CHAPTER 30

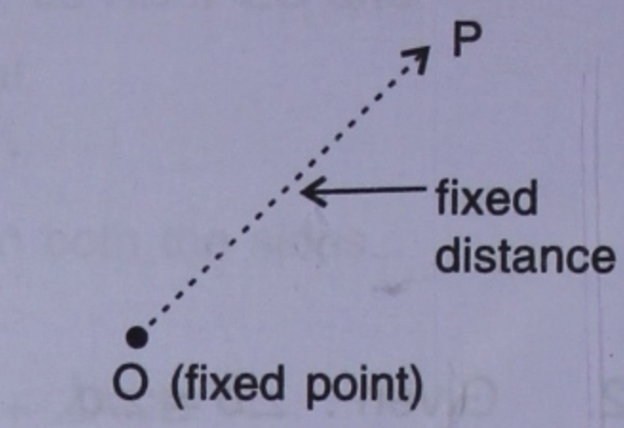
CIRCLES

30.1 DESCRIBING A CIRCLE

Consider a fixed point O and a moving point P such that :

(i) points O and P lie in the same plane.

(ii) the moving point P is always at the fixed (same) distance from the fixed point O.

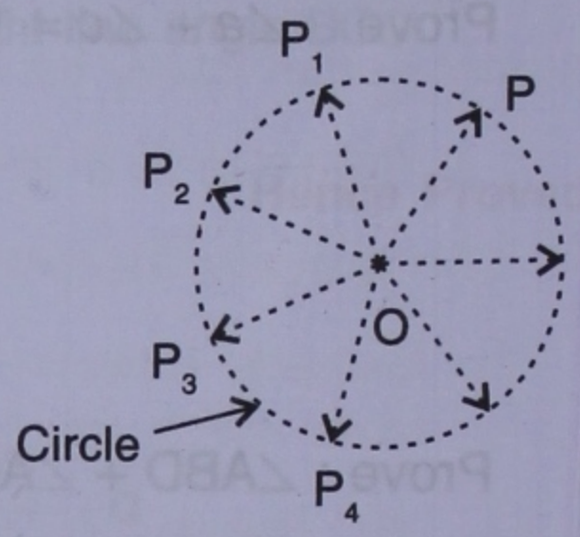


As P moves around fixed point O, mark different positions P_1, P_2, P_3, ..., etc., of it such that

$OP_1 = OP_2 = OP_3 = ... = OP$

Now, draw a free-hand curve passing through the points P, P_1, P_2, P_3, ..., etc.

The figure so obtained is called a **circle.**



The path traced by a moving point, which always remains at a fixed distance from a fixed point, is called a **circle**.

Infact, path traced by the moving point is the boundary of the circle and is called **circumference.**

Whereas, the area of the plane, enclosed by the circumference, is called **circle.**

But, in general, whenever we speak about circle we mean its circumference.

30.2 TERMS ASSOCIATED WITH CIRCLES

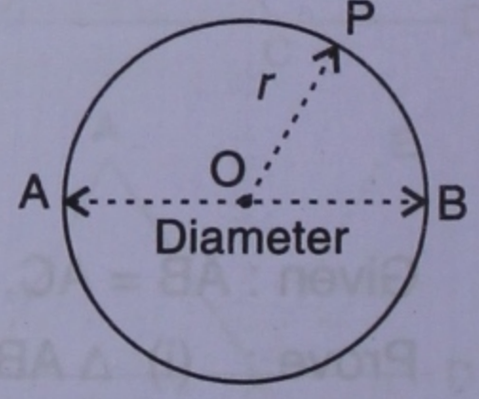
1. **Centre of a circle (O) :**

   It is the fixed point about which the moving point P moves.

2. **Radius of a circle (*r*) :**

   It is the fixed distance between the fixed point O (the centre) and the moving point P.

   In the figure, O is the centre and OP is the radius.



3. **Diameter (AB) :**

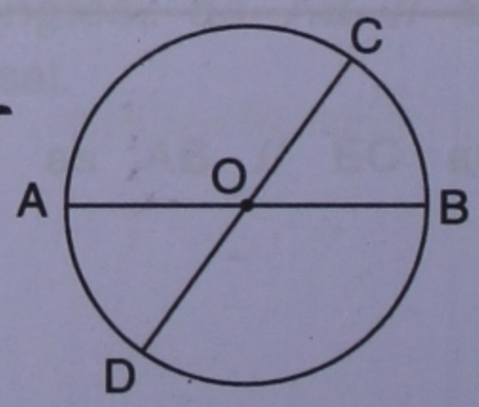
   It is the line segment which passes through the centre of the circle and terminates at both ends by the circumference of the circle.

   The adjoining figure shows the diameter AB which :

   (i) passes through the centre O of the circle.

   (ii) terminates at both the ends A and B at the circumference of the circle.

   Similarly, CD is also a diameter of the circle.



For every circle : (i) Diameter = 2 × Radius

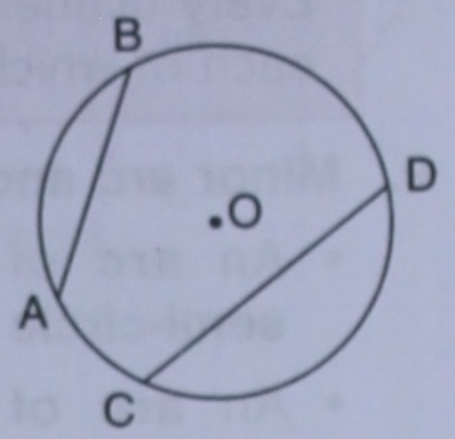
(ii) Radius = $\frac{\text{Diameter}}{2}$

4. Chord :

It is the line segment joining any two points on the circumference of a circle.

In the given figure; AB and CD are two chords. Similarly, more chords can be drawn.

The **largest chord** of a circle is its **diameter**.

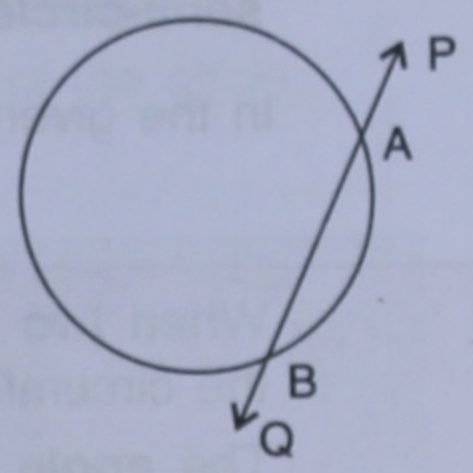


5. Secant :

It is the line which meets the circle at two points.

In the given figure, line PQ is a secant as this line meets the circle at two points A and B.

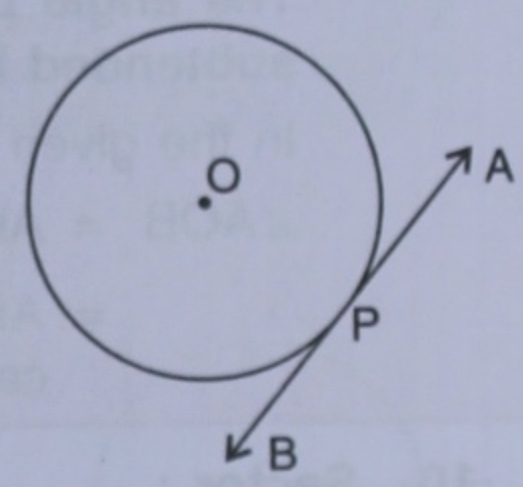
A line can meet the circle at the most at two points.



6. Tangent :

It is a line which meets the circle at one point only.

In the given figure, line AB is a tangent as AB meets the given circle only at one point *i.e.* at point P.



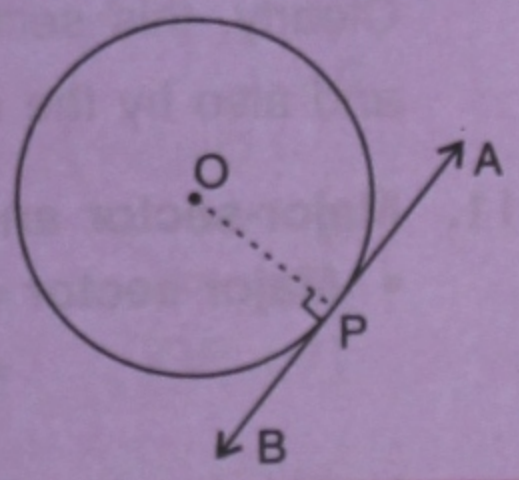
- The point on the circle, at which a tangent meets the circle, is called the **point of contact**.

  In the adjoining figure, point P is the point of contact.
- At the point of contact, angle between the radius and the tangent is 90°.

  In other words, at the point of contact, the radius and the tangent are perpendicular to each other.

  In the adjoining figure, at the point of contact P, the radius OP and the tangent AB are perpendicular to each other.

  For this reason; ∠OPA = ∠OPB = 90°.

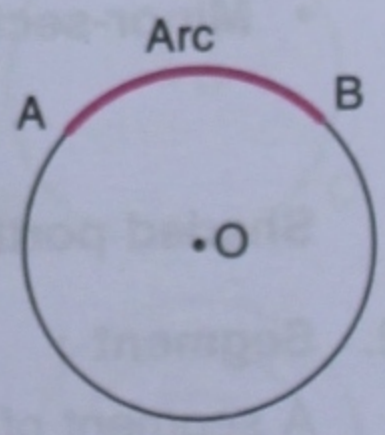


7. Arc :

Arc of a circle is a continuous part of its circumference.

In the given figure, AB is a continuous part of the circumference of the given circle, so $\overset{\frown}{AB}$ is an arc of this circle.

Arc AB = $\overset{\frown}{AB}$, arc CD = $\overset{\frown}{CD}$, arc PQ = $\overset{\frown}{PQ}$ and so on.

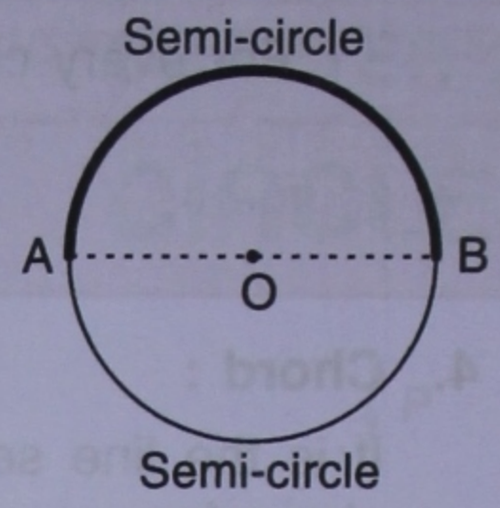


The length of the whole arc of a circle = Circumference of the circle

8. Semi-circle :

- It is one-half of the whole circle.
- It is one-half of the circumference of a circle.

Every diameter divides a circle into two equal parts, each of which is a semi-circle.

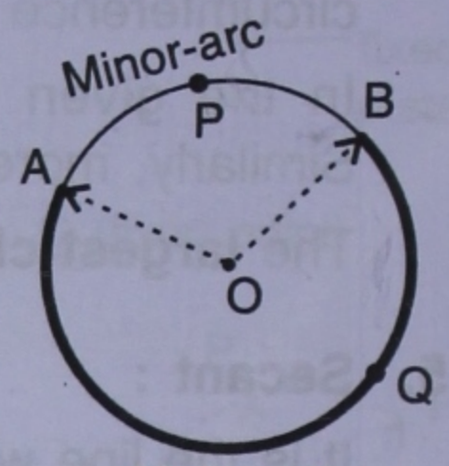


9. Minor arc and major arc :

- An **arc** of a circle which is **smaller than** its **semi-circle** is called a **minor-arc**.
- An **arc** of a circle which is **greater than** its **semi-circle** is called a **major-arc**.

In the given figure : (i) $\overset{\frown}{APB}$ is a minor-arc.

(ii) $\overset{\frown}{AQB}$ is a major-arc.



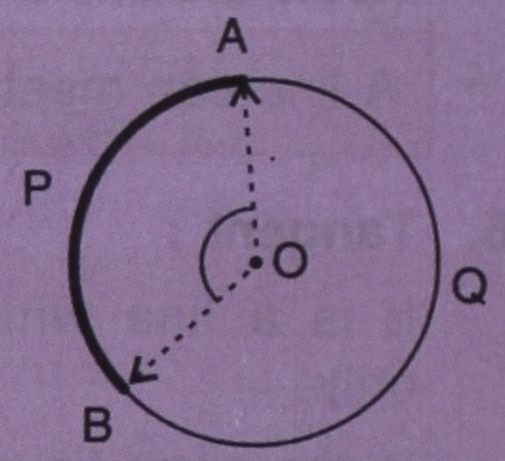
When two radii of a circle are drawn, they divide the circumference of the circle into two arcs.

The **angle between** these **two radii** is the **angle subtended by the** arc at the centre.

In the given figure :

$\angle AOB$ = Angle between radii OA and OB.

= Angle subtended by arc $\overset{\frown}{APB}$ at the centre of the circle.

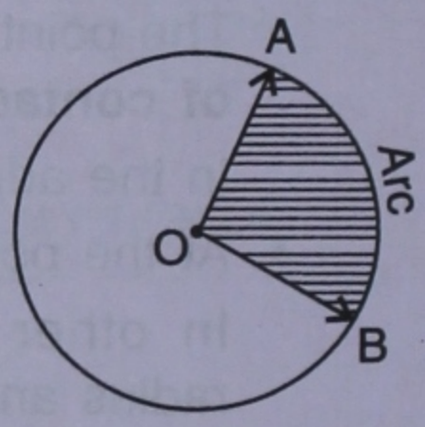


10. Sector :

A sector of a circle is the part of the circle bounded by two of its radii and the arc cut by these radii.

In the given figure, the shaded portion of the circle is a sector of this circle.

Clearly, this sector is bounded by radii OA and OB and also by the arc $\overset{\frown}{AB}$ cut by these radii.



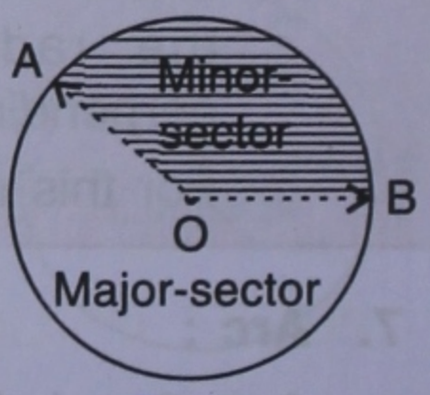
11. Major-sector and minor-sector :

- **Major-sector** = Sector greater than semi-circle.
  = Sector bounded by two radii and the major arc.

Unshaded portion of the given circle is major-sector.

- **Minor-sector** = Sector smaller than semi-circle.
  = Sector bounded by two radii and the minor-arc.

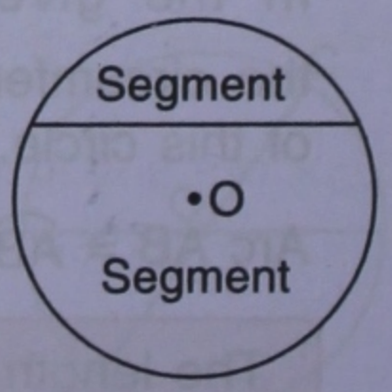
Shaded portion of the given circle is minor-sector.



12. Segment :

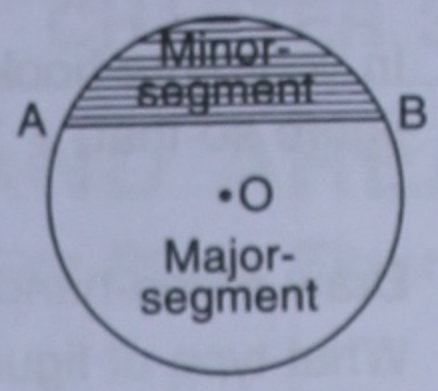
A segment of a circle is the part of the circle bounded by one of its chord and arc of the circle.

Infact, every chord of a circle divides the circle into two parts and each of these parts is a **segment**.



13. Major-segment and minor-segment :

In the given figure, chord AB divides a circle into two segments.



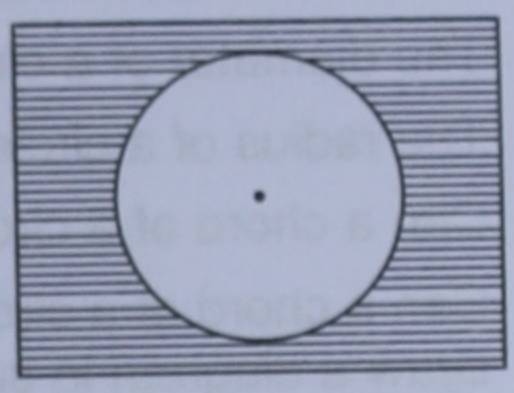
- **Major-segment** = Segment greater than semi-circle.
  = Segment bounded by the chord and the major-arc.
  = Segment which contains centre of the circle.

Unshaded portion of the given circle is major-segment.

- **Minor-segment** = Segment smaller than semi-circle.
  = Segment bounded by the chord and the minor-arc.
  = Segment which does not contain centre of the circle.

Shaded portion of the given circle is minor-segment.

14. Interior and exterior of a circle :

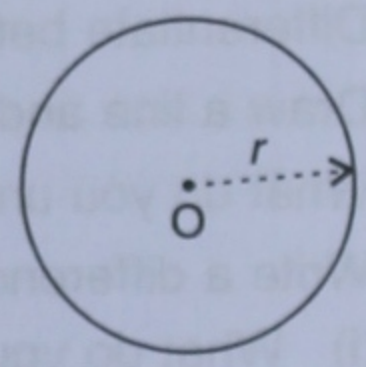


- The **space inside a circle** is called its **interior**.
- The **space outside a circle** is called its **exterior**.
- In the given figure, the **unshaded portion** is **interior** of the circle and the **shaded portion** is **exterior** of the circle.

15. Position of a point :

Draw a circle with centre O and radius *r*.

Now, consider a point P in the same plane as that of the circle, such that :

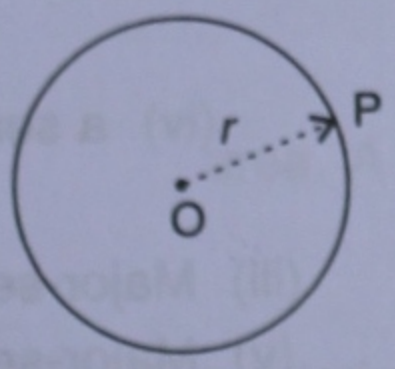


(i) **OP = radius** ; *the point P lies on the circumference of the circle.*

(ii) **OP > radius**; *the point P lies outside the circle.*

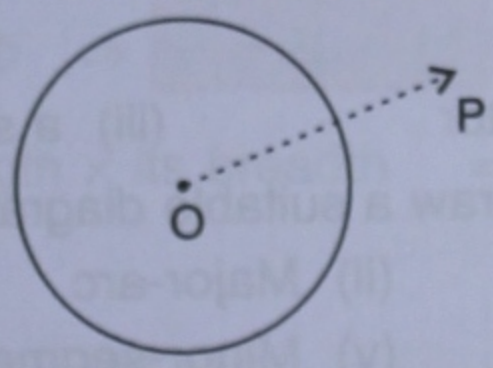
(iii) **OP < radius**; *the point P lies inside the circle.*

(i)



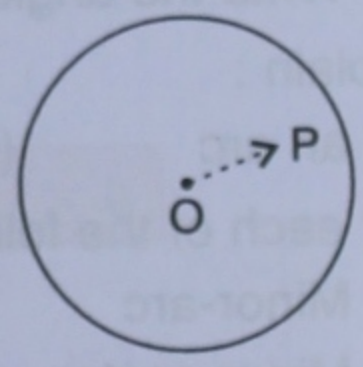
[OP = *r*]

(ii)



[OP > *r*]

(iii)

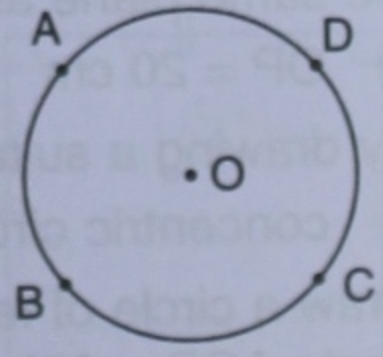


[OP < *r*]

16. Concyclic points :

Three or more points, which lie on the circumference of the same circle, are called **concyclic points**.

In the given figure; A, B, C and D are concyclic points as they lie on the circumference of the same circle with centre O.



18. Concentric circles :

The circles which have the same centre but different radii are called **concentric circles.**

The given figure shows concentric circles as all these circles have same centre but different radii.

EXERCISE 30

1. In your note-book, mark a point O. Then mark six points P, Q, R, S, T and U in the same figure so that,

   $$OP = OQ = OR = OS = OT = OU = 5 \text{ cm}$$

   Draw a free-hand curve passing through points P, Q, R, S, T and U.

   What type of figure do you get ?

2. What do you understand by :

   (i) the circumference of a circle ? (ii) a circle ?

   Draw a diagram representing a circle and its circumference.

3. Centre of a circle is a fixed point inside the circle. Is this statment true ?

4. The distance between the centre of a circle and any point on its circumference is equal to the radius of the circle. State, ***true*** or ***false***.

5. Write the relation between the lengths of the radius of a circle and its diameter.

6. The diameter of a circle is 8·4 cm; find the length of its radius.

7. The radius of a circle is 8·4 cm; find the length of its diameter.

8. Can a chord of a circle be more than its diameter ? Give reason.

9. Can a chord of a circle be smaller than its radius ?
   Draw a diagram in support of your answer.

10. In a circle of radius 5 cm; a chord is drawn that passes through the centre of the circle. Find the length of this chord and write a special name of it.

11. Differentiate between a chord and a secant of a circle.

12. Draw a line and a circle three times to show the different positions between them.

13. What do you understand by a tangent of a circle ?

    Write a difference between a secant and a tangent.

14. (i) What do you understand by the point of contact ?

    (ii) Write the angle between a tangent and a radius of the circle at the point of contact.

15. Explain :

    (i) an arc (ii) a sector (iii) a segment (iv) a semi-circle

16. For each of the following, draw a suitable diagram :

    (i) Minor-arc (ii) Major-arc (iii) Major-sector

    (iv) Minor-sector (v) Minor-segment (v) Major-segment.

17. O is the centre of a circle with radius 10 cm. State the location of a point P which lies in the same plane as that of the circle such that :

    (i) OP = 20 cm (ii) OP = 5 cm (iii) OP = 10 cm

18. By drawing a suitable diagram explain :

    (i) concentric circles (ii) concyclic points.

19. Draw a circle of radius 5 cm with centre O. Draw radii OA and OB of this circle such that angle AOB = 60°.

    Shade the major-sector by horizontal lines and the minor-sector by vertical lines.

20. Draw a circle of radius 4·4 cm and a chord of it with length 6 cm.

    Shade the minor-segment of it with horizontal lines and the major segment of it with vertical lines.