + 11y = 5x + 11y. \end{aligned}$$

Note This is the horizontal method of subtraction.

Alternative method

Steps 1. Write the subtrahend (that is, the expression to be subtracted) below the minuend (that is, the expression from which to subtract) in such a way that the like terms are in the same column.

2. Change the sign of each term of the subtrahend.

3. Then add the expressions.

Example

$$\begin{array}{r} 12x + 6y \\ 7x - 5y \\ - \quad + \\ \hline 5x + 11y \end{array}$$

(changing the sign of each term)

$(\because\ 12 - 7 = 5, 6 + 5 = 11)$

Solved Examples

EXAMPLE 1 **Add $5x^3 - 2x^2 + 6x - 3$, $-3x^3 + 2x^2 + 2$ and $-5x^3 + 7x + 15$.**

Solution

$$\begin{array}{rrrr} 5x^3 & -2x^2 & +6x & -3 \\ -3x^3 & +2x^2 & & +2 \\ -5x^3 & & +7x & +15 \\ \hline -3x^3 & & +13x & +14 \end{array}$$

EXAMPLE 2 **Add $-2a + 4b - 3c + 10d$, $6a - 3b + 2d$, $-3b - 4c + 6d$ and $4a - 8d$.**

Solution

$$\begin{array}{rrrr} -2a & +4b & -3c & +10d \\ 6a & -3b & & +2d \\ & -3b & -4c & +6d \\ 4a & & & -8d \\ \hline 8a & -2b & -7c & +10d \end{array}$$

EXAMPLE 3 **Subtract $2x^3 + 7x^2 - 7x + 3$ from $-5x^3 - 4x^2 + 10x - 4$.**

Solution

$$\begin{array}{rrrr} -5x^3 & -4x^2 & +10x & -4 \\ 2x^3 & +7x^2 & -7x & +3 \\ - & - & + & - \\ \hline -7x^3 & -11x^2 & +17x & -7 \end{array}$$

EXAMPLE 4 **Subtract $3x^4 + 4x^2 - 23$ from $15x^4 - 3x^2 - 5x - 20$.**

Solution

$$\begin{array}{rrrr} 15x^4 & -3x^2 & -5x & -20 \\ 3x^4 & +4x^2 & & -23 \\ - & - & & + \\ \hline 12x^4 & -7x^2 & -5x & +3 \end{array}$$

EXAMPLE 5 **Subtract $6x^2 + 7x - 5$ from the sum of $-2x^2 + 3x + 6$ and $4x^2 + 3x - 2$.**

Solution First, we find the sum of $-2x^2 + 3x + 6$ and $4x^2 + 3x - 2$.

$$\begin{array}{rrr} -2x^2 & +3x & +6 \\ 4x^2 & +3x & -2 \\ \hline 2x^2 & +6x & +4 \end{array}$$

Then, we subtract $6x^2 + 7x - 5$ from $2x^2 + 6x + 4$.

$$\begin{array}{rrr} 2x^2 & +6x & +4 \\ 6x^2 & +7x & -5 \\ - & - & + \\ \hline -4x^2 & -x & +9 \end{array}$$

EXAMPLE 6 **What should be subtracted from $5ab - 4bc + 3cd + 6abc$ to get $-3ab + 3cd + 9abc$?**

Solution To obtain the required expression, we have to subtract $-3ab + 3cd + 9abc$ from $5ab - 4bc + 3cd + 6abc$.

$$\begin{array}{rrrr} 5ab & -4bc & +3cd & +6abc \\ -3ab & & +3cd & +9abc \\ + & & - & - \\ \hline 8ab & -4bc & & -3abc \end{array}$$

$\therefore$ the required expression is $8ab - 4bc - 3abc$.

EXERCISE 1B

1. Add:

(i) $2ab, -3ab, 5ab$ and $\frac{1}{2}ab$

(ii) $3x^2, 6x^2, -9x^2$ and $\frac{5}{3}x^2$

(iii) $8ab^4, -6ab^4, 16ab^4, -20ab^4$ and $2ab^4$

(iv) $3a^2b^4, -2a^2b^2, 5a^2b^2, 12a^2b^4, 3a^2b^2$ and $5a^2b^4$

2. Subtract:

(i) $\frac{3}{2}xy$ from $\frac{5}{2}xy$

(ii) $-2abc$ from $6abc$

(iii) $5a-9b$ from $7a+10b$

(iv) $-6x^2+2y^2$ from $9x^2-3y^2$

3. Find the sum of each of the following.

(i) $7a+6b, -3a+9b$ and $6a-3b$

(ii) $-6x^2+7x+3, 3x^2+x+3$ and $5x^2+2x-9$

(iii) $3x^2+4x-3, 2x^2+2x+1$ and $-x^2+x-6$

(iv) $12ab+2bc+4cd-3, -2ab+3cd-2$ and $5ab-7bc+12$

4. Add the following.

(i) $2l+3m-6n+4p, 3l-5m+16n-4p, 12l-6m-4n-2p$ and $l-2m+3n-4p$

(ii) $7x^3+3x+9, -2x^3-3x^2-15, 3x^3-6x^2+4x-6$ and $12x^2-6$

(iii) $5a^2-7ab+9b^2, 4a^2-2b^2-9ab-6, 4-3b^2+2ab+6a^2$ and $12ab-3a^2-9b^2$

5. Subtract:

(i) $4a-2b-c$ from $-a+2b+3c$

(ii) $-3x^2+7y^2-z^2$ from $2x^2-5y^2-7z^2$

(iii) $-2ab+4bc-5cd$ from $3ab-4bc+17cd$

(iv) $2x^3-16x^2+14x-25$ from $-6x^3+4x^2-3x+5$

(v) $20a^4+12a^3+3a-25$ from $17a^4-2a^3-12a-16$

6. (i) Subtract $ab+cd+7ef$ from the sum of $3ab+4cd-3ef$ and $2ab-3cd-ef$.

(ii) Subtract $2a^3-4a^2+13$ from the sum of $7a^3+3a+7$ and $3a^3-3a^2+4a+5$.

7. What should be subtracted from $14x^4-3x^3+25x-42$ to get $3x^4+17x^3-12x-65$?

8. How much smaller is $5x-8y+9z$ than $12x-10y-3z+16$?

9. How much bigger is $7x^2y^2-16xy^2-8x^2y$ than $-3x^2+4x^2y-9xy$?

10. If the perimeter of a triangle is $(4y-3x+2z)$ cm and two sides of the triangle measure $(4x+2y+z)$ cm and $(3x+7y-2z)$ cm, find the length of the third side of the triangle.

ANSWERS

1. (i) $\frac{9}{2}ab$ (ii) $\frac{5}{3}x^2$ (iii) 0 (iv) $20a^2b^4+6a^2b^2$ **2.** (i) xy (ii) $8abc$ (iii) $2a+19b$ (iv) $15x^2-5y^2$

3. (i) $10a+12b$ (ii) $2x^2+10x-3$ (iii) $4x^2+7x-8$ (iv) $15ab-5bc+7cd+7$

4. (i) $18l - 10m + 9n - 6p$ (ii) $8x^3 + 3x^2 + 7x - 18$ (iii) $12a^2 - 2ab - 5b^2 - 2$

5. (i) $-5a + 4b + 4c$ (ii) $5x^2 - 12y^2 - 6z^2$ (iii) $5ab - 8bc + 22cd$ (iv) $-8x^3 + 20x^2 - 17x + 30$
(v) $-3a^4 - 14a^3 - 15a + 9$

6. (i) $4ab - 11ef$ (ii) $8a^3 + a^2 + 7a - 1$ **7.** $11x^4 - 20x^3 + 37x + 23$

8. $7x - 2y - 12z + 16$ **9.** $7x^2y^2 - 12x^2y - 16xy^2 + 3x^2 + 9xy$ **10.** $(-10x - 5y + 3z)$ cm

Multiplication

You know that the product of a literal multiplied by itself is a power of the literal and that the index of the power is the number of times the literal is multiplied by itself.

Thus, $x \times x = x^2$ and $x \times x \times x = x^3$.

Then, $x^2 \times x^3 = x \times x \times x \times x \times x = x^5 = x^{2+3}$.

We can generalise this as follows.

$$x^m \times x^n = x^{m+n}$$

We can express this as, the product of two powers (x^m and x^n) of a variable x is another power (x^{m+n}) of the variable x whose index ($m+n$) is the sum of their indices (m and n).

Similarly, $$x^m \times x^n \times x^p = x^{m+n} \times x^p = x^{m+n+p}$$

Multiplication of two or more monomials

To multiply two or more monomials, remember

Product of two monomials = (product of numerical factors) × (product of variable factors)

The sign of the product is determined by the usual convention.

$(+) \times (+) = (+)$ $(+) \times (-) = (-)$ $(-) \times (-) = (+)$ $(-) \times (+) = (-)$

EXAMPLE **Find the product of $3xy$ and $-9x^2yz$.**

Solution Product $= 3xy \times (-9x^2yz) = \{3 \times (-9)\} \times (xy \times x^2yz)$
$= -27 \times (x \times x^2) \times (y \times y) \times z = -27x^{1+2}y^{1+1}z = -27x^3y^2z$.

EXAMPLE **Find $(6a^2bc) \times (-9a^4b^2c^3) \times (3a^7bc^4)$.**

Solution Product $= \{6 \times (-9) \times 3\} \times \{(a^2bc) \times (a^4b^2c^3) \times (a^7bc^4)\}$
$= -162 \times (a^2 \times a^4 \times a^7) \times (b \times b^2 \times b) \times (c \times c^3 \times c^4)$
$= -162a^{2+4+7}b^{1+2+1}c^{1+3+4} = -162a^{13}b^4c^8$.

Multiplication of a polynomial by a monomial

To find the product of a polynomial and a monomial, multiply each term of the polynomial by the monomial and add the products.

Thus, the law is $a \times (b + c + d + \ldots) = a \times b + a \times c + a \times d + \ldots$ (distributive law)

$$(b + c + d + \ldots) \times a = b \times a + c \times a + d \times a + \ldots.$$

EXAMPLE **Multiply $7ax - 8by$ by 3.**

Solution Product $= 3 \times (7ax - 8by)$

$= 3 \times 7ax + 3 \times (-8)by = 21ax - 24by.$

EXAMPLE **Multiply $a^2 - 2ab + 3b^2$ by $6a$.**

Solution Product $= 6a \times (a^2 - 2ab + 3b^2) = 6a \times a^2 + 6a \times (-2ab) + 6a \times (3b^2)$

$= 6a^{1+2} + 6 \times (-2) \times a \times ab + 6 \times 3 \times ab^2$

$= 6a^3 + (-12)a^{1+1}b + 18ab^2 = 6a^3 - 12a^2b + 18ab^2.$

EXAMPLE **Find $5a^2b \times (3a^2b - 4ab^2 + 2ab - 5)$.**

Solution Product $= 5a^2b \times (3a^2b - 4ab^2 + 2ab - 5)$

$= 5a^2b \times 3a^2b + 5a^2b \times (-4ab^2) + 5a^2b \times 2ab + 5a^2b \times (-5)$

$= 5 \times 3 \times a^2 \times a^2 \times b \times b + 5 \times (-4) \times a^2 \times a \times b \times b^2 + 5 \times 2 \times a^2 \times a \times b \times b$
$+ 5 \times (-5)a^2b$

$= 15a^{2+2}b^{1+1} + (-20)a^{2+1}b^{1+2} + 10a^{2+1}b^{1+1} + (-25)a^2b$

$= 15a^4b^2 - 20a^3b^3 + 10a^3b^2 - 25a^2b.$

EXAMPLE **Simplify $-7xy \times (5x^3 - 2x^2y + 3xy^2 - 7xy - 2)$.**

Solution Product $= (-7xy) \times 5x^3 + (-7xy) \times (-2x^2y) + (-7xy) \times 3xy^2 + (-7xy) \times (-7xy)$
$+ (-7xy) \times (-2)$

$= (-7 \times 5) \times x^{1+3} \times y + (-7) \times (-2) \times x^{1+2} \times y^{1+1} + (-7 \times 3) \times x^{1+1} \times y^{1+2}$
$+ (-7) \times (-7) \times x^{1+1} \times y^{1+1} + (-7) \times (-2) \times x \times y$

$= -35x^4y + 14x^3y^2 - 21x^2y^3 + 49x^2y^2 + 14xy.$

Multiplication of two polynomials

To find the product of two polynomials, multiply all the terms of one polynomial by each of the terms of the other polynomial and add the resulting products.

EXAMPLE **Find the product of $(2x - 5y)$ and $(3x + 4y)$.**

Solution We first multiply each term of $3x + 4y$ by $2x$. Then multiply each term of $3x + 4y$ by $(-5y)$. Finally, we add all the terms.

Thus, $(2x - 5y)(3x + 4y) = 2x \times (3x + 4y) + (-5y) \times (3x + 4y)$

$= 2x \times 3x + 2x \times 4y + (-5y) \times (3x) + (-5y) \times (4y)$

$= 6x^{1+1} + 8xy + (-15)xy + (-20)y^{1+1}$

$= 6x^2 + 8xy - 15xy - 20y^2 = 6x^2 - 7xy - 20y^2.$

Column method

$$\begin{array}{l} 3x + 4y \\ \underline{2x - 5y} \\ 6x^2 + 8xy \\ \underline{\qquad -15xy - 20y^2} \\ 6x^2 - 7xy - 20y^2 \end{array}$$

(multiplying each term of $3x + 4y$ by $2x$)
(multiplying each term of $3x + 4y$ by $-5y$ and writing like terms in the same column and adding)

Solved Examples

EXAMPLE 1 **Multiply $3x + 2x^2 - 5$ by $2x + 4$.**

Solution For convenience, we arrange the terms of the polynomial in descending powers of x.

$$\begin{array}{rrrrl} & 2x^2 & +3x & -5 & \\ & & 2x & +4 & \\ \hline 4x^3 & +6x^2 & -10x & & \text{(multiplying } 2x^2+3x-5 \text{ by } 2x) \\ & +8x^2 & +12x & -20 & \text{(multiplying } 2x^2+3x-5 \text{ by } 4) \\ \hline 4x^3 & +14x^2 & +2x & -20 & \text{(adding in columns)} \end{array}$$

EXAMPLE 2 **Multiply $3x^2 - x + 4$ by $4x^2 - 2x - 5$.**

Solution

$$\begin{array}{rrrrr} & & 3x^2 & -x & +4 \\ & & 4x^2 & -2x & -5 \\ \hline 12x^4 & -4x^3 & +16x^2 & & \\ & -6x^3 & +2x^2 & -8x & \\ & & -15x^2 & +5x & -20 \\ \hline 12x^4 & -10x^3 & +3x^2 & -3x & -20 \end{array}$$

EXAMPLE 3 **Find the product $(5a^2 - 7ab - 4b^2) \times (5a + 2b - 6)$.**

Solution

$$\begin{array}{l} 5a^2 - 7ab - 4b^2 \\ 5a + 2b - 6 \\ \hline 25a^3 - 35a^2b - 20ab^2 \\ \qquad 10a^2b - 14ab^2 - 8b^3 \\ \qquad\qquad -30a^2 + 42ab + 24b^2 \\ \hline 25a^3 - 25a^2b - 34ab^2 - 8b^3 - 30a^2 + 42ab + 24b^2 \end{array}$$

EXAMPLE 4 **Simplify $(x + 6)(x - 3)(x + 5)$.**

Solution $(x-3)(x+5) = x(x+5) + (-3)(x+5) = x \times x + x \times 5 + (-3) \times x + (-3) \times 5$

$= x^2 + 5x - 3x - 15 = x^2 + 2x - 15$

$\therefore \quad (x+6)(x-3)(x+5) = (x+6)(x^2+2x-15) = x(x^2+2x-15) + 6(x^2+2x-15)$

$= x \times x^2 + x \times (2x) + x \times (-15) + 6 \times x^2 + 6 \times 2x + 6 \times (-15)$

$= x^3 + 2x^2 - 15x + 6x^2 + 12x - 90 = x^3 + 8x^2 - 3x - 90.$

Remember These

1. $x^m \times x^n = x^{m+n}, \quad x^m \times x^n \times x^p = x^{m+n+p}$

2. (i) The product of two monomials = (the product of their numerical coefficients) × the product of their literal coefficients)

(ii) For a monomial and an algebraic expression:

$a \times (b + c) = a \times b + a \times c, \quad a \times (b + c + d) = a \times b + a \times c + a \times d$

(iii) For two binomials or polynomials:

$(a + b) \times (c + d) = a \times (c + d) + b \times (c + d) = a \times c + a \times d + b \times c + b \times d$

$(a + b + c) \times (x + y + z) = a \times x + b \times x + c \times x + a \times y + b \times y + c \times y + a \times z + b \times z + c \times z$

EXERCISE 1C

1. Find the product of each of the following.

(i) $3a^2$ and $\frac{4}{3}ab^3$ (ii) $-\frac{4}{7}x^2y^2$ and $-\frac{2}{5}x^2y^2z$

(iii) $-\frac{1}{3}ab^2, \frac{3}{4}a^2b$ and $4a^2b^2c$ (iv) $\frac{1}{27}xyz, -\frac{1}{2}x^2yz, 6xyz^3$ and $-9xy^3$

2. Multiply:

(i) $ax - by + cz$ by $2a$ (ii) $\frac{2}{3}xy - \frac{3}{4}y^2z + \frac{4}{5}x^2z$ by xyz

(iii) $-5a^3 + \frac{6}{7}ab^3c - \frac{4}{15}ab^2$ by $-abc$ (iv) $2l^4 - \frac{3}{5}lmn + 5m^2n - \frac{6}{7}n^3 - n$ by $-6l^3m^3n^3$

3. Simplify the following.

(i) $(2x-1)(3x+5)$ (ii) $(2x+3)(3x+4)$ (iii) $(ax+b)(bx+c)$

(iv) $(3x^2+1)(2x-3)$ (v) $\left(x+\frac{1}{2}\right)\left(x-\frac{1}{3}\right)$ (vi) $(6x-7y)(7x-5y)$

4. Find the product of

(i) x^2+x and $2x+1$ (ii) $x-y-1$ and $2x+y$ (iii) x^2-3x+1 and $3x-4$

(iv) $2x^3-3x^2+3$ and $3x-4$ (v) $\frac{1}{2}x^3 - \frac{3}{2}x^2 - \frac{1}{2}x - \frac{1}{4}$ and $8x-16$

5. Multiply:

(i) $x-3y+4$ and $5x+y-2$ (ii) $2x^2-x-2$ by x^2+x-1

(iii) $3a^2+2a-5$ by $2-3a-6a^2$ (iv) $5a^2+15ab+10b^2$ by $a^2-2ab+2b^2$

(v) $6d^3+4d^2-3d+5$ by d^2+d-1

6. Simplify:

(i) $(x+1)(x+2)(x+4)$ (ii) $(x+1)(x-1)(x+2)$ (iii) $(x+2)(x-2)(x+3)(x-3)$

ANSWERS

1. (i) $4a^3b^3$ (ii) $\frac{8}{35}x^4y^4z$ (iii) $-a^5b^5c$ (iv) $x^5y^6z^5$

2. (i) $2a^2x - 2aby + 2acz$ (ii) $\frac{2}{3}x^2y^2z - \frac{3}{4}xy^3z^2 + \frac{4}{5}x^3yz^2$ (iii) $5a^4bc - \frac{6}{7}a^2b^4c^2 + \frac{4}{15}a^2b^3c$

(iv) $-12l^7m^3n^3 + \frac{18}{5}l^4m^4n^4 - 30l^3m^5n^4 + \frac{36}{7}l^3m^3n^6 + 6l^3m^3n^4$

3. (i) $6x^2+7x-5$ (ii) $6x^2+17x+12$ (iii) $abx^2+(ac+b^2)x+bc$ (iv) $6x^3-9x^2+2x-3$

(v) $x^2+\frac{x}{6}-\frac{1}{6}$ (vi) $42x^2-79xy+35y^2$

4. (i) $2x^3+3x^2+x$ (ii) $2x^2-2x-xy-y^2-y$ (iii) $3x^3-13x^2+15x-4$ (iv) $6x^4-17x^3+12x^2+9x-12$

(v) $4x^4-20x^3+20x^2+6x+4$

5. (i) $5x^2-14xy-3y^2+18x+10y-8$ (ii) $2x^4+x^3-5x^2-x+2$ (iii) $-18a^4-21a^3+30a^2+19a-10$

(iv) $5a^4+5a^3b-10a^2b^2+10ab^3+20b^4$ (v) $6d^5+10d^4-5d^3-2d^2+8d-5$

6. (i) $x^3+7x^2+14x+8$ (ii) x^3+2x^2-x-2 (iii) x^4-13x^2+36

Division

You know that $x^3 = x \times x \times x$ and $x^2 = x \times x$.

$\therefore \quad \frac{x^3}{x^2} = \frac{x \times x \times x}{x \times x} = x = x^{3-2}$.

Similarly, $\frac{x^5}{x^3} = \frac{x \times x \times x \times x \times x}{x \times x \times x} = x \times x = x^2 = x^{5-3}$.

We can generalise this as

$$x^m \div x^n = x^{m-n}$$

Hence, $x^m \div x^m = 1 = x^{m-m} = x^0$, that is, $x^0 = 1$.

The convention for signs in the case of division is the same as that for multiplication.

$(+) \div (+) = (+) \qquad (+) \div (-) = (-) \qquad (-) \div (-) = (+) \qquad (-) \div (+) = (-)$

Division of a monomial by a monomial

To divide one monomial by another, divide the numerical coefficient of the dividend by the numerical coefficient of the divisor and the powers of variables in the dividend by the corresponding powers in the divisor. Then multiply all the quotients.

Quotient of two monomials = (quotient of numerical factors) × (quotient of literal factors)

EXAMPLE **Divide $12a^2b$ by $2ab$.**

Solution $12a^2b \div 2ab = \frac{12a^2b}{2ab} = \left(\frac{12}{2}\right) \times \left(\frac{a^2}{a}\right) \times \left(\frac{b}{b}\right) = 6 \times a^{2-1} \times b^{1-1} = 6ab^0 = 6a \times 1 = 6a.$

EXAMPLE **Divide $25a^5b^8c^3$ by $5a^4b^3c^2$.**

Solution $25a^5b^8c^3 \div 5a^4b^3c^2 = \frac{25a^5b^8c^3}{5a^4b^3c^2} = \frac{25}{5} \times \frac{a^5}{a^4} \times \frac{b^8}{b^3} \times \frac{c^3}{c^2}$

$= 5a^{5-4}b^{8-3}c^{3-2} = 5ab^5c.$

EXAMPLE **Divide $-18ab^3c^2d^5$ by $27a^6b^3cd^6$.**

Solution $-18ab^3c^2d^5 \div 27a^6b^3cd^6 = \frac{-18ab^3c^2d^5}{27a^6b^3cd^6} = -\frac{18}{27} \times \frac{a}{a^6} \times \frac{b^3}{b^3} \times \frac{c^2}{c} \times \frac{d^5}{d^6}$

$= -\frac{2}{3} \times \frac{b^{3-3} \times c^{2-1}}{a^{6-1} \times d^{6-5}} = -\frac{2c}{3a^5d}.$

Division of a polynomial by a monomial

To divide a polynomial by a monomial, divide each term of the polynomial by the monomial and then add all the partial quotients thus obtained.

EXAMPLE **Divide $5a^5 - 4a^3 + 6a^2 - 8a + 16$ by $4a^2$.**

Solution

$$(5a^5 - 4a^3 + 6a^2 - 8a + 16) \div 4a^2 = \frac{5a^5 - 4a^3 + 6a^2 - 8a + 16}{4a^2}$$

$$= \frac{5a^5}{4a^2} - \frac{4a^3}{4a^2} + \frac{6a^2}{4a^2} - \frac{8a}{4a^2} + \frac{16}{4a^2}$$

$$= \frac{5}{4}a^{5-2} - a^{3-2} + \frac{3}{2}a^{2-2} - \frac{2}{a^{2-1}} + \frac{4}{a^2}$$

$$= \frac{5}{4}a^3 - a + \frac{3}{2}a^0 - \frac{2}{a} + \frac{4}{a^2}$$

$$= \frac{5}{4}a^3 - a + \frac{3}{2} - \frac{2}{a} + \frac{4}{a^2}.$$

EXAMPLE **Divide $20x^4y^2 + 12x^3y - 24x^2y^2 + 16xy^3$ by $-4x^4y^4$.**

Solution

$$\frac{20x^4y^2 + 12x^3y - 24x^2y^2 + 16xy^3}{-4x^4y^4} = \frac{20x^4y^2}{-4x^4y^4} + \frac{12x^3y}{-4x^4y^4} - \frac{24x^2y^2}{-4x^4y^4} + \frac{16xy^3}{-4x^4y^4}$$

$$= -\frac{5x^{4-4}}{y^{4-2}} - \frac{3}{x^{4-3}y^{4-1}} + \frac{6}{x^{4-2}y^{4-2}} - \frac{4}{x^{4-1}y^{4-3}}$$

$$= \frac{-5}{y^2} - \frac{3}{xy^3} + \frac{6}{x^2y^2} - \frac{4}{x^3y}.$$

Division of a polynomial by a polynomial

To divide one polynomial by another, take the following steps.

Steps

1. Arrange the terms of the dividend and the divisor in descending powers of a variable.
2. Divide the first term of the dividend by the first term of the divisor to obtain the first term of the quotient.
3. Multiply all the terms of the divisor by the first term of the quotient and subtract the product from the dividend.
4. Treat this remainder as the new dividend. Divide the new dividend by the divisor using Steps 2 and 3.
5. Continue until the remainder is either zero or an expression of lower degree than the degree of the divisor.

EXAMPLE **Divide $x^2 - 7x + 13$ by $x - 4$.**

Solution

$$\begin{array}{l} x-4)\,x^2 - 7x + 13\,(x-3 \\ \quad\;\; x^2 - 4x \\ \quad\;\; -\;\;\; + \\ \hline \qquad\;\; -3x + 13 \\ \qquad\;\; -3x + 12 \\ \qquad\;\;\; + \quad - \\ \hline \qquad\qquad\quad 1 \end{array}$$

$[\because\ x^2 \div x = x$ and $x(x-4) = x^2 - 4x]$

$[\because\ -3x \div x = -3$ and $-3(x-4) = -3x + 12]$

$\therefore$ quotient $= x - 3$ and remainder $= 1$.

Verification of answer

Divisor × quotient = $(x-4)\times(x-3) = x^2 - 4x - 3x + 12 = x^2 - 7x + 12$

∴ divisor × quotient + remainder = $(x^2 - 7x + 12) + 1 = x^2 - 7x + 13$ = dividend.

∴ the answer is correct.

Solved Examples

EXAMPLE 1 **Divide $2x^3 - 5x^2 + 3x + 4$ by $2x^2 - x + 1$.**

Solution

$$\begin{array}{l} 2x^2 - x + 1)\ 2x^3 - 5x^2 + 3x + 4\ (x - 2 \\ \qquad\qquad 2x^3 - x^2 + x \qquad \left[\because \frac{2x^3}{2x^2} = x \text{ and } (2x^2 - x + 1)x = 2x^3 - x^2 + x\right] \\ \qquad\qquad - \quad + \quad - \\ \hline \qquad\qquad\quad -4x^2 + 2x + 4 \\ \qquad\qquad\quad -4x^2 + 2x - 2 \qquad \left[\because \frac{-4x^2}{2x^2} = -2, (-2)(2x^2 - x + 1) = -4x^2 + 2x - 2\right]. \\ \qquad\qquad\quad + \quad - \quad + \\ \hline \qquad\qquad\qquad\qquad 6 \end{array}$$

∴ quotient = $x - 2$ and remainder = 6.

EXAMPLE 2 **Divide $12x^4 - 10x^3 + 3x^2 - 3x - 20$ by $3x^2 - x + 4$.**

Solution

$$\begin{array}{l} 3x^2 - x + 4)\ 12x^4 - 10x^3 + 3x^2 - 3x - 20\ (4x^2 - 2x - 5 \\ \qquad\qquad 12x^4 - 4x^3 + 16x^2 \\ \qquad\qquad - \quad + \quad - \\ \hline \qquad\qquad\qquad -6x^3 - 13x^2 - 3x - 20 \\ \qquad\qquad\qquad -6x^3 + 2x^2 - 8x \\ \qquad\qquad\qquad + \quad - \quad + \\ \hline \qquad\qquad\qquad\qquad -15x^2 + 5x - 20 \\ \qquad\qquad\qquad\qquad -15x^2 + 5x - 20 \\ \qquad\qquad\qquad\qquad + \quad - \quad + \\ \hline \qquad\qquad\qquad\qquad\qquad 0 \end{array}$$

∴ quotient = $4x^2 - 2x - 5$ and remainder = 0.

EXAMPLE 3 **Divide $x^4 + 2x^3 + 4x^2 - 2x + 1$ by $x^2 + x + 2$.**

Solution

$$\begin{array}{l} x^2 + x + 2)\ x^4 + 2x^3 + 4x^2 - 2x + 1\ (x^2 + x + 1 \\ \qquad\qquad x^4 + x^3 + 2x^2 \\ \qquad\qquad - \quad - \quad - \\ \hline \qquad\qquad\qquad x^3 + 2x^2 - 2x + 1 \\ \qquad\qquad\qquad x^3 + x^2 + 2x \\ \qquad\qquad\qquad - \quad - \quad - \\ \hline \qquad\qquad\qquad\qquad x^2 - 4x + 1 \\ \qquad\qquad\qquad\qquad x^2 + x + 2 \\ \qquad\qquad\qquad\qquad - \quad - \quad - \\ \hline \qquad\qquad\qquad\qquad -5x - 1 \end{array}$$

∴ quotient = $x^2 + x + 1$ and remainder = $-5x - 1$.

EXAMPLE 4 **Divide $10x^3 - 13xy^2 - 23x^2y + 20y^3$ by $5x - 4y$.**

Solution The dividend in descending powers of $x = 10x^3 - 23x^2y - 13xy^2 + 20y^3$.

$$\begin{array}{r|l} 5x-4y) & 10x^3 - 23x^2y - 13xy^2 + 20y^3 \; (2x^2 - 3xy - 5y^2 \\ & 10x^3 - 8x^2y \\ & - \quad + \\ \hline & -15x^2y - 13xy^2 + 20y^3 \\ & -15x^2y + 12xy^2 \\ & + \quad - \\ \hline & -25xy^2 + 20y^3 \\ & -25xy^2 + 20y^3 \\ & + \quad - \\ \hline & 0 \end{array}$$

$\therefore$ quotient $= 2x^2 - 3xy - 5y^2$ and remainder $= 0$.

Remember These

1. $x^m \div x^n = x^{m-n}$
2. The quotient of two monomials = (quotient of their numerical coefficients) × (quotient of their literal coefficients)
3. $(a+b) \div x = (a \div x) + (b \div x)$
   $(a+b+c) \div x = (a \div x) + (b \div x) + (c \div x)$
   $(a+b+c+d) \div x = (a \div x) + (b \div x) + (c \div x) + (d \div x)$
4. Dividend = divisor × quotient + remainder.
   The result of a division can be verified by this formula.

EXERCISE 1D

1. Divide:
   (i) $6a^4$ by $-2a$
   (ii) $44a^2b^2c$ by $11abc^2$
   (iii) $-52ab^3c^5$ by $26a^3bc^2$
   (iv) $-\frac{4}{7}x^3y^5z^7$ by $-\frac{16}{49}x^5y^8z^7$
2. Find:
   (i) $(35x - 7y + 28z) \div 7$
   (ii) $(12x^3 - 6x^2 - 9x - 9) \div 3x$
   (iii) $(8y^2 - 12x^2 + 16x^2y^2) \div (-4xy)$
   (iv) $(8a^2b^3 - 6a^3b^2 + 4ab^2 - 2a + b) \div 2ab$
   (v) $\left(\frac{1}{4}x^9y^5 + \frac{1}{8}x^6y^2 - \frac{1}{16}x^4y - \frac{1}{4}xy^2 - xy\right) \div \frac{1}{4}x^2y^2$
3. Divide and verify the answer.
   (i) $6a^2 + a - 2$ by $2a - 1$
   (ii) $x^2 + 5x + 4$ by $x + 4$
   (iii) $6x^2 + 7x - 2$ by $2x + 3$
   (iv) $4x^2 - 4x - 17$ by $2x - 5$
   (v) $2x^2 - 7x - 3$ by $x - 7$

4. Divide:

(i) $10a^3 - 7a^2b - 16ab^2 + 12b^3$ by $5a - 6b$ (ii) $2l^3 + 9l^2m + 17lm^2 + 12m^3$ by $2l + 3m$

(iii) $c^4 - d^4$ by $c + d$ (iv) $2a^5 - 6a^4 + 8a^3 - 15a^2 + 6a + 10$ by $a^2 + 3$

(v) $x^5 - 2x^3 + 9x^2 - 3x + 19$ by $x^2 + 1$ (vi) $a^3 - 3a^2 - 9a - 5$ by $a - 5$

(vii) $4x^3 - 8x^2 - 9x + 8$ by $2x - 3$

5. Divide:

(i) $4x^3 - 10x^2 + 24x - 15$ by $x^2 - 2x + 5$

(ii) $2x^4 - 5x + 3x^3 - 3 + 3x^2$ by $2x^2 - x - 1$

(iii) $x^4 + 2x^3 + x^2 - 18x - 15$ by $x^2 - 1 - x$

(iv) $x^5 - 4x^4 - 6x^3 + 21x^2 - 24x + 26$ by $x^2 - 2x + 3$

ANSWERS

1. (i) $-3a^3$ (ii) $4\frac{ab}{c}$ (iii) $-2\frac{b^2c^3}{a^2}$ (iv) $\frac{7}{4x^2y^3}$

2. (i) $5x - y + 4z$ (ii) $4x^2 - 2x - 3 - \frac{3}{x}$ (iii) $-\frac{2y}{x} + \frac{3x}{y} - 4xy$ (iv) $4ab^2 - 3a^2b + 2b - \frac{1}{b} + \frac{1}{2a}$

(v) $x^7y^3 + \frac{1}{2}x^4 - \frac{1}{4}\frac{x^2}{y} - \frac{1}{x} - \frac{4}{xy}$

3. (i) quotient = $3a + 2$, remainder = 0 (ii) quotient = $x + 1$, remainder = 0

(iii) quotient = $3x - 1$, remainder = 1 (iv) quotient = $2x + 3$, remainder = -2

(v) quotient = $2x + 7$, remainder = 46

4. (i) quotient = $2a^2 + ab - 2b^2$, remainder = 0 (ii) quotient = $l^2 + 3lm + 4m^2$, remainder = 0

(iii) quotient = $c^3 - c^2d + cd^2 - d^3$, remainder = 0 (iv) quotient = $2a^3 - 6a^2 + 2a + 3$, remainder = 1

(v) quotient = $x^3 - 3x + 9$, remainder = 10 (vi) quotient = $a^2 + 2a + 1$, remainder = 0

(vii) quotient = $2x^2 - x - 6$, remainder = -10

5. (i) quotient = $4x - 2$, remainder = -5 (ii) quotient = $x^2 + 2x + 3$, remainder = 0

(iii) quotient = $x^2 + 3x + 5$, remainder = $-10x - 10$ (iv) quotient = $x^3 - 2x^2 - 13x + 1$, remainder = $17x + 23$

Simplification of Algebraic Expressions

The rules for the simplification of an algebraic expression are the same as the rules used for numbers.

Removal of brackets

Brackets in an algebraic expression are opened in the order: — (line bracket), ( ), { } and [ ]. The rules for opening brackets are as follows.

Rules 1. If there is a + (positive) sign outside a bracket, the bracket is removed without changing the signs of the terms inside the bracket.

2. If there is a – (negative) sign outside a bracket, the bracket is removed after changing the signs of all the terms inside the bracket.

EXAMPLE **Simplify $4xy - 8\{x^2 - 4x(y - 3x)\} - \{3y^2 - y(2x - y)\}$.**

Solution The given expression $= 4xy - 8\{x^2 - (4xy - 12x^2)\} - \{3y^2 - (2xy - y^2)\}$

$$= 4xy - 8\{x^2 - 4xy + 12x^2\} - \{3y^2 - 2xy + y^2\}$$
$$= 4xy - 8\{13x^2 - 4xy\} - \{4y^2 - 2xy\}$$
$$= 4xy - 104x^2 + 32xy - 4y^2 + 2xy$$
$$= -104x^2 + 38xy - 4y^2.$$

EXAMPLE **Simplify $18x - [-12y - \{9z - (7y - \overline{6x + 2z})\}]$.**

Solution The given expression $= 18x - [-12y - \{9z - (7y - 6x - 2z)\}]$

$$= 18x - [-12y - \{9z - 7y + 6x + 2z\}]$$
$$= 18x - [-12y - \{6x - 7y + 11z\}]$$
$$= 18x - [-12y - 6x + 7y - 11z]$$
$$= 18x - [-6x - 5y - 11z] = 18x + 6x + 5y + 11z$$
$$= 24x + 5y + 11z.$$

Rule of BODMAS

To simplify an algebraic expression containing various operations, we first open brackets (B) then operate 'of' (O), followed by division (D), multiplication (M), addition (A) and subtraction (S) in that order. This is called the rule of BODMAS.

Solved Examples

EXAMPLE 1 **Simplify $4x - 16xy \div 8y + \frac{1}{4}$ of $24x$.**

Solution $4x - 16xy \div 8y + \frac{1}{4}$ of $24x$

$= 4x - 16xy \div 8y + 6x$ [simplifying 'of']

$= 4x - \frac{16xy}{8y} + 6x$ [simplifying '÷']

$= 4x - 2x + 6x$

$= 10x - 2x$ [simplifying '+']

$= 8x.$ [simplifying '–']

EXAMPLE 2 **Simplify $x^2 \div 4x + \frac{1}{4}$ of $4x + 8x \div 8 - \frac{x}{4}$.**

Solution $x^2 \div 4x + \frac{1}{4}$ of $4x + 8x \div 8 - \frac{x}{4}$

$$= x^2 \div 4x + x + 8x \div 8 - \frac{x}{4}$$
$$= \frac{x^2}{4x} + x + \frac{8x}{8} - \frac{x}{4}$$
$$= \frac{x}{4} + x + x - \frac{x}{4}$$
$$= 2x.$$

EXAMPLE 3 **Simplify the expression $6ab$ of $2a \div 12 \times \frac{12}{ab} + 14a - a$.**

Solution

$$6ab \text{ of } 2a \div 12 \times \frac{12}{ab} + 14a - a$$
$$= 12a^2b \div 12 \times \frac{12}{ab} + 14a - a$$
$$= \frac{12a^2b}{12} \times \frac{12}{ab} + 14a - a$$
$$= a^2b \times \frac{12}{ab} + 14a - a$$
$$= 12a + 14a - a$$
$$= 26a - a = 25a.$$

EXERCISE 1E

Simplify the following expressions.

1. $3a - b\{a - (1 - a)\} - b(1 - 2a)$
2. $7(4a - 3) - 3a\{8 - 2a(1 + 2a)\}$
3. $2x - [y + \{y - (x + 2y)\}]$
4. $12x - [3x - 2y - \{y - 2(x - \overline{2x + y})\}]$
5. $2x + 4\{2(x + 2y)\} + 3\{5(4x - \overline{y + 4x})\}$
6. $10x^2 - 3(y^2 - \overline{x^2 - z^2}) - 2\{(x^2 - y^2 + z^2) - z^2 + y^2\}$
7. $5a - [2b - \{4c - (6a - \overline{2b + 3c - 2a})\}]$
8. $15x + 10x \div 2 - \frac{1}{3}$ of $12x$
9. $12a$ of $\frac{1}{2} - 6a^2 \div 3a + (a + a)$
10. $60a^6 \div (4a^3 + a^3) - \frac{3}{5}$ of $(6a^3 + 18a^4 \div 2a)$

ANSWERS

1. $3a$	2. $12a^3 + 6a^2 + 4a - 21$	3. $3x$	4. $11x + 5y$	5. $10x + y$
6. $11x^2 - 3y^2 - 3z^2$	7. $7c - 3a$	8. $16x$	9. $6a$	10. $3a^3$

❖

Revision Exercise 1

1. Write the terms of the polynomial $4x^2 - 9xy + 3y^2 - x - 2y + 7$. Also, find the numerical coefficients of xy and x^2.
2. Subtract $3x^3 - 7x^2 + 5x + 2$ from $2x^3 + 3x + 7$.
3. There are $(12a - 13x)$ members in a club. Each member contributes Rs $(3x + 7a)$. What is the total collection?
4. A person has $2x$ sons. If he wants to divide $(2x^3 - 6yx^2 + 12x)$ m^2 of land equally among his sons, how much land will each of them get?
5. The product of two numbers is $x^4 - y^4$. If one of them is $x - y$, find the other number.
6. Divide $x^8 + x^4y^4 + y^8$ by the product of $x^2 + xy + y^2$ and $x^2 - xy + y^2$.
7. A colony has $(a + 5b)$ buildings. Each building has $(a - 3b)$ flats and each flat has $(a + b)$ rooms. How many rooms are there in the colony?
8. Simplify:

   (i) $x^2 - [2y^2 + x^2 - \{-3x^2 + 5y^2 - (x^2 - 3 + y^2)\}] - (4x^2 + 2y^2 + 3)$

   (ii) $a + [9b - 2a - \{6a + 5b + (3a - \overline{7 - 2b})\}] - (-10a + 2b + 7)$

ANSWERS

1. Terms: $4x^2, -9xy, 3y^2, -x, -2y, 7$; coefficient of $xy = -9$; coefficient of $x^2 = 4$

2. $-x^3 + 7x^2 - 2x + 5$ 3. Rs $(84a^2 - 39x^2 - 55ax)$ 4. $(x^2 - 3xy + 6)$ m^2

5. $x^3 + x^2y + xy^2 + y^3$ 6. $x^4 - x^2y^2 + y^4$ 7. $a^3 + 3a^2b - 13ab^2 - 15b^3$

8. (i) $-8x^2$ (ii) 0

❖