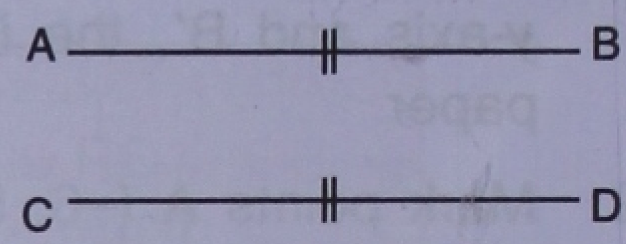
CHAPTER 26

CONGRUENCY : CONGRUENT TRIANGLES

26.1 MEANING OF CONGRUENCY

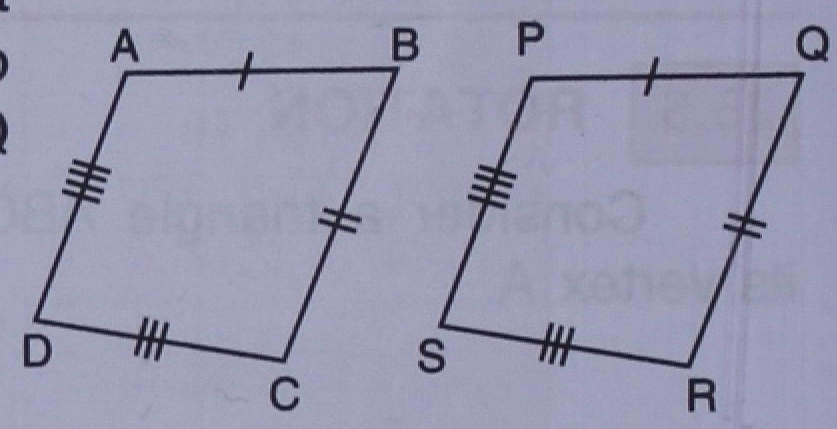
If two geometrical figures coincide exactly, by placing one over the other, the figures are said to be congruent to each other.

1. Two lines AB and CD are said to be congruent if, on placing AB on CD, or CD on AB, the two lines AB and CD exactly coincide.



It is possible only when AB and CD are equal in length.

2. Two figures ABCD and PQRS are said to be congruent if, on placing ABCD on PQRS or PQRS on ABCD the two figures exactly coincide, *i.e.,* A and P coincide, B and Q coincide, C and R coincide and D and *S* coincide.



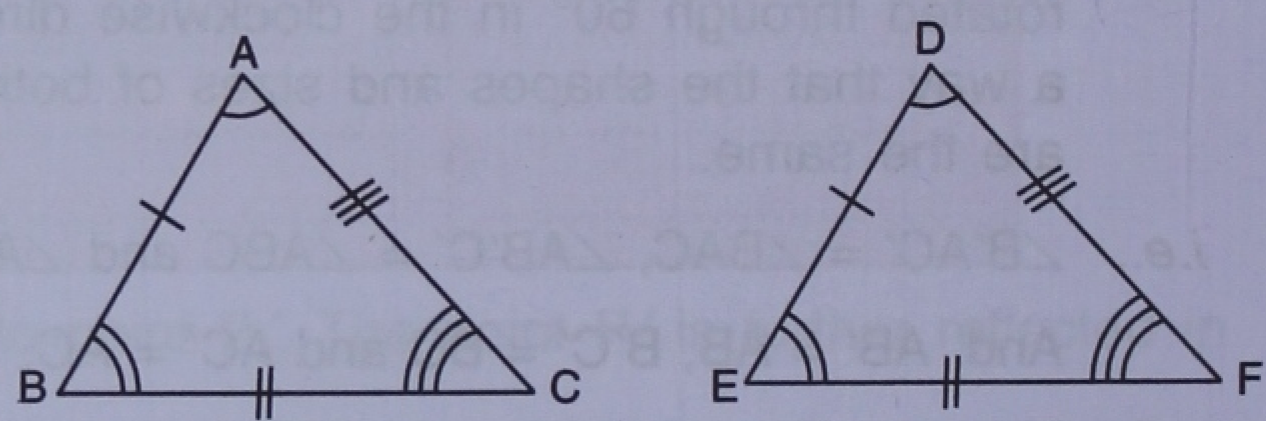
It is possible only when :

AB = PQ, BC = QR, CD = RS and AD = PS

Also, ∠A = ∠P, ∠B = ∠Q, ∠C = ∠R and ∠D = ∠S.

26.2 CONGRUENCY IN TRIANGLES

Let triangle ABC is placed over triangle DEF; such that, vertex A falls on vertex D and side AB falls on side DE, then if the two triangles coincide with each other in such a way that B falls on E, C falls on F; side BC coincides with side EF and side AC coincides with side DF, then the two triangles are congruent to each other.



The symbol used for congruency is "≡" or "≅".

∴ **Δ ABC is congruent to Δ DEF** is written as : **Δ ABC ≡ Δ DEF or Δ ABC ≅ Δ DEF.**

26.3 CORRESPONDING SIDES AND CORRESPONDING ANGLES

In case of congruent triangles ABC and DEF, as given above, *the sides* of the two triangles, *which coincide with each other, are called corresponding sides.*

Thus, the side AB and DE are corresponding sides, sides BC and EF are corresponding sides and sides AC and DF are also corresponding sides.

In the same way, *the angles* of the two triangles *which coincide with each other, are called corresponding angles.* Thus, three pairs of corresponding angles are ∠A and ∠D, ∠B and ∠E and also ∠C and ∠F.

The corresponding parts of congruent triangles are always equal (congruent).

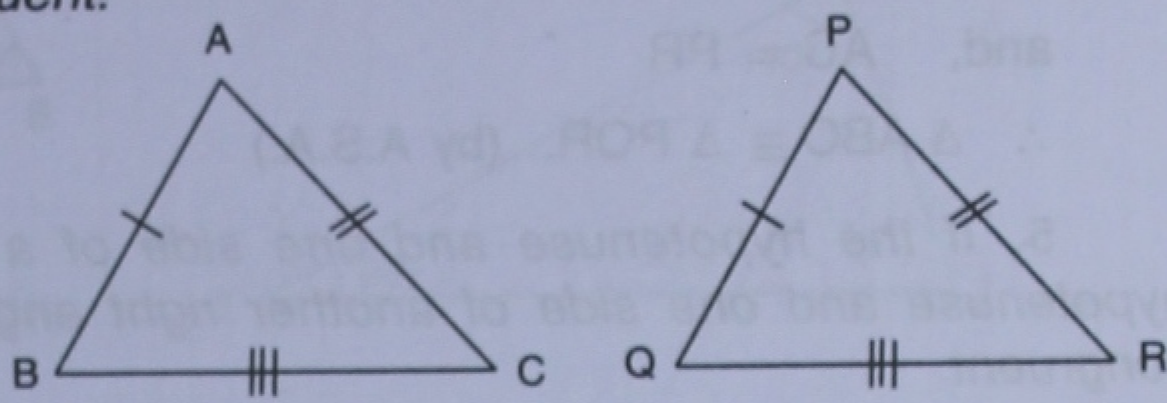
∴ (i) AB = DE, BC = EF and AC = DF, *i.e., corresponding sides are equal.*

Also, (ii) ∠A = ∠D, ∠B = ∠E and ∠C = ∠F, *i.e., corresponding angles are equal.*

26.4 CONDITIONS OF CONGRUENCY

1. *If three sides of one triangle are equal to three sides of the other triangle, each to each, then the two triangles are congruent.*

This condition is known as : **side, side, side** and is abbreviated as **S.S.S.**



In triangles ABC and PQR, given alongside :

AB = PQ,

BC = QR and AC = PR

So Δ ABC is congruent to Δ PQR, *i.e.,* Δ ABC ≅ Δ PQR by S.S.S.

Similarly, in congruent triangles, corresponding angles are equal.

∴ ∠A = ∠P, ∠B = ∠Q and ∠C = ∠R

2. *If two sides and the included angle of one triangle are equal to two sides and the included angle of the other triangle, each to each, then the triangles are congruent.*

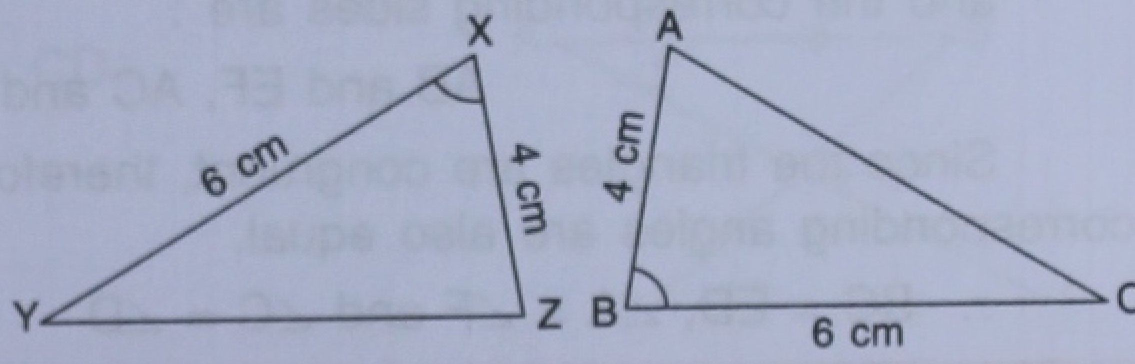
This condition is known as : **side, angle, side** and is abbreviated as **S.A.S.**

In the given triangles,

AB = XZ,

BC = XY and ∠ABC = ∠ZXY

∴ Δ ABC ≡ Δ ZXY (by S.A.S.)



Triangles will be congruent by S.A.S., only when the angles included by the corresponding equal sides are equal.

The pairs of corresponding sides of these two congruent triangles are :

AB and ZX, BC and XY, AC and ZY

The pairs of corresponding angles are :

∠B and ∠X, ∠A and ∠Z, ∠C and ∠Y.

3. *If two angles and the included side of one triangle are equal to the two angles and the included side of the other triangle, then the triangles are congruent.*

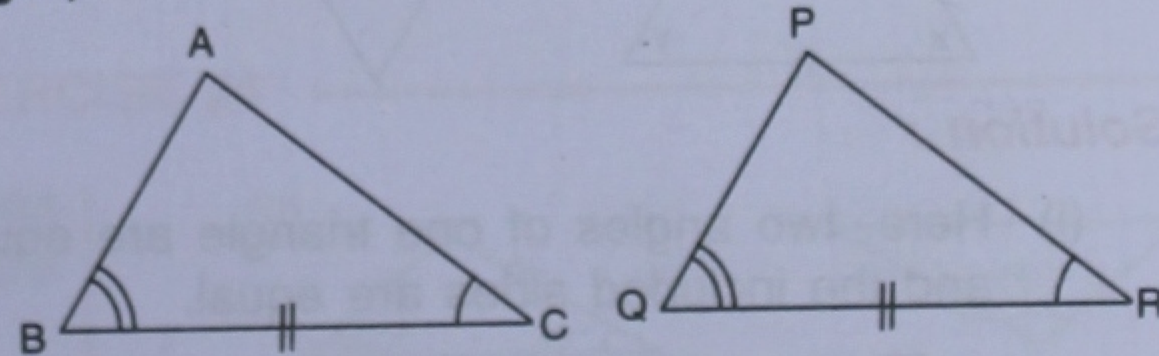
This condition is known as : **angle, side, angle** and is abbreviated as **A.S.A**.

In the given figure :

BC = QR,

∠B = ∠Q and ∠C = ∠R

∴ Δ ABC ≡ Δ PQR. (by A.S.A.)



4. *If any two angles and a side (not the included side) of one triangle are equal to two angles and the corresponding side of the other triangle; then the two triangles are congruent.*

This condition is known as : **angle, angle, side** and is abbreviated as : **A.A.S.**

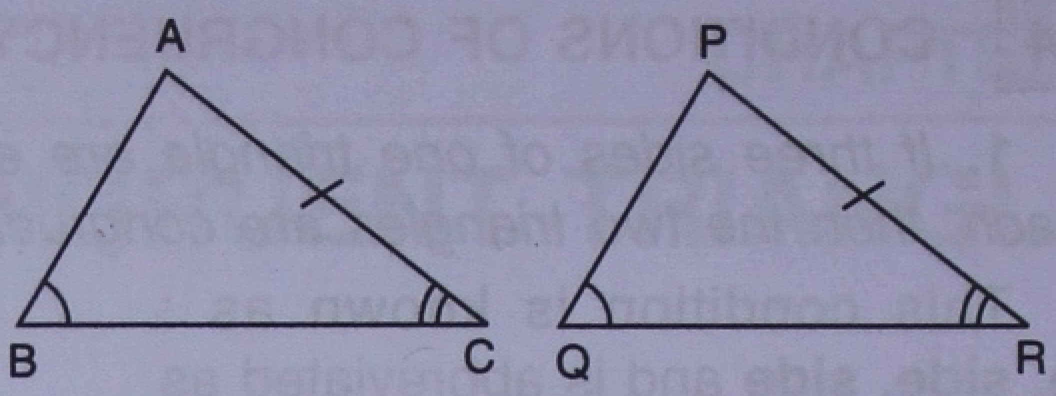
In the given figure :

$\angle B = \angle Q$,

$\angle C = \angle R$

and, $AC = PR$

$\therefore$ $\Delta ABC \cong \Delta PQR$. (by A.S.A.)



5. *If the hypotenuse and one side of a right angled triangle are equal to the hypotenuse and one side of another right angled triangle, then the two triangles are congruent.*

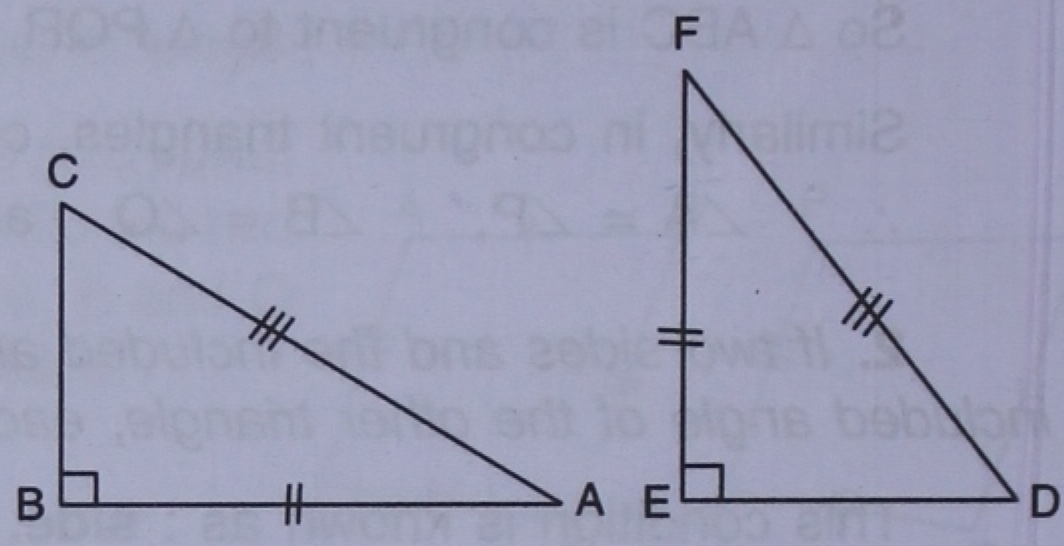
This condition is known as: **right angle, hypotenuse, side** and is abbreviated as **R.H.S.**

In the given figure :

$\angle B = \angle E = 90°$, $AB = FE$

and hypotenuse AC = hypotenuse FD

$\therefore$ $\Delta ABC \equiv \Delta FED$ (by R.H.S.)



The corresponding angles in this case are :

$\angle A$ and $\angle F$, $\angle B$ and $\angle E$, $\angle C$ and $\angle D$,

and the corresponding sides are :

AB and EF, AC and FD, BC and ED.

Since the triangles are congruent, therefore all its corresponding sides are equal and corresponding angles are also equal.

$\therefore$ $BC = ED$, $\angle A = \angle F$ and $\angle C = \angle D$

If three angles of a triangle are equal to the three angles of the other triangle, then the triangles are ***not necessarily congruent.***

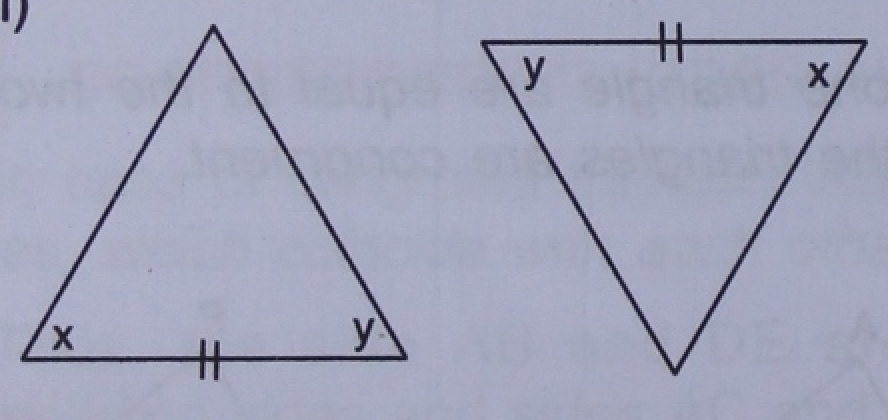
For congruency at least ***one pair of corresponding sides must be equal.***

$\therefore$ A.A.A. is not a test of congruency.

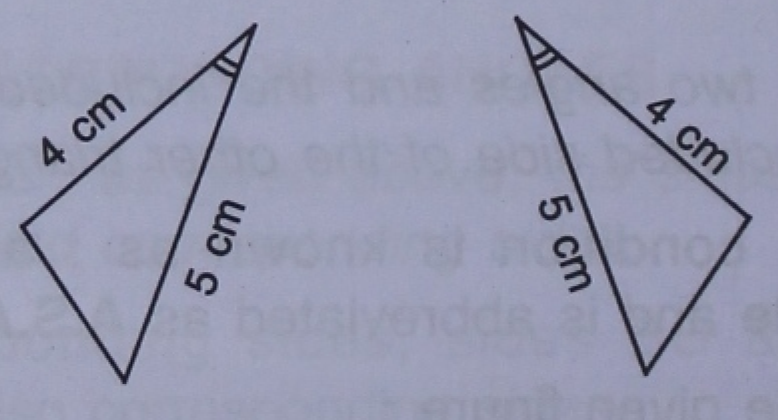
Example 1 :

State, by what test the following triangles are congruent.

(i)



(ii)



Solution :

(i) Here, two angles of one triangle are equal to the two angles of the other triangle and the included sides are equal.

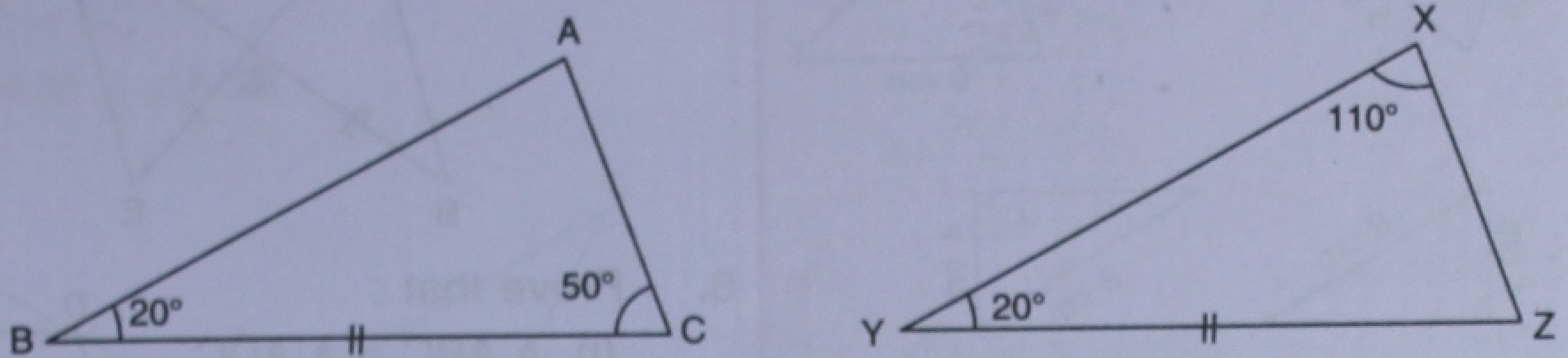
$\therefore$ The two **triangles are congruent by A.S.A.** **(Ans.)**

(ii) Since, two sides of one triangle are equal to the two sides of the other triangle and the included angles are equal.

$\therefore$ The **triangles are congruent by S.A.S.** **(Ans.)**

Example 2 :

State, whether or not, the following triangles are congruent.



Solution :

Here, $\angle A = 180° - (20° + 50°) = 110°$

Also, $\angle Z = 180° - (110° + 20°) = 50°$

We see that in Δ ABC and Δ XYZ,

$\angle C = \angle Z = 50°$

$\angle B = \angle Y = 20°$ and BC = YZ and they are included sides.

$\therefore$ **Δ ABC $\cong$ Δ XYZ** by A.S.A. **(Ans.)**

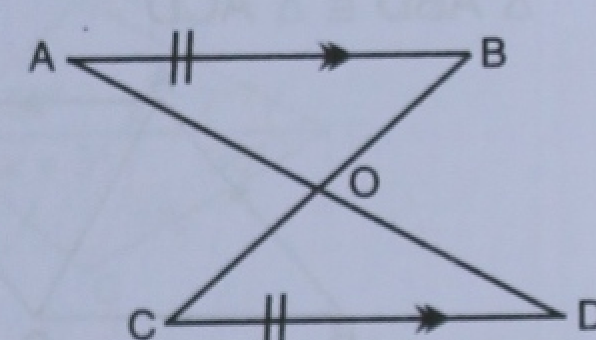
Example 3 :

In the given figure, AB // CD and AB = CD.

Prove that : (i) Δ AOB $\cong$ Δ DOC

(ii) AO = DO

(iii) BO = CO



Solution :

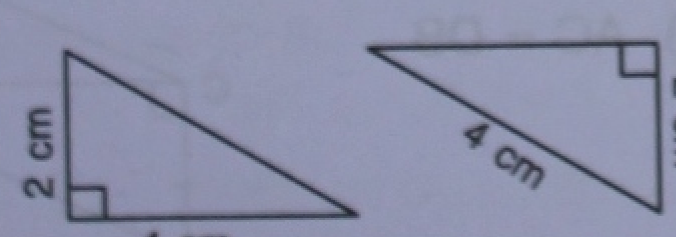
Statement	Reason
In triangles AOB and COD	
AB = CD	Given
$\angle BAO = \angle CDO$	Alternate angles, as AB // CD
$\angle ABO = \angle DCO$	Alternate angles, as AB // CD
$\therefore$ (i) Δ AOB $\cong$ Δ DOC	A.S.A.
(ii) AO = DO	Corresponding sides of congruent triangles.
also, (iii) BO = CO	Corresponding sides of congruent triangles.

Hence proved.

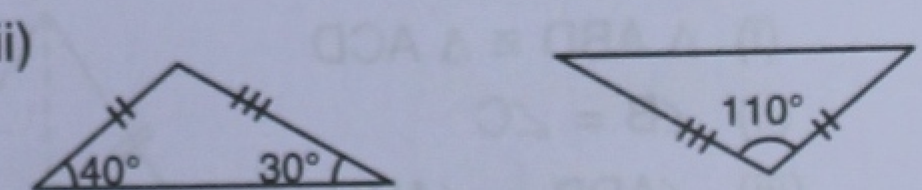
EXERCISE 26

1. State, whether the pairs of triangles given in the following figures are congruent or not :

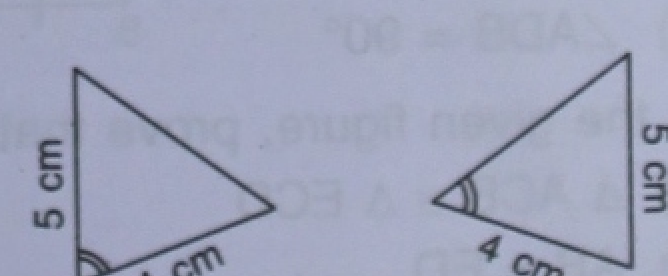
(i)

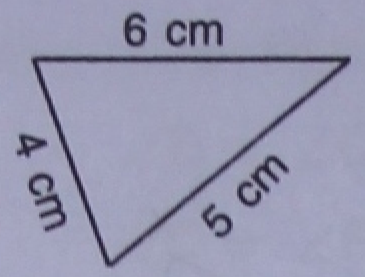


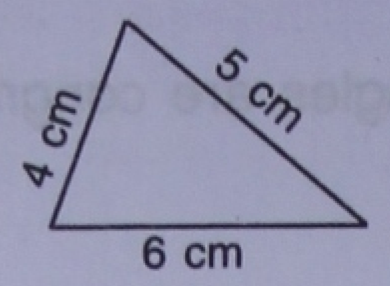
(ii)

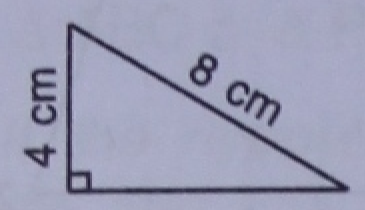


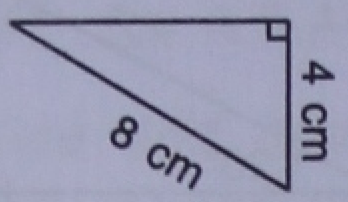
(iii)

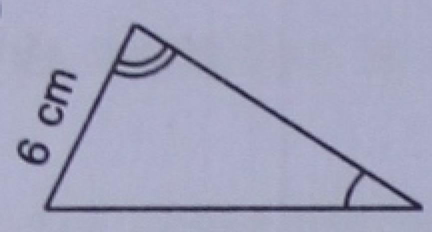


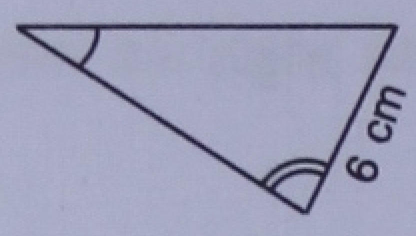
(iv) 



(v) 

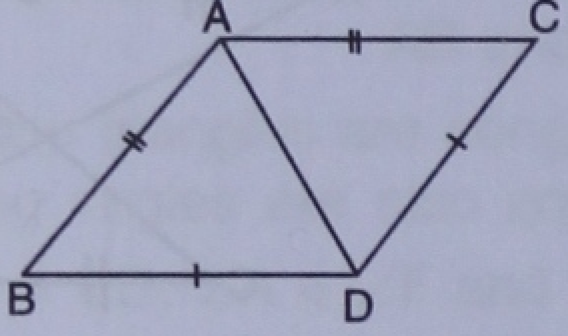


(vi) 

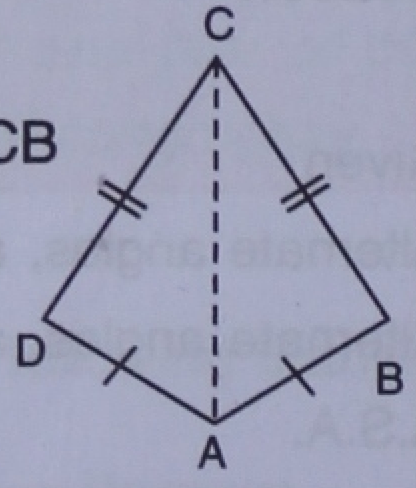


(vii) Δ ABC in which AB = 2 cm, BC = 3.5 cm and ∠C = 80°.
and, Δ DEF in which DE = 2 cm, DF = 3.5 cm and ∠D = 80°.

2. In the given figure, prove that :
Δ ABD ≅ Δ ACD

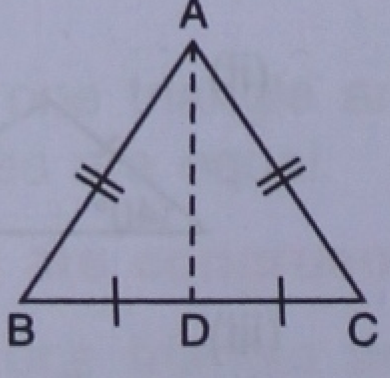


3. Prove that :
(i) Δ ABC ≅ Δ ADC
(ii) ∠B = ∠D
(iii) AC bisects angle DCB

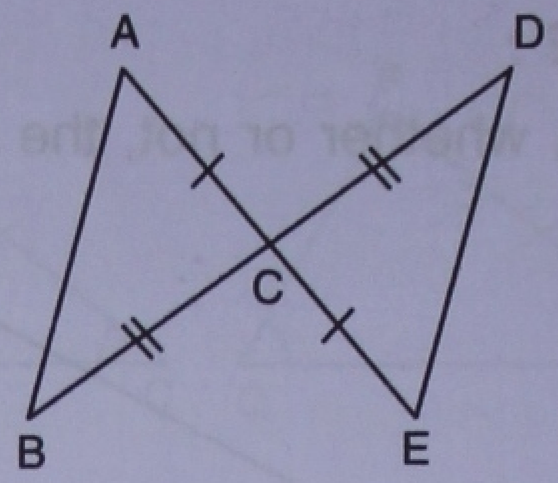


(iii) As Δ ABC ≅ Δ ADC
⇒ ∠BCA = ∠DCA
and so AC bisects angle DCB.

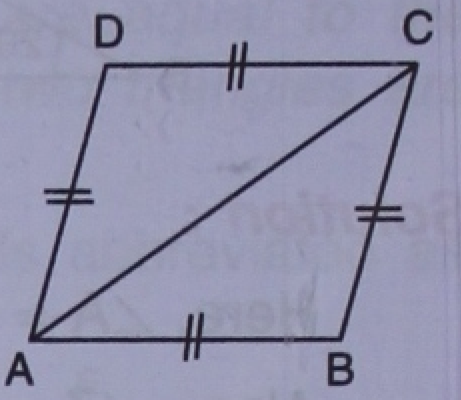
4. Prove that :
(i) Δ ABD ≅ Δ ACD
(ii) ∠B = ∠C
(iii) ∠ADB = ∠ADC
(iv) ∠ADB = 90°



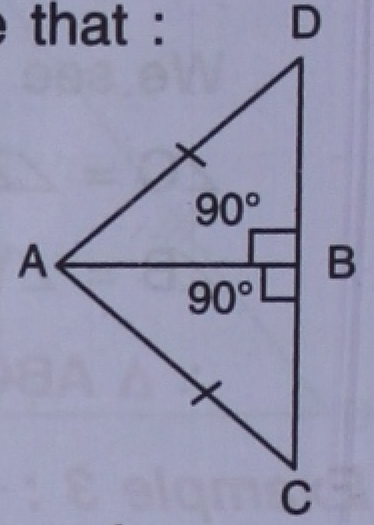
5. In the given figure, prove that :
(i) Δ ACB ≅ Δ ECD
(ii) AB = ED



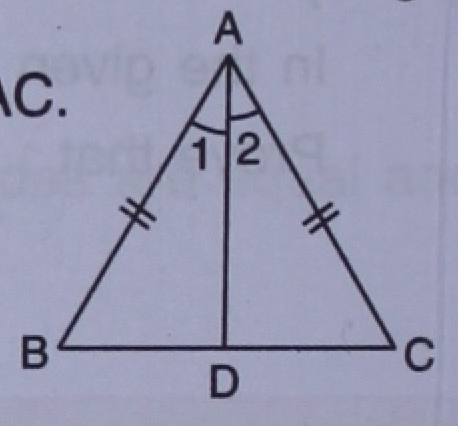
6. Prove that :
(i) Δ ABC ≅ Δ ADC
(ii) ∠B = ∠D



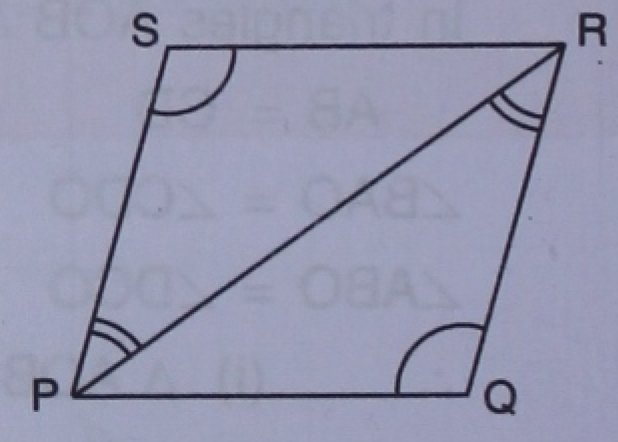
7. In the given figure, prove that :
BD = BC.



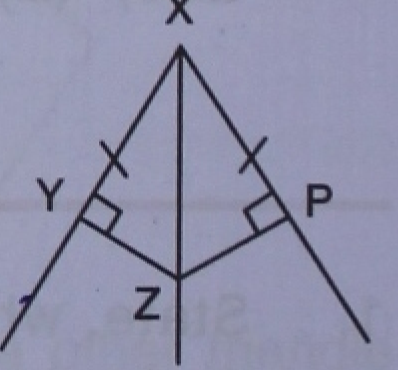
8. In the given figure,
∠1 = ∠2 and AB = AC.
Prove that :
(i) ∠B = ∠C
(ii) BD = DC
(iii) AD is perpendicular to BC.



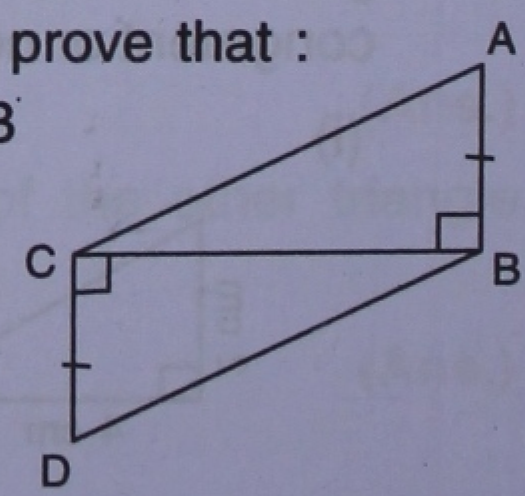
9. In the given figure, prove that :
(i) PQ = RS
(ii) PS = QR



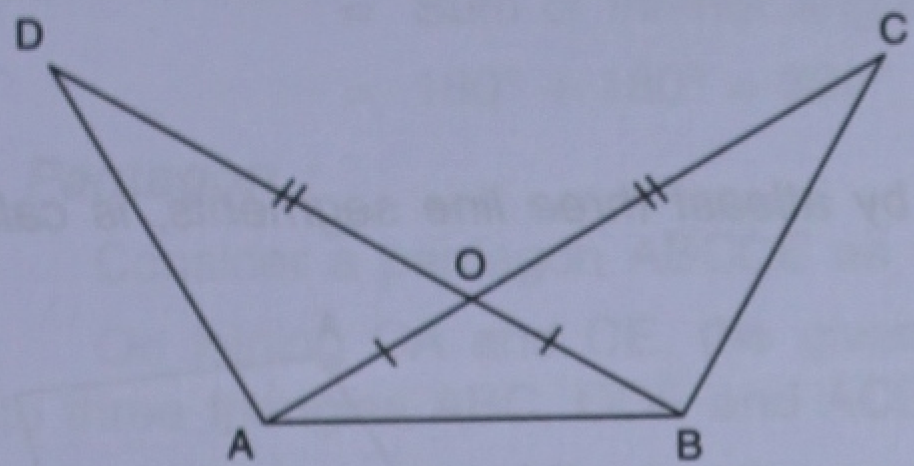
10. In the given figure, prove that :
(i) Δ XYZ ≅ Δ XPZ
(ii) YZ = PZ
(iii) ∠YXZ = ∠PXZ



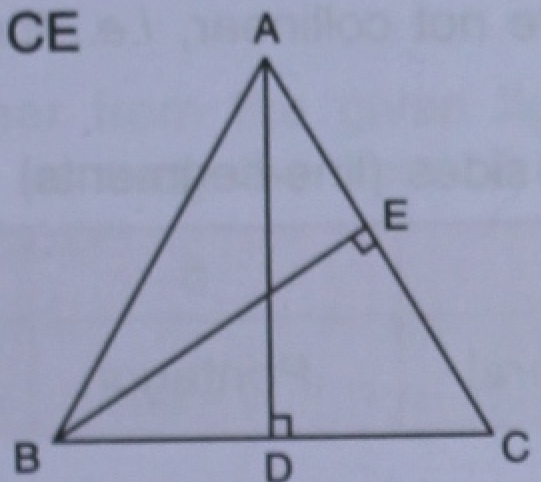
11. In the given figure, prove that :
(i) Δ ABC ≅ Δ DCB
(ii) AC = DB



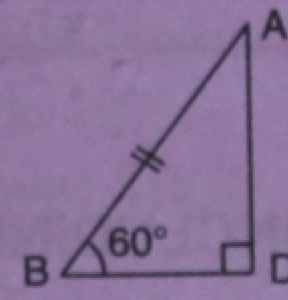
12. In the given figure, prove that :
   (i) Δ AOD ≅ Δ BOC
   (ii) AD = BC
   (iii) ∠ADB = ∠ACB

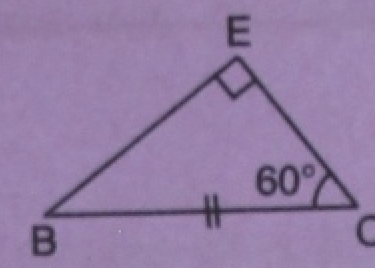


13. ABC is an equilateral triangle, AD and BE are perpendiculars to BC and AC respectively. Prove that :
   (i) AD = BE
   (ii) BD = CE



Consider Δ ADB and Δ CEB :





- AB = BC (sides of same equilateral triangle)
- ∠ADB = ∠BEC (each 90°)
- ∠ABD = ∠BCE (each 60°)

∴ Δ ADB ≅ Δ BEC by A.A.S.

Since, the corresponding parts of the triangles are congruent, therefore:
(i) AD = BE and (ii) BD = CE

14. Use the informations given in the following figure to prove triangles ABD and CBD are congruent.
   Also, find the values of *x* and *y*.