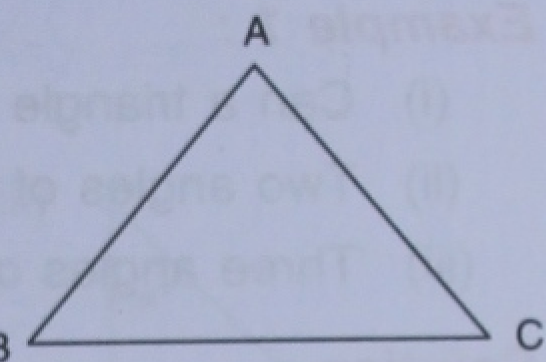
CHAPTER 24

TRIANGLES

24.1 REVIEW

1. Definition of a triangle :

A closed figure, having 3 sides, is called ***a triangle*** and is usually denoted by the Greek letter Δ (delta).



The figure, given alongside, shows a triangle ABC (Δ ABC) bounded by three sides AB, BC and CA.

2. Vertex :

The point, where any two sides of a triangle meet, is called a ***vertex.***

Clearly, the given triangle has three vertices, namely : A, B and C.

[Vertices is the plural of vertex]

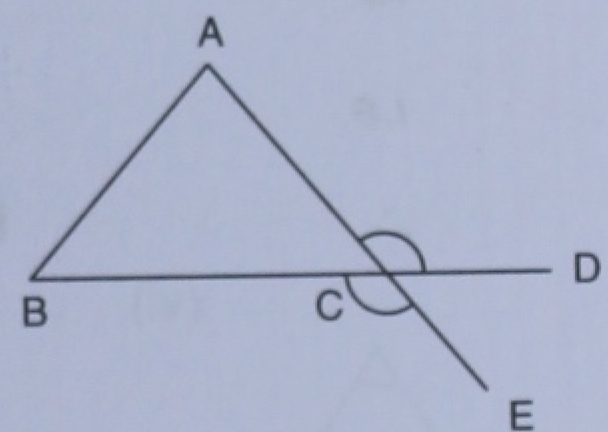
3. Interior angles :

In Δ ABC (given above), the angles BAC, ABC and ACB are called its interior angles as they lie inside the Δ ABC.

[The sum of interior angles of a triangle is always 180°]

4. Exterior angles :

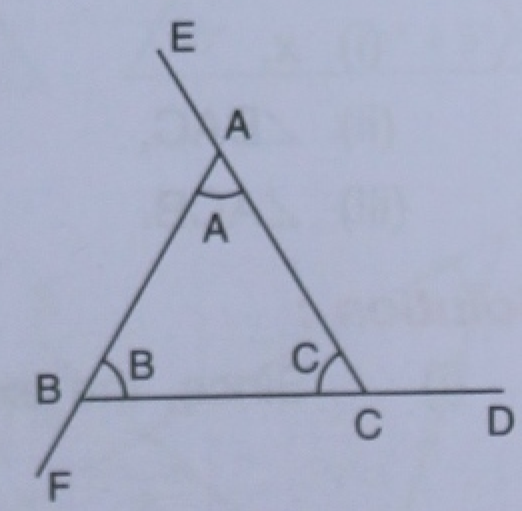
When any side of a triangle is produced the angle so formed, outside the triangle and at its vertex, is called its ***exterior angle.***



For a given triangle ABC, if side BC is produced to the point D, then ∠ACD is its exterior angle. And, if side AC is produced to the point E, then the exterior angle would be ∠BCE.

Thus, at every vertex, two exterior angles can be formed and that these two angles being vertically opposite angles, are always equal.

Also, at each vertex of a triangle, the sum of the exterior angle and its corresponding interior angle is 180°.



In ΔABC, given alongside,

Exterior angle + Interior angle = 180°

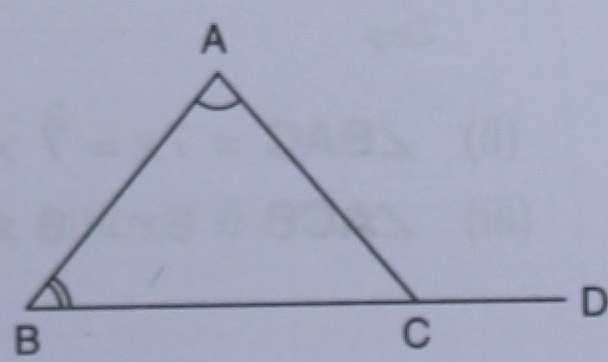
⇒ At vertex A : ∠BAE + ∠A = 180°

At vertex B : ∠CBF + ∠B = 180° and

At vertex C : ∠ACD + ∠C = 180°

5. Interior opposite angles :

When any side of a triangle is produced, an exterior angle is formed. The two interior angles of this triangle, that are opposite to the exterior angle formed, are called its ***interior opposite angles.***



In the given figure, side BC of Δ ABC is produced to the point D, so that the exterior ∠ACD is formed. Then the two interior opposite angles are ∠BAC and ∠ABC.

6. Relation between exterior angle and interior opposite angles :

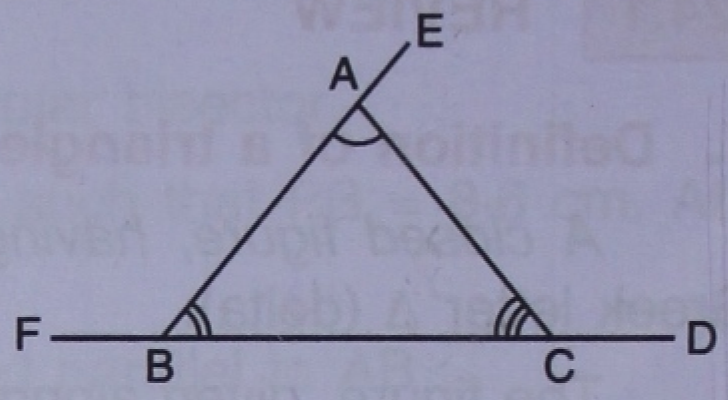
Exterior angle of a triangle is always equal to the sum of its two interior opposite angles.

Thus in the figure, given above, $\angle ACD = \angle BAC + \angle ABC$.

Similarly; in the triangle ABC, drawn alongside,

Exterior angle CAE = $\angle B + \angle C$

and exterior angle ABF = $\angle A + \angle C$.



Example 1 :

(i) Can a triangle have angles 60°, 70° and 70° ?

(ii) Two angles of a triangle are 48° and 73°, find its third angle.

(iii) Three angles of a triangle are $(2x + 20)°$, $(x + 30)°$ and $(2x - 10)$. Find the angles.

Solution :

(i) Since, 60° + 70° + 70° = 200°

$\Rightarrow$ **A triangle can not have angles 60°, 70° and 70°** **(Ans.)**

[**Remember :** Sum of the angles of a triangle is always 180°]

(ii) Sum of two given angles = 48° + 73° = 121°

$\Rightarrow$ **The third angle** = 180° – 121° = **59°** **(Ans.)**

(iii) Since, the sum of the interior angles of a triangle = 180°

$\therefore \quad (2x + 20) + (x + 30) + (2x - 10) = 180°$

$\Rightarrow \quad 5x + 40 = 180°$

i.e. $\quad 5x = 180 - 40 = 140$ and $x = \frac{140}{5} = 28$

$\therefore$ **Required angles** = $(2x + 20)°$, $(x + 30)°$ and $(2x - 10)°$

$= (2 \times 28 + 20)°, (28 + 30)°$ and $(2 \times 28 - 10)°$

= **76°**, **58°** and **46°** **(Ans.)**

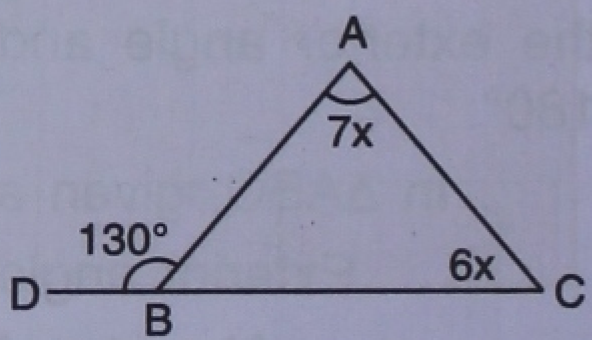
Example 2 :

Use the figure, given alongside, to find the value of :

(i) x,

(ii) $\angle BAC$,

(iii) $\angle ACB$.



Solution :

(i) Since, the exterior angle of a Δ = sum of its two interior opposite angles

$\therefore \quad 130° = 7x + 6x$

$\Rightarrow \quad 13x = 130°$

$\Rightarrow \quad$ $\boldsymbol{x} = \frac{130°}{13} = $ **10°** **(Ans.)**

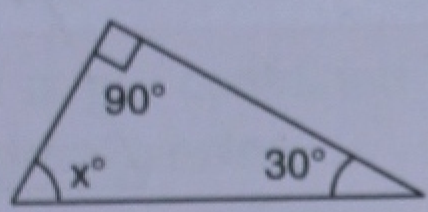
(ii) $\angle$**BAC** = $7x = 7 \times 10°$ = **70°** **(Ans.)**

(iii) $\angle$**ACB** = $6x = 6 \times 10°$ = **60°** **(Ans.)**

EXERCISE 24(A)

1. State, if the triangles are possible with the following angles :
   - (i) 20°, 70° and 90°
   - (ii) 40°, 130° and 20°
   - (iii) 60°, 60° and 50°
   - (iv) 125°, 40° and 15°
2. If the angles of a triangle are equal, find its angles.
3. In a triangle ABC, ∠A = 45° and ∠B = 75°, find ∠C.
4. In a triangle PQR, ∠P = 60° and ∠Q = ∠R, find ∠R.
5. Calculate the unknown marked angles in each figure :

(i)



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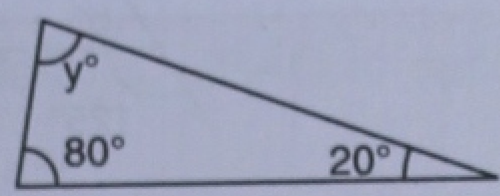

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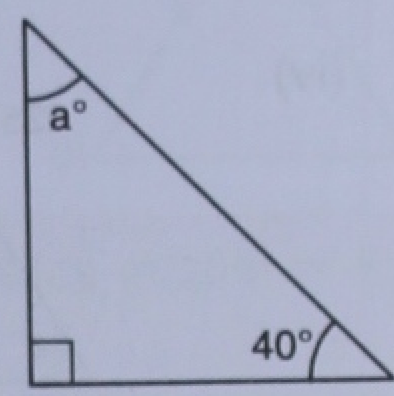
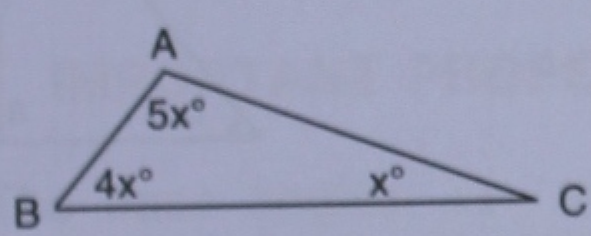


6. Find the value of each angle in the given figures :

(i)



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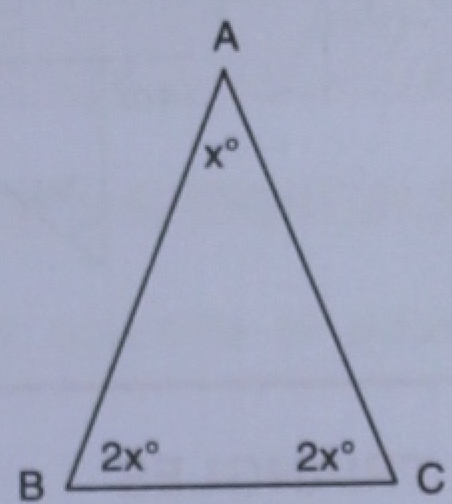
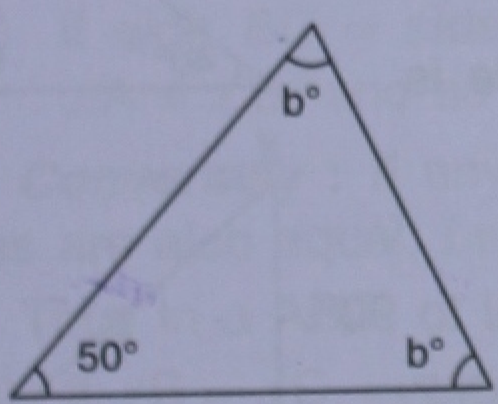


7. Find the unknown marked angles in the given figures :

(i)



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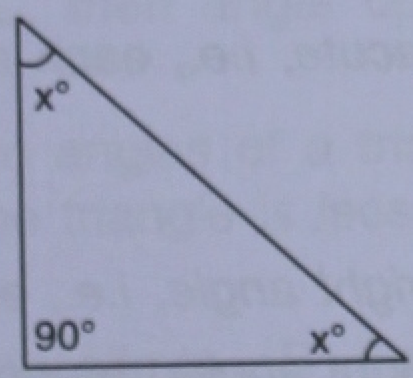

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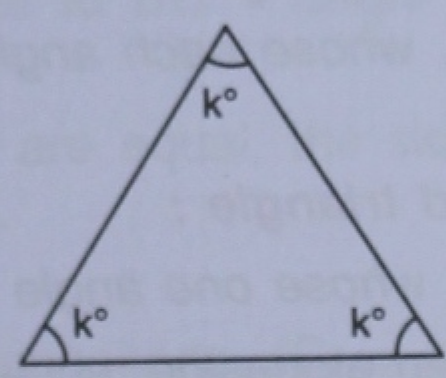

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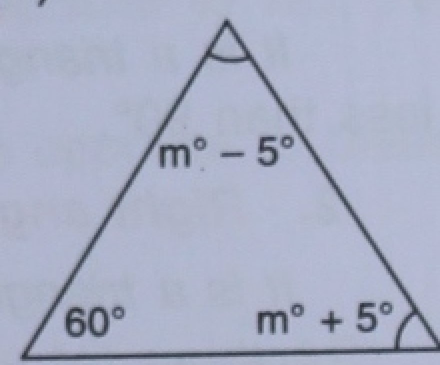
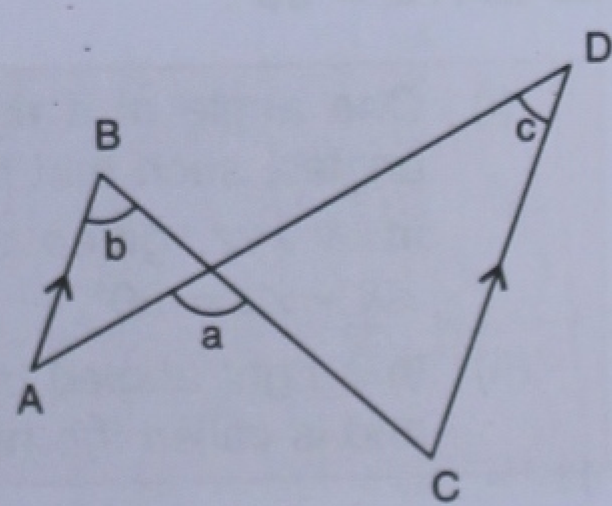


8. In the given figure, show that : ∠a = ∠b + ∠c.
   - (i) If ∠b = 60° and ∠c = 50°, find ∠a.
   - (ii) If ∠a = 100° and ∠b = 55°, find ∠c.
   - (iii) If ∠a = 108° and ∠c = 48°, find ∠b.

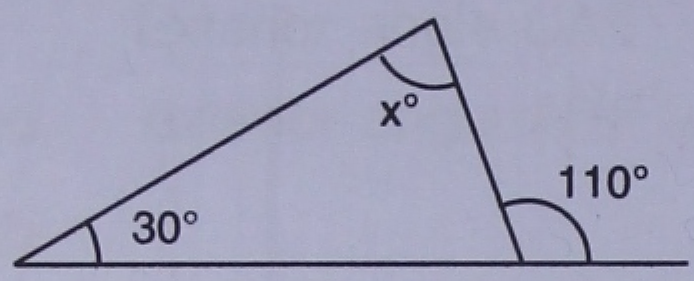


9. Calculate the angles of a triangle, if they are in the ratio 4 : 5 : 6.
10. One angle of a triangle is 60°. The other two angles are in the ratio of 5 : 7. Find the two angles.

11. One angle of a triangle is 61° and the other two angles are in the ratio $1\frac{1}{2} : 1\frac{1}{3}$. Find these angles.

12. Find the unknown marked angles in the given figures.

(i)



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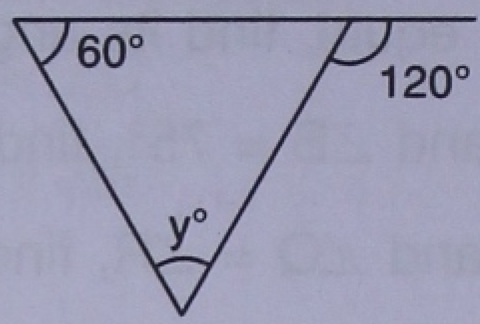

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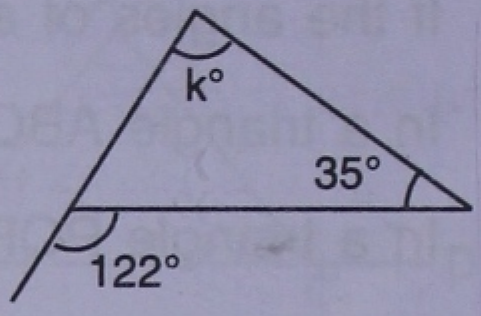

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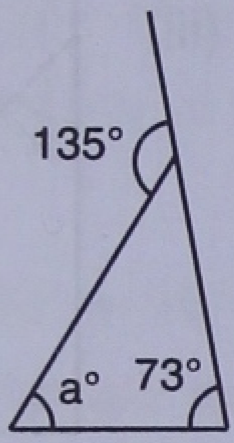

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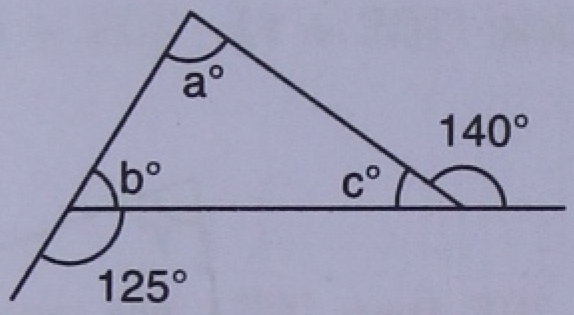

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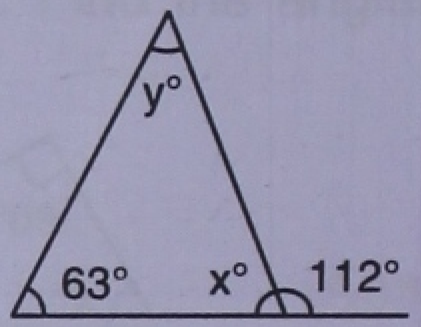

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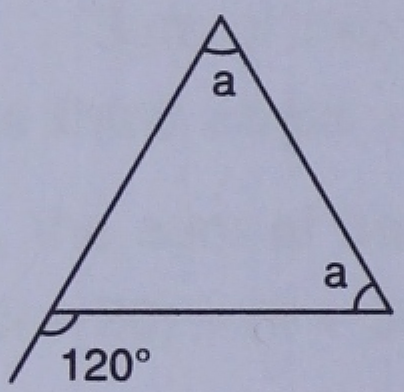

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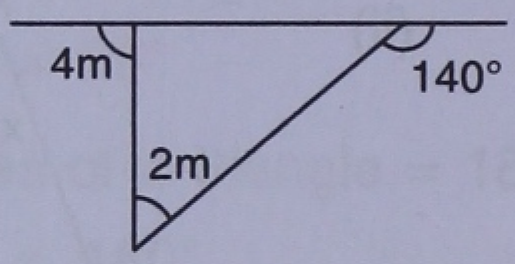

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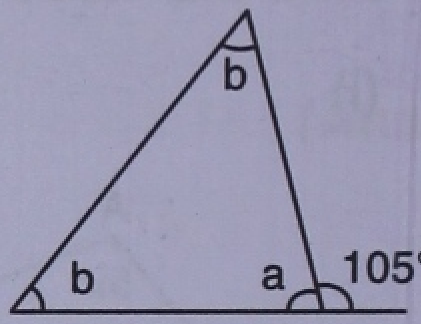
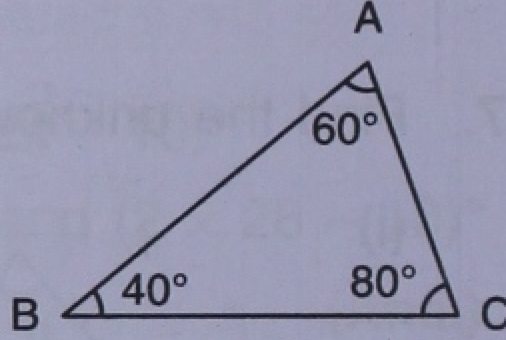


24.2 CLASSIFICATION OF TRIANGLES

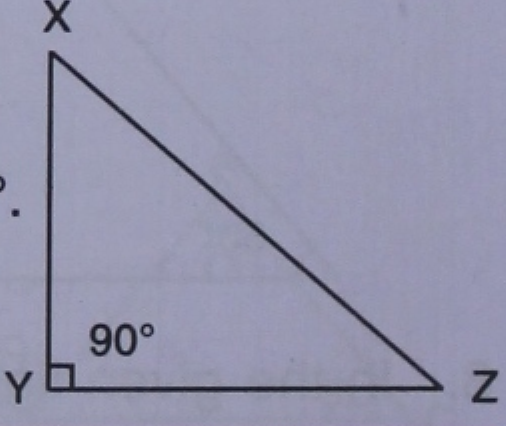


(a) With regard to their angles :

1. ***Acute angled triangle :***

It is a triangle, whose each angle is acute, i.e., each angle is less than 90°.

2. ***Right angled triangle :***



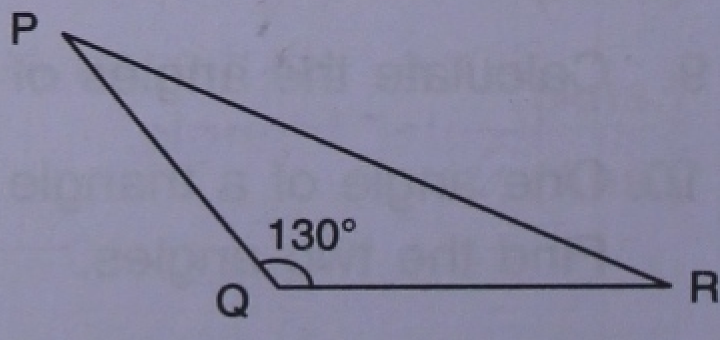
It is a triangle, whose one angle is a right angle, i.e., equal to 90°.

The figure, given alongside, shows a right angled triangle XYZ as ∠XYZ = 90°.

(i) One angle of a right angled triangle is 90° and the other two angles of it are acute angles, such that their sum is always 90°.
In Δ *xyz*, given above, ∠*y* = 90° and each of ∠*x* and ∠*z* is acute such that ∠*x* + ∠*z* = 90°.

(ii) In a right angled triangle, the side opposite to the right angle is largest of all its sides and is called the **hypotenuse.** In given right angled Δ XYZ, side XZ is the hypotenuse.

3. ***Obtuse angled triangle :***

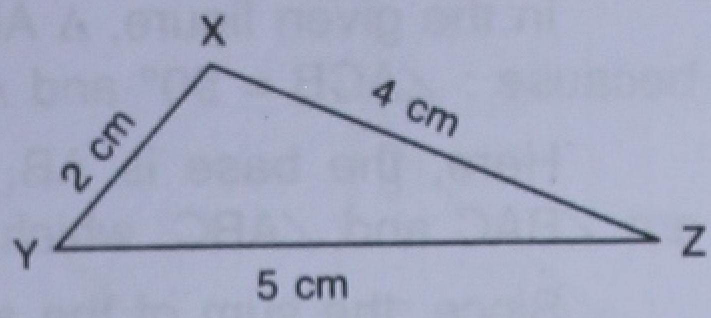


If one angle of a triangle is more than 90°, it is called an obtuse angled triangle.

In case of an obtuse angled triangle, each of the other two angles is always acute and their sum is less than 90°.

(b) With regard to their sides :

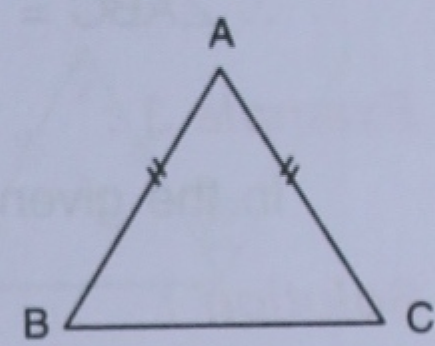
1. *Scalene triangle :*



If all the sides of a triangle are unequal, it is called a ***scalene triangle.***

In a scalene triangle, all its angles are also unequal.

2. *Isosceles triangle :*



If atleast two sides of a triangle are equal, it is called an ***isosceles triangle****.*

In Δ ABC, shown alongside, side AB = side AC.

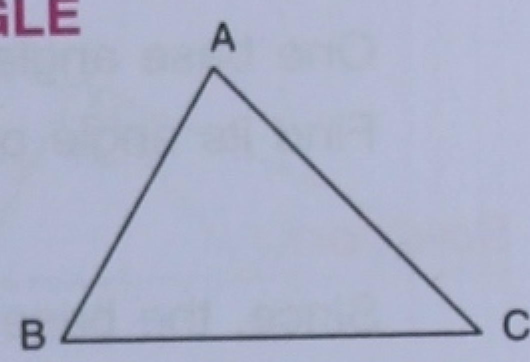
∴ Δ ABC is an isosceles triangle.

(i) The angle contained by equal sides *i.e.* ∠BAC is called the **vertical angle** or the **angle of vertex**.

(ii) The third side (the unequal side) is called the **base** of the isosceles triangle.

(iii) The two other angles (other than the angle of vertex) are called the **base angles** of the triangle.

24.3 IMPORTANT PROPERTIES OF AN ISOSCELES TRIANGLE



The base angles, *i.e.*, the angles opposite to equal sides of an isosceles triangle are always equal.

In given triangle ABC,

(i) if side AB = side BC, then angle opposite to AB = angle opposite to BC, *i.e.*, ∠C = ∠A.

(ii) if side BC = side AC, then angle opposite to BC = angle opposite to AC, *i.e.* ∠A = ∠B and so on.

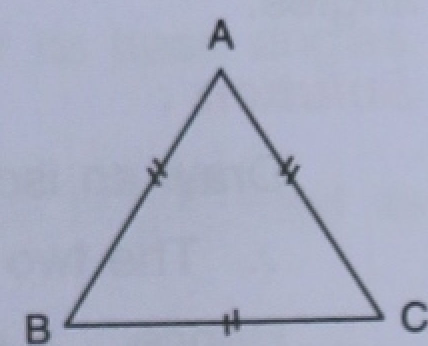
Conversely **:** If any two angles of a triangle are equal, the sides opposite to these angles are also equal, *i.e.*, the triangle is isosceles.

Thus in Δ ABC,

(i) if ∠B = ∠C ⇒ side opposite to ∠B = side opposite to ∠C ⇒ side AC = side AB.

(ii) if ∠A = ∠B ⇒ side BC = side AC and so on.

3. *Equilateral triangle :*



If all the three sides of a triangle are equal, it is called an ***equilateral triangle****.*

In the given figure, Δ ABC is equilateral, because AB = BC = CA

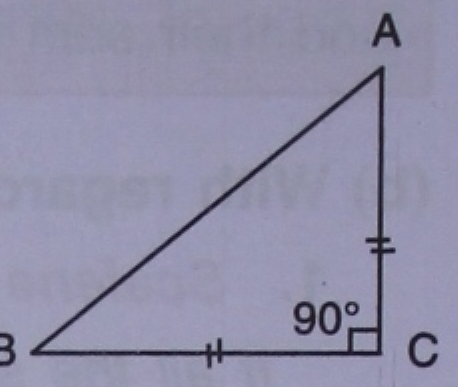
Also, all the angles of an equilateral triangle are equal to each other and so each angle = 60° [∵ 60° + 60° + 60° = 180°]

Since, all the angles of an equilateral triangle are equal, it is also known as **equiangular triangle.**

An equilateral triangle is always an isosceles triangle, but its converse is not always true.

4. ***Isosceles right angled triangle :***

If one angle of an isosceles triangle is 90°, it is called an Isosceles right angled triangle.



In the given figure, Δ ABC is an isosceles right angled triangle, because : ∠ACB = 90° and AC = BC.

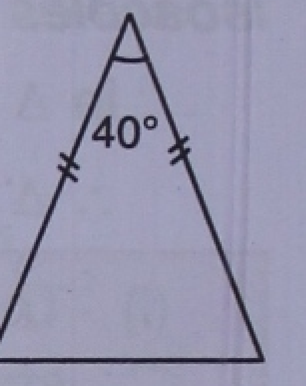
Here, the base is AB, the vertex is C and the base angles are ∠BAC and ∠ABC, which are equal.

Since, the sum of the angles of a triangle = 180°

∴ ∠ABC = ∠BAC = 45° [45° + 45° + 90° = 180°]

Example 3 :

In the given isosceles triangle, find the base angles.



Solution :

Let each of the base angles be x.

∴ $x + x + 40° = 180°$ [Sum of the angles of a Δ = 180°]

⇒ $2x + 40° = 180°$

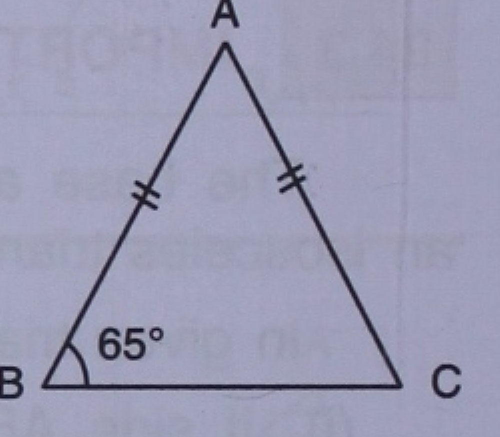
⇒ $2x = 180° - 40°$

⇒ $x = \frac{140°}{2} = 70°$ ∴ **Each base angle is 70°** **(Ans.)**

Example 4 :

One base angle of an isosceles triangle is 65°.

Find its angle of vertex.



Solution :

Since, the base angles of an isosceles triangle are equal.

∴ Other base angle is also 65°.

Let the angle of vertex be x.

∴ $x + 65° + 65° = 180°$ [Sum of the angles of a Δ = 180°]

⇒ $x + 130° = 180°$

⇒ $x = 180° - 130° = 50°$ **(Ans.)**

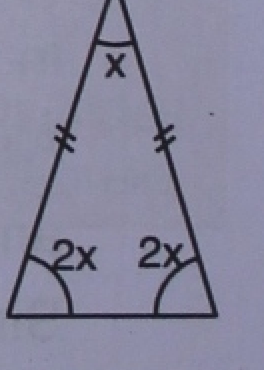
Example 5 :

If one base angle of an isosceles triangle is double of the vertical angle, find all its angles.

Solution :

Draw an isosceles triangle in which mark the vertical angle as x.

∴ The two base angles will be $2x$ each.



Hence, $x + 2x + 2x = 180°$

⇒ $5x = 180°$ and $x = \frac{180°}{5} = 36°$

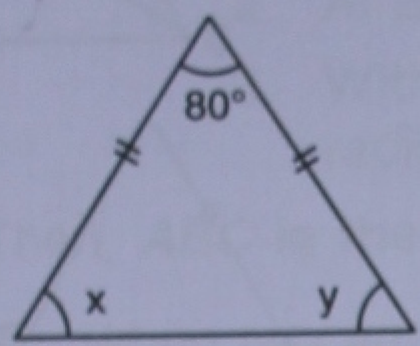
⇒ $2x = 2 \times 36° = 72°$

∴ **Vertical angle** = **36°** and **each base angle = 72°** **(Ans.)**

EXERCISE 24(B)

1. Find the unknown angles in the given figures :

(i)



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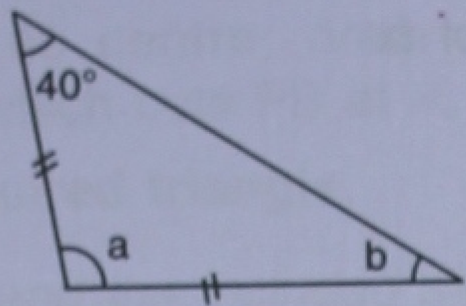

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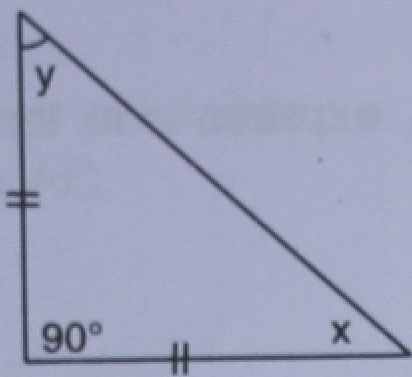

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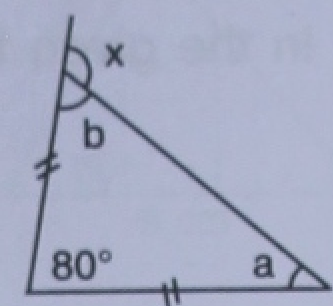

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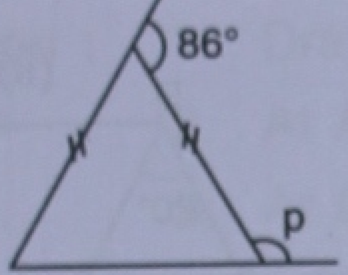

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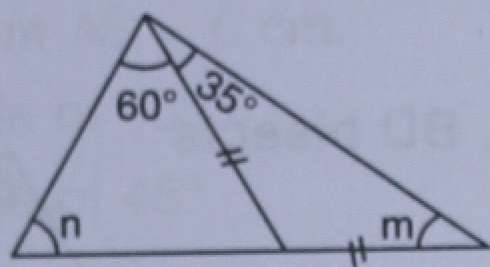

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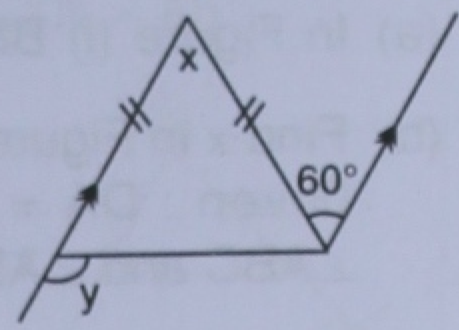
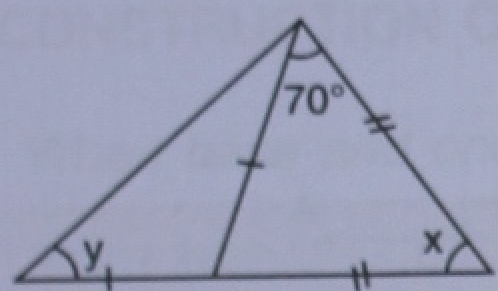


2. Apply the properties of isosceles and equilateral triangles to find the unknown angles in the given figures :

(i)



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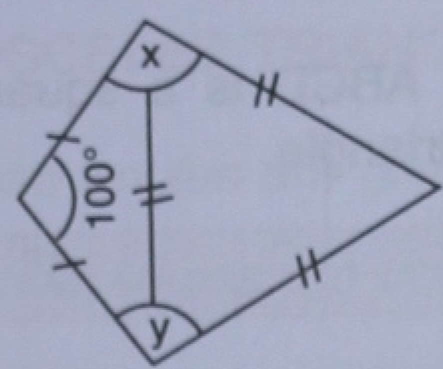

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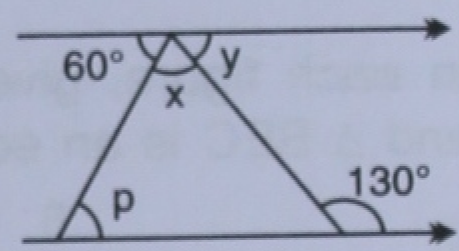

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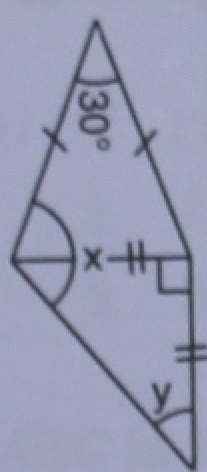
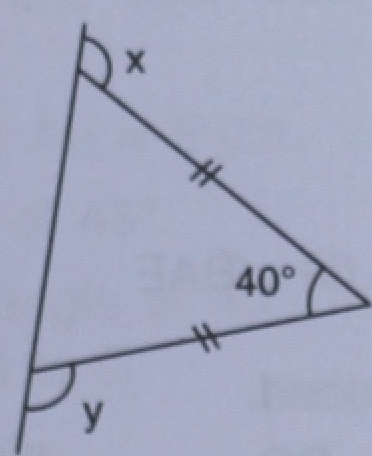
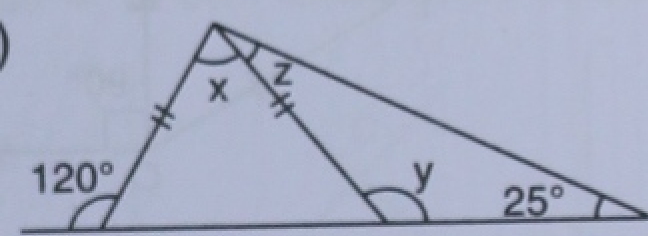


(v)

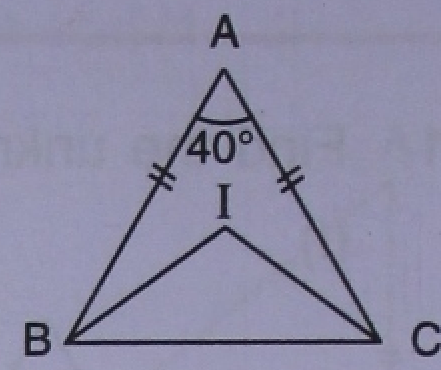


(vi)

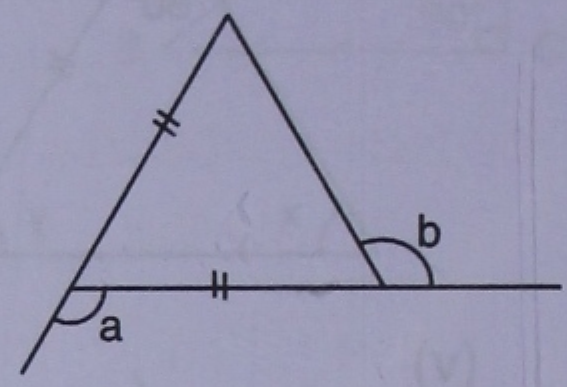


3. The angle of vertex of an isosceles triangle is 100°. Find its base angles.
4. One of the base angles of an isosceles triangle is 52°. Find its angle of vertex.
5. In an isosceles triangle, each base angle is four times of its vertical angle. Find all the angles of the triangle.
6. The vertical angle of an isosceles triangle is 15° more than each of its base angles. Find each angle of the triangle.
7. The base angle of an isosceles triangle is 15° more than its vertical angle. Find its each angle.
8. The vertical angle of an isosceles triangle is three times the sum of its base angles. Find each angle.
9. The ratio between a base angle and the vertical angle of an isosceles triangle is 1 : 4. Find each angle of the triangle.

10. In the given figure, BI is the bisector of ∠ABC and CI is the bisector of ∠ACB. Find ∠BIC.

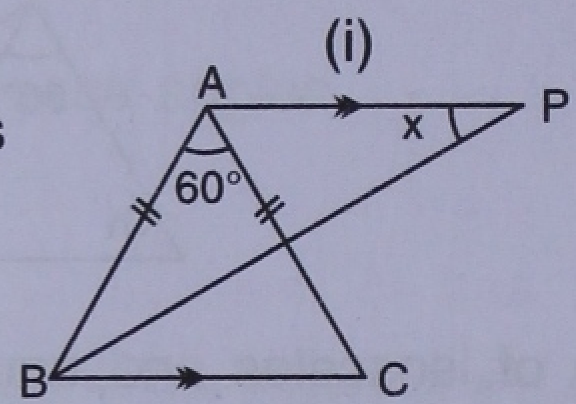


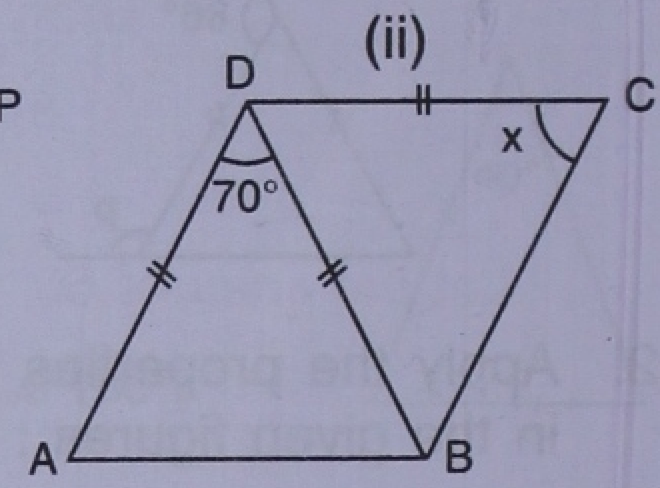
11. In the given figure, express *a* in terms of *b*.



12. (a) In Figure (i) BP bisects ∠ABC and AB = AC. Find *x*.

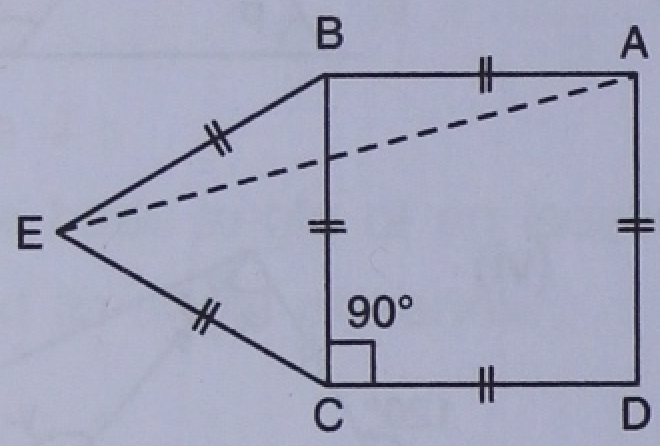
(b) Find x in Figure (ii).
Given : DA = DB = DC, BD bisects ∠ABC and ∠ADB = 70°.



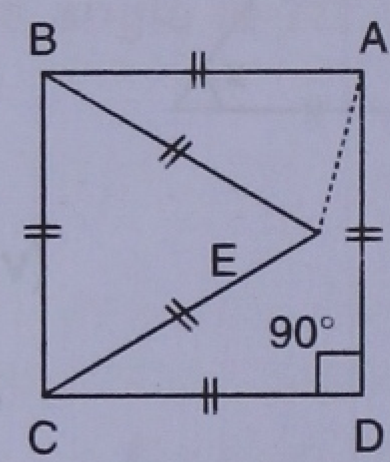


13. In each figure, given below, ABCD is a square and Δ BEC is an equilateral triangle.

(i)

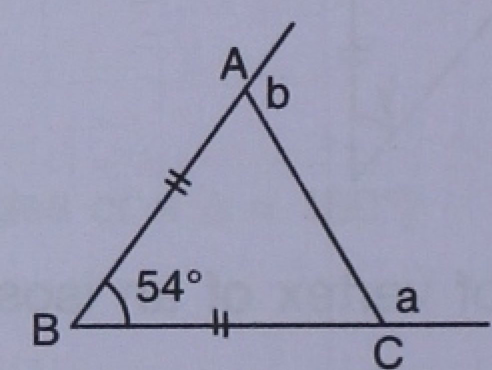


(ii)



Find, in each case : (i) ∠ABE (ii) ∠BAE

14. In Δ ABC, BA and BC are produced. Find the angles *a* and *b*, if AB = BC.

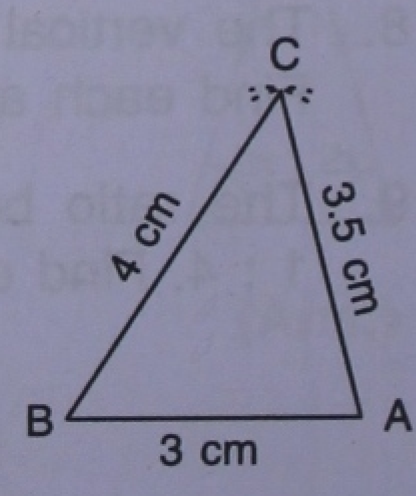


24.4 CONSTRUCTION OF TRIANGLES

1. *When the lengths of three sides are given :*

Let us construct a Δ ABC, such that AB = 3 cm, BC = 4 cm and CA = 3·5 cm.

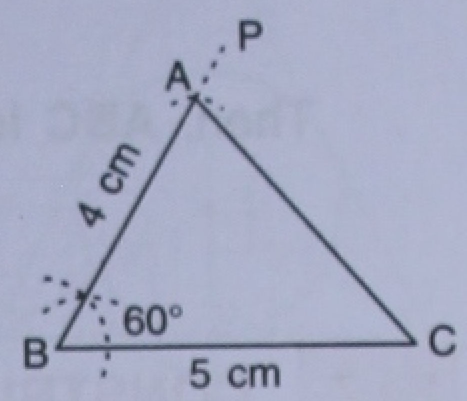
Steps :
1. Draw a line segment *AB* = 3 cm.
2. With A as centre, draw an arc of radius 3·5 cm and with B as centre draw another arc with radius 4 cm. Let these arcs meet at C.
3. Join BC and AC.



Then, **triangle ABC so obtained is the required triangle.**

2. *When the lengths of two sides and the included angle are given :*

Let us construct a Δ ABC, such that AB = 4 cm, BC = 5 cm and ∠ABC = 60°.



Steps : 1. Draw a line segment BC = 5 cm.

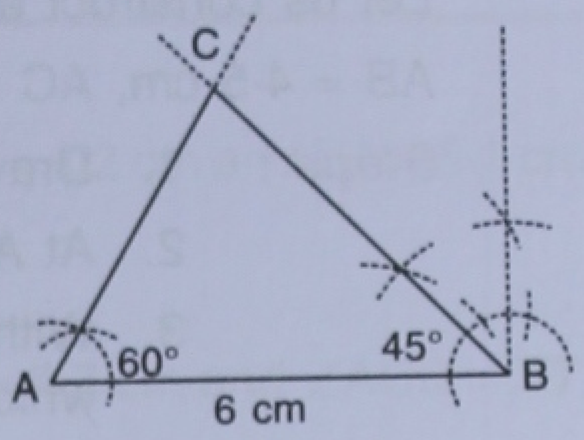
2. At B, draw an angle PBC = 60°.

3. With B as centre, draw an arc of 4 cm radius, which cuts PB at *A*. Join AC.

Then, ***ABC* is the required triangle.**

3. *When two angles and the included side are given :*

Let us construct a Δ ABC, such that AB = 6 cm, ∠A = 60° and ∠B = 45°.



Steps : 1. Draw a line segment AB = 6 cm.

2. At A, draw an angle of 60°.

3. At B, draw an angle of 45°.

Let the lines of 60° and 45° meet at C.

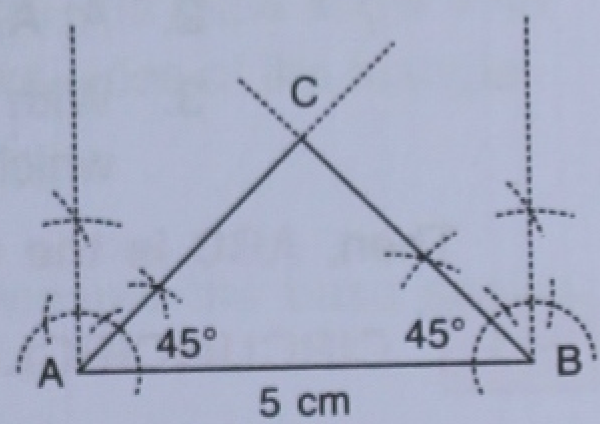
Then, ***ABC* is the required triangle.**

24.5 CONSTRUCTION OF AN ISOSCELES TRIANGLE

1. *When base and one of the base angles are given :*

In an isosceles triangle, the base angles are equal.

Let us construct an isosceles Δ ABC such that, base AB = 5 cm and each base angle = 45°.



Steps : 1. Draw a line segment AB = 5 cm.

2. At A, draw an angle of 45°.

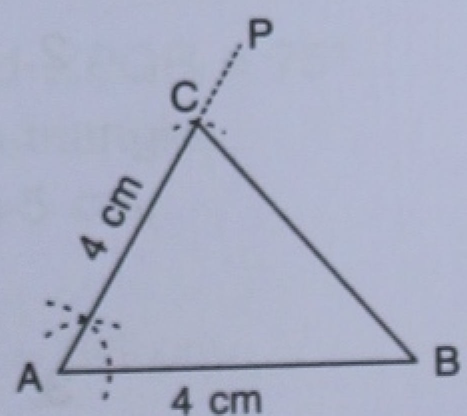
3. At B also, draw an angle of 45°. Let these 45° lines meet at C.

Then, ***ABC* is the required triangle.**

2. *When one of the equal sides and the vertex angle are given :*

Vertex angle is the angle between two equal sides.

Let us construct an isosceles triangle ABC such that :
AB = AC = 4 cm and ∠BAC = 60°.



Steps : 1. Draw a line segment AB = 4 cm.

2. At A, draw AP so that angle BAP = 60°.

3. With A as centre draw an arc of 4 cm radius, which cuts AP at C. Join C and *B*.

Then, **ABC is the required isosceles triangle.**

24.6 CONSTRUCTION OF AN EQUILATERAL TRIANGLE

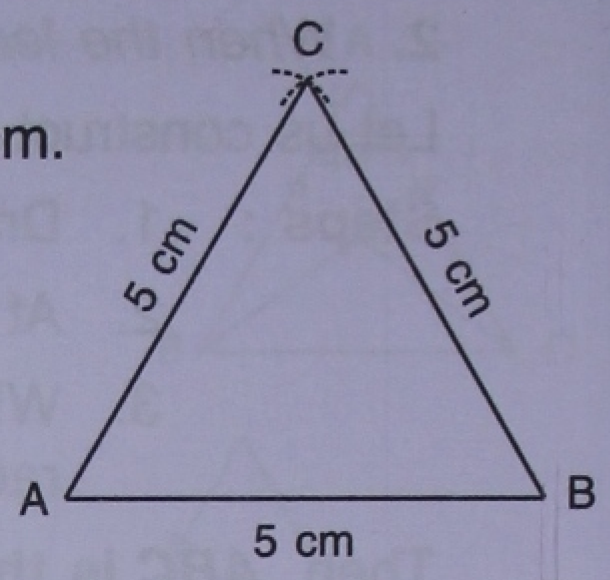
An equilateral triangle can be drawn when one of its sides is given.

Let to construct an equilateral Δ ABC with each side equal to 5 cm.

Steps : 1. Draw a line AB = 5 cm.

2. With A as centre, draw an arc of radius 5 cm.
3. With B as centre, draw another arc of radius 5 cm. Let, the two arcs meet at C. Join AC and BC.

Then, **ABC is the required triangle.**



24.7 CONSTRUCTION OF A RIGHT ANGLED TRIANGLE

1. *When the lengths of the sides, containing the right angle, are given :*

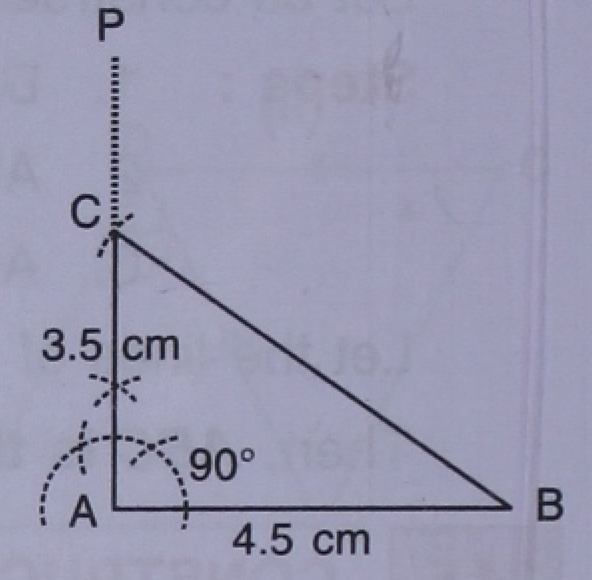
Let us construct a right angled Δ ABC such that,

AB = 4·5 cm, AC = 3·5 cm and ∠A = 90°.

Steps : 1. Draw a line segment AB = 4·5 cm.

2. At A, draw AP so that angle PAB = 90°.
3. With A as centre, draw an arc of radius 3·5 cm which cuts AP at point C. Join BC.

Then, ***ABC*** **is the required triangle.**



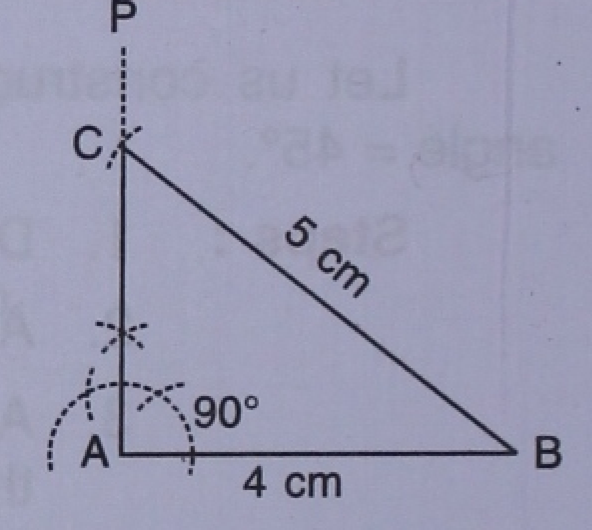
2. *When the lengths of one side and the hypotenuse are given :*

Let us construct a right angled Δ ABC such that AB = 4 cm, ∠A = 90° and BC (hypotenuse) = 5 cm.

Steps : 1. Draw a line segment AB = 4 cm.

2. At A, draw AP so that angle PAB = 90°.
3. With B as centre and radius = 5 cm, draw an arc which cuts AP at point C. Join BC.

Then, **ABC is the required triangle.**



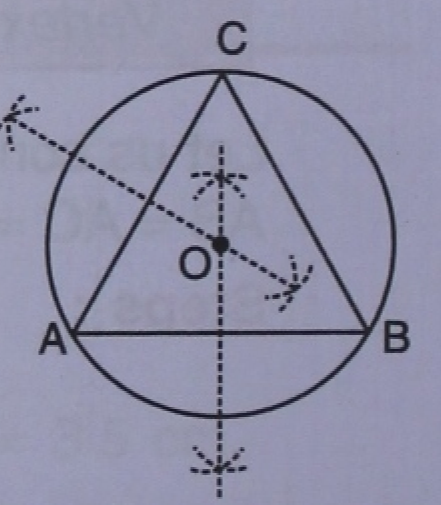
24.8 CIRCUMCIRCLE AND INCIRCLE

1. **Circumcircle :** *If a circle passes through all the three vertices of a triangle, it is called the* ***circumcircle*** *of the triangle.* Its centre is called **circumcentre** and its radius is called **circumradius.**

To construct the circumcircle of a triangle :

Steps : 1. Construct the ΔABC with the given measurements.

2. Draw the perpendicular bisectors of any two sides of the triangle.
   Here, the perpendicular bisectors of the sides AB and AC are drawn. These bisectors intersect each other at point O.
3. Taking O as centre and OA or OB or OC as radius, draw a circle.
   The circle so drawn passes through the vertices A, B and C.

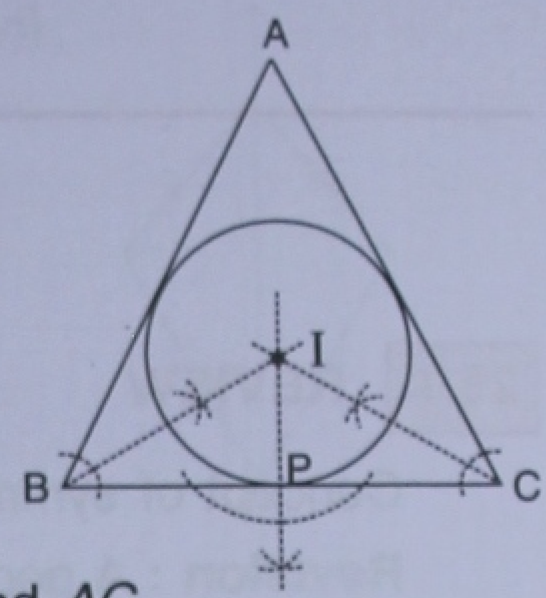


[Here, the **centre O** is the **circumcentre** of Δ ABC, whereas **OA** = **OB** = **OC** = its **circumradius**].

2. **Incircle :** *If a circle is drawn, inside a triangle, such that it touches all the three sides of the triangle, it is called the* ***incircle*** *of that triangle.*

The centre of this circle is called the **incentre** of the triangle.

To construct the incircle of a triangle :



Steps : 1. Construct the Δ *ABC* according to the given measurements.

2. Bisect any two of its angles. Let these two bisectors meet at *I*.
3. From *I*, draw a perpendicular on *BC*. This perpendicular meets *BC* at point *P*.
4. With *I* as centre and *IP* as radius, draw a circle.

The circle so drawn will touch the sides *BC, AB* and *AC.*

[Here, **I** is the **incentre** of the triangle].

EXERCISE 24(C)

1. Construct a Δ ABC such that :
   (i) AB = 6 cm, BC = 4 cm and CA = 5·5 cm (ii) CB = 6·5 cm, CA = 4·2 cm and BA = 5·1 cm
   (iii) BC = 4 cm, AC = 5 cm and AB = 3·5 cm
2. Construct a Δ ABC such that :
   (i) AB = 7 cm, BC = 5 cm and ∠ABC = 60° (ii) BC = 6 cm, AC = 5·7 cm and ∠ACB = 75°
   (iii) AB = 6·5 cm, AC = 5·8 cm and ∠A = 45°
3. Construct a Δ PQR such that :
   (i) PQ = 6 cm, ∠Q = 60° and ∠P = 45°. Measure ∠R.
   (ii) QR = 4·4 cm, ∠R = 30° and ∠Q = 75°. Measure PQ and PR.
   (iii) PR = 5·8 cm, ∠P = 60° and ∠R = 45°. Measure ∠Q and verify it by calculations.
4. Construct an isosceles Δ ABC such that :
   (i) base BC = 4 cm and base angle = 30° (ii) base AB = 6·2 cm and base angle = 45°
   (iii) base AC = 5 cm and base angle = 75°. Measure the other two sides of the triangle.
5. Construct an isosceles Δ ABC such that :
   (i) AB = AC = 6·5 cm and ∠A = 60°
   (ii) One of the equal sides = 6 cm and vertex angle = 45°. Measure the base angles.
   (iii) BC = AB = 5·8 cm and ∠B = 30°. Measure ∠A and ∠C.
6. Construct an equilateral Δ ABC such that :
   (i) AB = 5 cm. Draw the perpendicular bisectors of BC and AC. Let P be the point of intersection of these two bisectors. Measure PA, PB and PC.
   (ii) Each side is 6 cm.
7. (i) Construct a Δ ABC such that AB = 6 cm, BC = 4·5 cm and AC = 5·5 cm. Construct a circumcircle of this triangle.
   (ii) Construct an isosceles Δ PQR such that PQ = PR = 6·5 cm and ∠PQR = 75°. Using ruler and compasses only construct a circumcircle to this triangle.
   (iii) Construct an equilateral triangle ABC such that its one side = 5·5 cm. Construct a circumcircle to this triangle.
8. (i) Construct a Δ ABC such that AB = 6 cm, BC = 5·6 cm and CA = 6·5 cm. Inscribe a circle to this triangle and measure its radius.
   (ii) Construct an isosceles Δ MNP such that base MN = 5·8 cm, base angle MNP = 30°. Construct an incircle to this triangle and measure its radius.
   (iii) Construct an equilateral Δ DEF whose one side is 5·5 cm. Construct an incircle to this triangle.
   (iv) Construct a Δ PQR such that PQ = 6 cm, ∠QPR = 45° and angle PQR = 60°. Locate its incentre and then draw its incircle.