CHAPTER 28

OPERATIONS ON SETS

28.1 INTRODUCTION

Operations on sets means combining two or more sets to get a single set satisfying the given conditions.

Some combining operations discussed in this chapter are :

(i) Union of sets

(ii) Intersection of sets

(iii) Difference of two sets

28.2 UNION OF SETS

Union of two given sets is the smallest set that contains all the elements of the two given sets.

To find the union of two given sets, the smallest set with the elements of both the sets is so formed that every element of the two sets is present in this new set but no element is repeated.

For example :

Let set A = { 2, 4, 5, 6 }

and set B = { 4, 6, 7, 8 }

The elements of the new set formed must belong to either A or B or to both.

Taking every element of the two sets A and B without repeating any of them, we get a new set = { 2, 4, 5, 6, 7, 8 }.

This new set contains all the elements of set A and all the elements of set B, with no repetition of elements, and is named **Union of sets A and B.**

The symbol used for the union of two sets is " $\cup$ ".

Therefore, (i) Union of sets A and B = A union B

= $A \cup B$

(ii) Union of sets P and Q = $P \cup Q$ and so on.

Example 1 :

If set P = { 2, 3, 4, 5, 6, 7 }, set Q = { 0, 3, 6, 9, 12 } and set R = { 2, 4, 6, 8 }, find : (i) union of sets P and Q. (ii) $P \cup R$. (iii) $Q \cup R$.

Solution :

(i) **Union of sets P and Q** = $P \cup Q$

= The smallest set that contains all the elements of set P and all the elements of set Q, without repetition

= **{ 0, 2, 3, 4, 5, 6, 7, 9, 12 }** **(Ans.)**

Similarly,

(ii) **$P \cup R$** = Union of sets P and R = **{ 2, 3, 4, 5, 6, 7, 8 }** **(Ans.)**

(iii) **$Q \cup R$** = Union of sets Q and R = **{ 0, 2, 3, 4, 6, 8, 9, 12 }** **(Ans.)**

28.3 INTERSECTION OF SETS :

Intersection of two given sets is the largest set that contains all the elements which are common to both the sets.

For example :

Let set A = { 2, 3, 4, 5, 6 }
and set B = { 3, 5, 7, 9 }

In these two sets, the elements 3 and 5 are common.

The set containing the elements common to the two sets A and B, *i.e.* {3, 5}, is the **Intersection of sets A and B.**

The symbol used for the intersection of two sets is "$\cap$".

Therefore, (i) Intersection of sets A and B = A intersection B
= $A \cap B$

(ii) Intersection of sets P and Q = $P \cap Q$ and so on.

Example 2 :

If set A = { 4, 6, 8, 10, 12 }, set B = { 3, 6, 9, 12, 15, 18 }
and set C = { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 }

Find : (i) intersection of sets A and B.
(ii) $B \cap C$ (iii) $A \cap C$.

Solution :

(i) **Intersection of sets A and B**
= $A \cap B$
= Set of all the elements which are common to set A and set B
= **{ 6, 12 }** **(Ans.)**

Similarly,

(ii) **$B \cap C$** = Intersection of sets B and C = **{ 3, 6, 9 }** **(Ans.)**

(iii) **$A \cap C$** = Intersection of sets A and C = **{ 4, 6, 8, 10 }** **(Ans.)**

If the given sets are in description form or set-builder form, convert them into roster form and then do the required operation.

Example 3 :

If P = { multiples of 3 between 1 and 20 }
and Q = { even natural numbers upto 15 }
Find : (i) $P \cup Q$ (ii) $P \cap Q$

Solution :

On converting both the sets into roster form

Here, $P = \{3, 6, 9, 12, 15, 18\}$

and $Q = \{2, 4, 6, 8, 10, 12, 14\}$

$\therefore$ (i) $P \cup Q$ = Union of sets P and Q

= The smallest set containing all the elements of set P and all the elements of set Q, without repetition of any element.

= $\{2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 18\}$ **(Ans.)**

(ii) $P \cap Q$ = Intersection of sets P and Q

= The largest set containing only those elements that are common to both the given sets.

= $\{6, 12\}$ **(Ans.)**

Example 4 :

If M = The empty set and N = { ◊, Δ, O }, find :

(i) $M \cup N$ and (ii) $M \cap N$

Solution :

Given : M = { } and N = { ◊, Δ, O }

$\therefore$ (i) $M \cup N$ = { ◊, Δ, O } = N **(Ans.)**

and (ii) $M \cap N$ = { } = ϕ **(Ans.)**

$A \cap \phi = \phi$, *i.e. intersection of any set with the empty set, is always the empty set.*
And, $A \cup \phi = A$, *i.e. union of any set with the empty set, is always the set itself.*

EXERCISE 28(A)

1. State **true** or **false** :
   (i) If A = { 5, 6, 7 } and B = { 6, 8, 10, 12 }, $A \cup B$ = { 5, 6, 7, 8, 10, 12 }.
   (ii) If P = { a, b, c } and Q = { b, c, d }, P intersection Q = { b, c }.
   (iii) Union of two given sets is the set of elements which are common to both the sets.
   (iv) Two disjoint sets have at least one element in common.
   (v) Two overlapping sets have all elements in common.
   (vi) If two given sets have no element common to the two sets, the sets are said to be disjoint.
   (vii) If A and B are two disjoint sets, $A \cap B$ = { }, the empty set.
   (viii) If M and N are two overlapping sets, intersection of sets M and N is not the empty set.

2. Let A, B and C be three sets such that :
   set A = { 2, 4, 6, 8, 10, 12 }, set B = { 3, 6, 9, 12, 15 }
   and set C = { 1, 4, 7, 10, 13, 16 }.

Find : (i) $A \cup B$ (ii) $B \cap C$

(iii) $B \cap A$ (iv) $B \cup A$

(v) $B \cup C$

Is $A \cup B = B \cup A$?

Is $B \cap C = B \cup C$?

3. If A = { 2, 3, 4, 5 }, B = { 1, 3, 5, 7 } and C = { 4, 5, 6, 7 }; find :

   (i) $A \cup B$ (ii) $A \cup C$

   (iii) $(A \cup B) \cap (A \cup C)$

4. If A = { a, b, c, d }, B = { c, d, e, f } and C = { b, d, f, g }; find :

   (i) $A \cap B$ (ii) $A \cap C$

   (iii) $(A \cap B) \cup (A \cap C)$

5. Let A = Set of natural numbers less than 8

   B = { even natural numbers less than 12 }

   C = { multiples of 3 between 5 and 15 }

   and D = { multiples of 4 greater than 6 and less than 20 }. Find :

   (i) $B \cup C$ (ii) $A \cup D$

   (iii) $C \cup D$ (iv) $A \cap C$

   (v) $(B \cap C) \cup A$ (vi) $(D \cup A) \cap B$

28.4 DIFFERENCE OF TWO SETS

For any two sets A and B, their difference is represented by :
(i) A – B or *(ii) B – A*, where

(i) A – B = { Elements of set A that do not belong to set B }.

And

(ii) B – A = { Elements of set B that do not belong to set A}.

Example 5 :

Given three sets P, Q and R such that :

P = { x : x is a natural number between 10 and 16 }

Q = Set of even numbers between 8 and 20

and R = { 7, 9, 11, 14, 18, 20 }, find :

(i) P – Q (ii) Q – R (iii) R – P (iv) Q – P

Solution :

According to the given statement :

P = { 11, 12, 13, 14, 15 }

Q = { 10, 12, 14, 16, 18 }

and R = { 7, 9, 11, 14, 18, 20 }

$\therefore$ (i) **P – Q** = {Those elements of set P that are not in set Q}
= { 11, 13, 15 } **(Ans.)**

(ii) **Q – R** = { Elements of set Q not belonging to set R }
= { 10, 12, 16 } **(Ans.)**

Similarly,

(iii) **R – P** = { 7, 9, 18, 20 } **(Ans.)**

(iv) **Q – P** = { 10, 16, 18 } **(Ans.)**

Example 6 :

Given : A = { letters in the word 'MANDAKINI' }
and B = { letters in the word 'DAMINI' }, find :
(i) A – B (ii) B – A.

Solution :

Clearly, A = { m, a, n, d, k, i }
and B = { d, a, m, i, n }

Converting the given sets A and B into roster form.

(i) **A – B** = { letters in set A but not in set B }
= { k } **(Ans.)**

(ii) **B – A** = { letters in set B but not in set A }
= { }, *i.e.* ϕ. **(Ans.)**

EXERCISE 28(B)

1. If set A = { 3, 4, 5, 6 } and set B = { 2, 4, 6, 8 }, find (i) A – B (ii) B – A
2. Given set A = { 2, 4, 6, 8, 10, 12 }, set B = { 3, 6, 9, 12, 15, 18 } and set C = { 0, 6, 12, 18 }, find :
   (i) A – B (ii) B – C (iii) C – A (iv) A – C
3. Given P = { a, c, d, m }, Q = { c, e, m, x } and R = { a, e, i, o }, find :
   (i) P – R (ii) Q – P (iii) R – Q
4. If A = { counting numbers between 30 and 40 },
   B = { counting numbers between 20 and 50 that are divisible by 4 },
   find : (i) A – B
   (ii) B – A
5. If P = { letters in the word 'BANARAS' }
   and Q = { letters in the word 'BHARAT' }
   and R = { letters in the word 'BHATINDA' }
   find : (i) P – Q
   (ii) R – Q
   (iii) P – R.

Revision Exercise (Chapter 28)

1. If A = { 2, 4, 6, 8, 10, 12 } and B = { 3, 6, 9, 12, 15 }
   Find : (i) $A \cup B$ (ii) $B \cap A$ (iii) $A - B$ (iv) $B - A$
2. If A = { 3, 5, 6, 8, 11 }, B = { 3, 6, 9, 12, 13, 15 } and C = { 5, 7, 9, 11 }
   Find : (i) $B \cup A$ (ii) $B \cap C$ (iii) $A \cap C$
3. If A = { letters of the word MORADABAD } and B = { letters of the word MADRAS }
   Find : (i) $A \cup B$ (ii) $A \cap B$ (iii) $A - B$ (iv) $B - A$
4. If M = { x : x is an even number up to 24 } and
   N = { x : x is a factor of 24 },
   find : (i) $M \cap N$ (ii) $N - M$
5. If A = { natural numbers divisible by 4 and up to 36 }
   B = { x : x is a natural number that is a multiple of 3 and is up to 36 } and
   C = { x : x is a natural number that is a multiple of 6 and is up to 36 },
   find : (i) $A \cup B$ (ii) $(A \cup B) \cap C$
   (iii) $A \cap B$ (iv) $(A \cap B) \cup C$
   (v) $(A \cup B) \cup (A \cap B)$
6. If set A = {5, 6, 8} and set B = {2, 3, 4, 5, 6, 7, 8, 9}, show that :
   (i) $A \cap B = A$ (ii) $A \cup B = B$
7. If set M = {4, 6, 8, 10, 12} and set N = {4, 8, 12}, show that :
   (i) $M \cup N = M$ (ii) $M \cap N = N$
8. If set P = {1, 3, 5, 7, 9, 11} and set Q = {3, 7, 9, 11, 13}, find :
   (i) $P \cup Q$ (ii) $P \cap Q$ (iii) $P - Q$
   (iv) $Q - P$ (v) $(P \cup Q) - P$ (vi) $Q - (P \cap Q)$
   (vii) $(P \cup Q) - (P \cap Q)$