CHAPTER 20

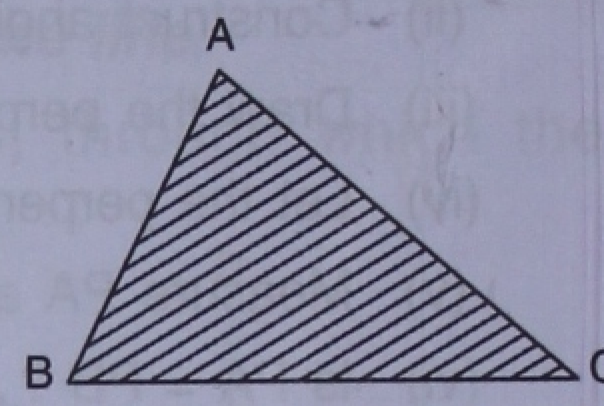
TRIANGLES

(Including Types, Properties and Constructions)

20.1 TRIANGLE :

A triangle is a plane closed figure bounded by three line segments.

In the adjoining figure, the line segments AB, BC and CA form the triangle ABC.



The three line segments *AB, BC and CA are the sides of the triangle ABC.*

A triangle is denoted by the Greek letter Δ (delta).

Thus, triangle ABC can be written as Δ ABC.

20.2 VERTEX :

Vertex of a triangle is a point where any two of its sides meet.

In the figure given above, the sides AB and AC meet at point A.

$\therefore$ A is a vertex of Δ ABC.

Similarly, vertex B = the point where the sides BC and AB meet.

And, vertex C = the point where the sides AC and BC meet.

The plural of vertex is vertices.

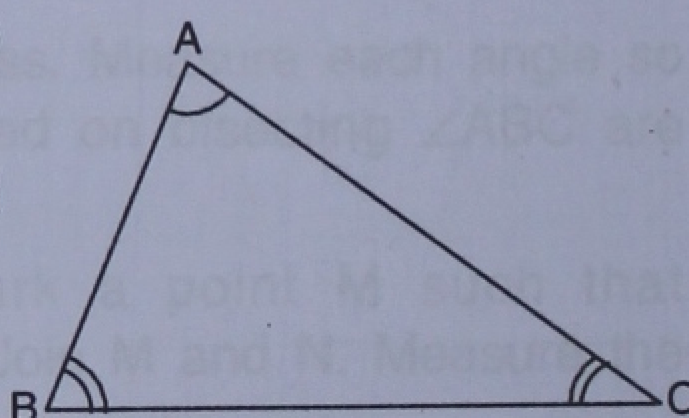
Thus, A, B and C are the three vertices of the triangle ABC.

BC is the side opposite to vertex A and A is the vertex opposite to side BC. The same is true for vertex B and side AC, and for vertex C and side AB.

20.3 ANGLES (INTERIOR ANGLES) OF A TRIANGLE :

Every triangle has three angles.

In the triangle ABC drawn alongside, the three angles (interior angles) are : $\angle$BAC, $\angle$ABC and $\angle$ACB.



(i) An interior angle of a triangle can also be denoted by the letter representing the corresponding vertex. Consider $\angle$ABC, since it is formed at vertex B, it can be written as $\angle$B.

Thus, $\angle ABC = \angle B$, $\angle BCA = \angle C$ and $\angle BAC = \angle A$, all are interior angles of the triangle ABC.

(ii) The sum of the interior angles of a triangle is always 180°, *i.e.* two right angles.

$\therefore$ In Δ ABC, $\angle A + \angle B + \angle C = 180°$,

and in Δ PQR, $\angle P + \angle Q + \angle R = 180°$ and so on.

1. Each triangle has three sides, three vertices and three angles (interior angles).
2. Δ ABC can also be written as Δ BAC or Δ BCA or Δ CAB, or Δ ACB or Δ CBA *i.e.* the three letters representing a triangle can be written in any order.

20.4 EXTERIOR ANGLE OF A TRIANGLE :

*When any side of a triangle is extended, the angle formed outside the triangle is called an **exterior angle**.*

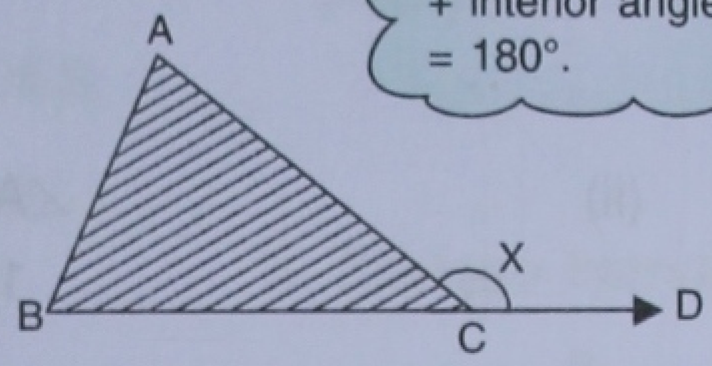
20.5 SOME IMPORTANT RESULTS :

1. *An exterior angle of a triangle is an adjacent and supplementary angle to the corresponding interior angle of the triangle.*

For example :

At each vertex, exterior angle + interior angle = 180°.

In the figure given alongside, the side BC of Δ ABC is extended up to point D, thus forming an exterior angle ACD. The exterior angle ACD is adjacent and supplementary to the corresponding interior ∠ACB of the Δ ABC *i.e.* ∠ACD + ∠ACB = 180°.



2. *An exterior angle of a triangle is always equal to the sum of its two opposite interior angles.*

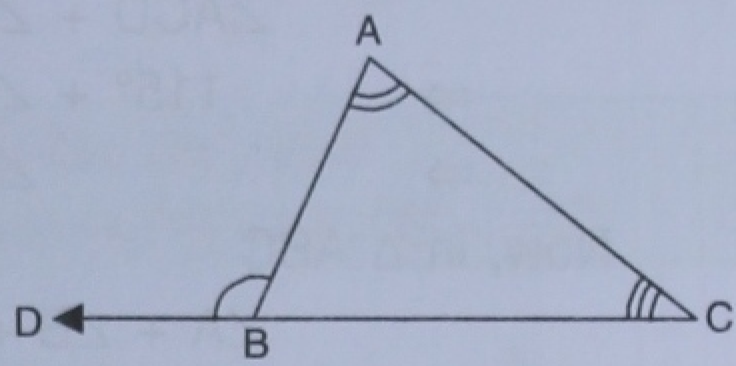
For example :

In the given figure, exterior angle ABD is formed by extending the side CB of the triangle ABC.

∴ Exterior angle ABD

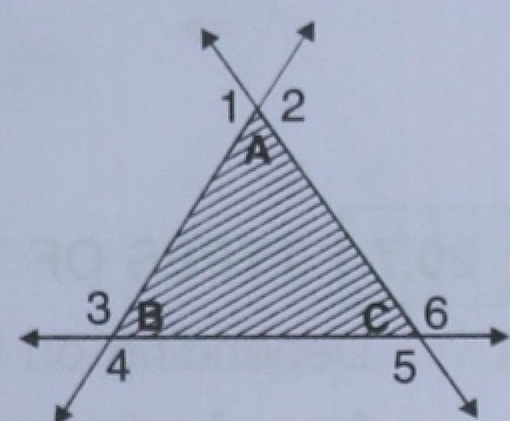
= Sum of interior opposite angles A and C,

i.e. ∠ABD = ∠A + ∠C.



3. *On extending the sides of a triangle, six exterior angles are formed, two at each vertex.*

The adjoining figure shows the six exterior angles formed by extending the sides of the triangle ABC.



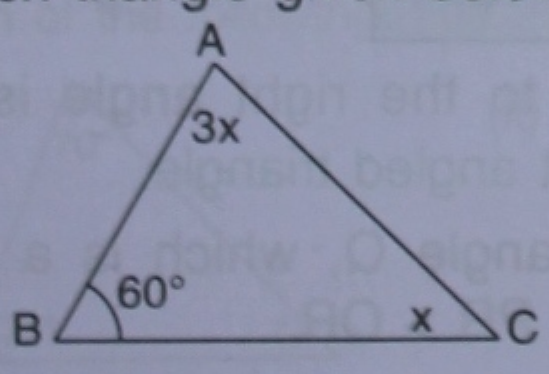
20.6 CONSIDER THE FOLLOWING TABLE :

Exterior Angle	Adjacent Interior Angle	Interior opposite Angles	Relation between an exterior angle and its adjacent interior angle	Relation between an exterior angle and the interior opposite angles
∠1	∠A	∠B and ∠C	∠1 + ∠A = 180°	∠1 = ∠B + ∠C
∠2	∠A	∠B and ∠C	∠2 + ∠A = 180°	∠2 = ∠B + ∠C
∠3	∠B	∠A and ∠C	∠3 + ∠B = 180°	∠3 = ∠A + ∠C
∠4	∠B	∠A and ∠C	∠4 + ∠B = 180°	∠4 = ∠A + ∠C
∠5	∠C	∠A and ∠B	∠5 + ∠C = 180°	∠5 = ∠A + ∠B
∠6	∠C	∠A and ∠B	∠6 + ∠C = 180°	∠6 = ∠A + ∠B

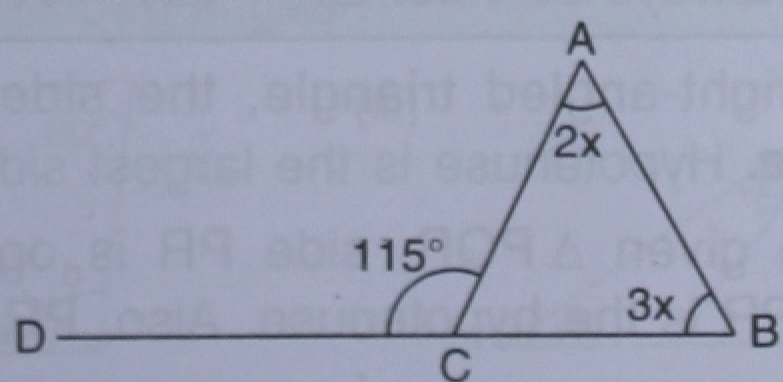
Example 1 :

For each triangle given below, find the value of x :

(i)



(ii)



Solution :

(i)

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow \quad 3x + 60^\circ + x = 180^\circ$$

$$4x = 180^\circ - 60^\circ$$

$$4x = 120^\circ$$

$$\therefore \quad x = \frac{120^\circ}{4} = 30^\circ \quad \textbf{(Ans.)}$$

Sum of the angles of a Δ is 180°

(ii)

$$\angle ACD = \angle A + \angle B$$

$$\Rightarrow \quad 115^\circ = 2x + 3x$$

$$5x = 115^\circ$$

$$\therefore \quad x = \frac{115^\circ}{5} = 23^\circ \quad \textbf{(Ans.)}$$

Exterior angle of a Δ = sum of interior opposite angles.

Alternative method :

$$\angle ACD + \angle ACB = 180^\circ$$

$$\Rightarrow \quad 115^\circ + \angle ACB = 180^\circ$$

$$\Rightarrow \quad \angle ACB = 180^\circ - 115^\circ = 65^\circ$$

At each vertex, exterior angle + interior angle = 180°

Now, in Δ ABC

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow \quad 2x + 3x + 65^\circ = 180^\circ$$

$$5x = 180^\circ - 65^\circ = 115^\circ \quad \therefore\ x = \frac{115^\circ}{5} = 23^\circ \quad \textbf{(Ans.)}$$

20.7 TYPES OF TRIANGLES ACCORDING TO ANGLES :

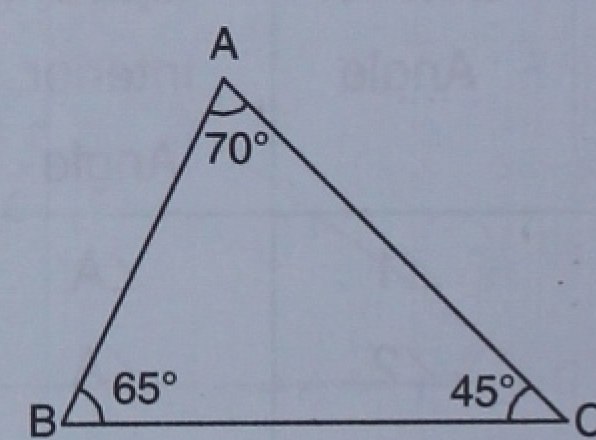
Depending on the sizes of its angles, a triangle can be classified as :

1. Acute-angled triangle
2. Right-angled triangle
3. Obtuse-angled triangle.

1. Acute-angled triangle :

*If each angle of a triangle is acute (less than 90°), it is called an **acute-angled triangle**.*

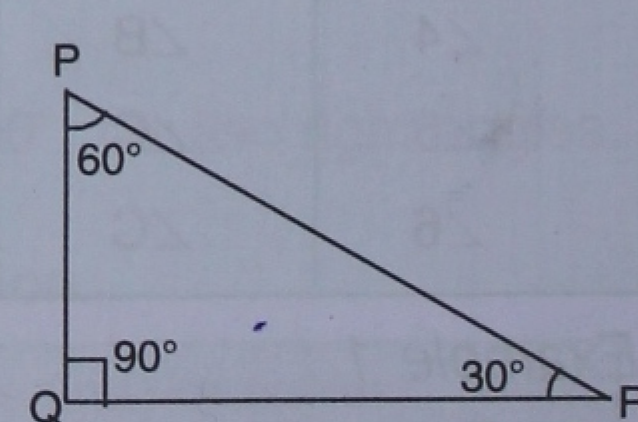
The adjoining figure shows an acute-angled triangle; each of its angles is less than 90°.



2. Right-angled triangle :

*If one of the angles of a triangle is a right angle i.e. 90° it is called a **right-angled triangle**.*

The figure given alongside shows a right angled triangle PQR, as ∠PQR = 90°.



> Sum of the two acute angles of a right angled triangle is always 90°, *i.e.* ∠P + ∠R = 90°.

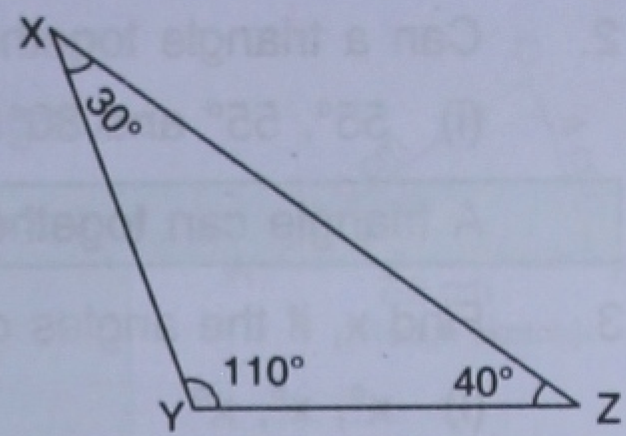
In a right-angled triangle, the side opposite to the right angle is called the ***hypotenuse.*** Hypotenuse is the largest side of a right angled triangle.

In the given Δ PQR, side PR is opposite to angle Q, which is a right angle. Therefore, PR is the hypotenuse. Also, PR > PQ and PR > QR.

3. Obtuse-angled triangle :

*If an angle of a triangle is obtuse (more than 90°), the triangle is called an **obtuse-angled triangle**.*

In the adjoining figure, Δ XYZ is an obtuse-angled triangle, as $\angle XYZ = 110°$, *i.e.* $\angle XYZ$ is an obtuse angle.



20.8 TYPES OF TRIANGLES ACCORDING TO SIDES :

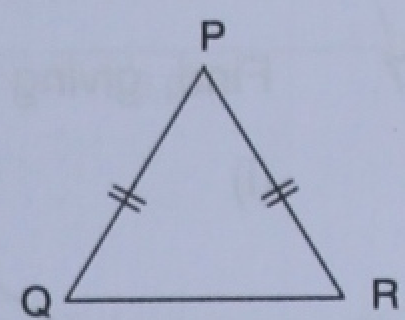
On the basis of sides, a triangle can be classified as :

1. Isosceles triangle 2. Equilateral triangle 3. Scalene triangle.

1. Isosceles triangle :

A triangle, with atleast two sides equal is called an isosceles triangle.

In the given figure, PQR is an isosceles triangle, as PQ = PR.

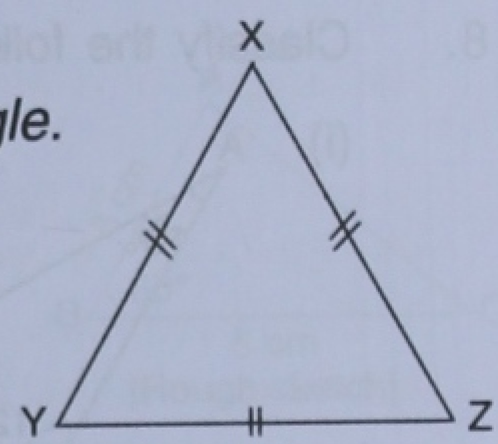


In an isosceles triangle, the angles opposite to the equal sides are equal.
Thus, PQ = PR $\Rightarrow$ Angle opposite to PQ = Angle opposite to PR, i.e. $\angle R = \angle Q$.

2. Equilateral triangle :

A triangle, with all its sides equal is called an equilateral triangle.

In the figure given alongside, XYZ is an equilateral triangle as :
side XY = side YZ = side XZ.



In an equilateral triangle, all angles are equal, *i.e.* $\angle XYZ = \angle YXZ = \angle XZY$.

Since the sum of all the three interior angles of every triangle is 180°

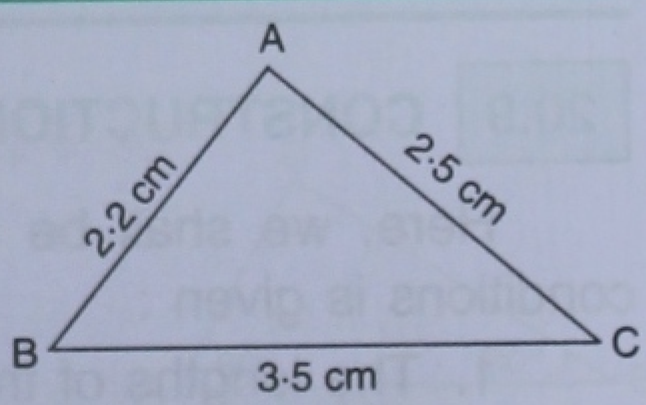
therefore each interior angle of an equilateral triangle $= \dfrac{180°}{3} = 60°$.

Thus, in an equilateral Δ XYZ :
(i) XY = YZ = ZX and (ii) $\angle XYZ = \angle YXZ = \angle XZY = 60°$.

Every equilateral triangle is isosceles; but the converse is not always true.

3. Scalene triangle :

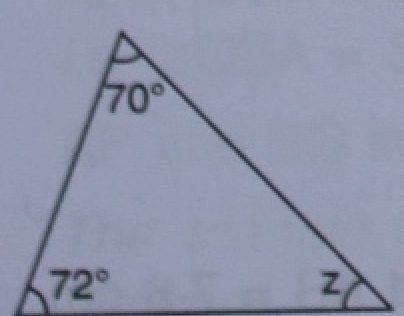
If the three sides of a triangle are unequal, *i.e.* if the sides are of different lengths, the triangle is called a *scalene triangle*.



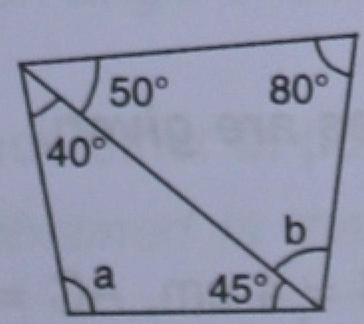
EXERCISE 20(A)

1. In each of the following, find the marked unknown angles :

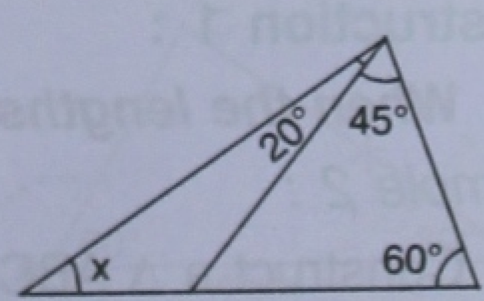
(i)



(ii)



(iii)



2. Can a triangle together have the following angles ?

   (i) 55°, 55° and 80° (ii) 33°, 74° and 73° (iii) 85°, 95° and 22°

A triangle can together have the given angles if the sum of these angles is 180°.

3. Find x, if the angles of a triangle are :

   (i) x°, x°, x° (ii) x°, 2x°, 2x° (iii) 2x°, 4x°, 6x°

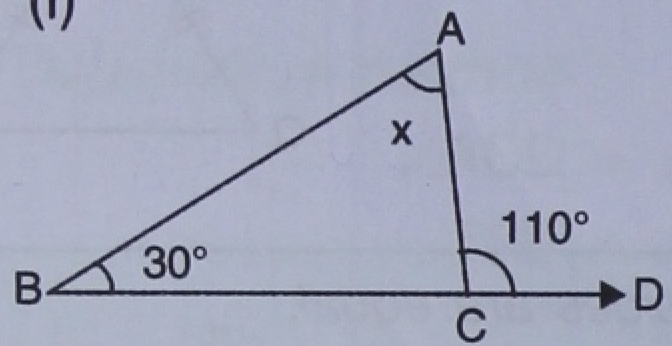
4. One angle of a right-angled triangle is 70°. Find the other acute angle.

5. In Δ ABC, ∠A = ∠B = 62°; find ∠C.

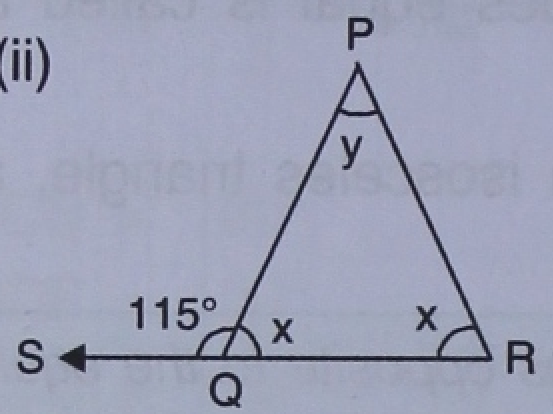
6. In Δ ABC, ∠B = ∠C and ∠A = 100°; find ∠B.

7. Find, giving reasons, the unknown marked angles in each triangle drawn below :

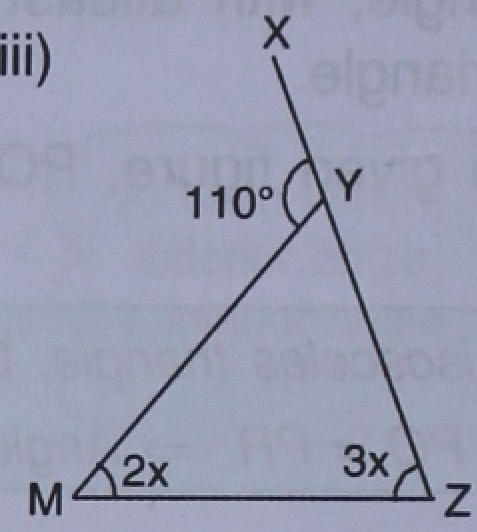
(i)



(ii)

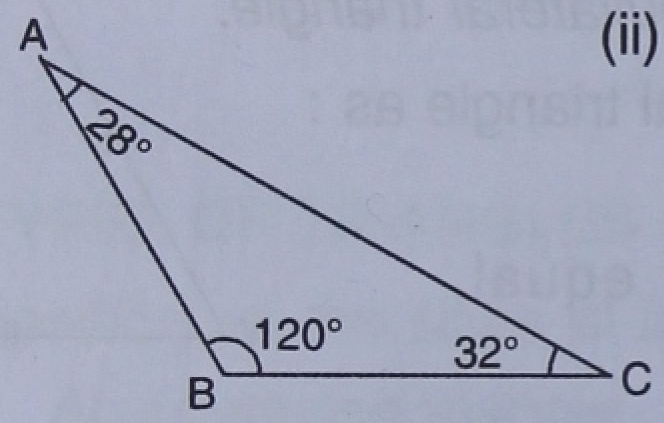


(iii)

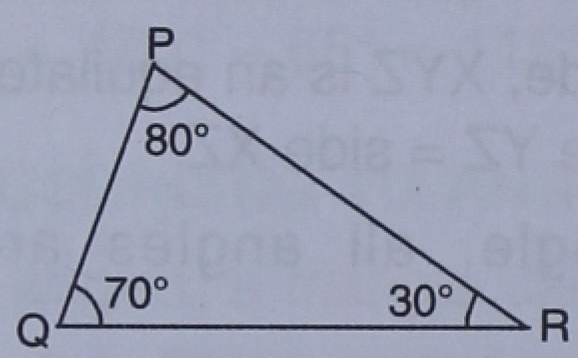


8. Classify the following triangles according to angle :

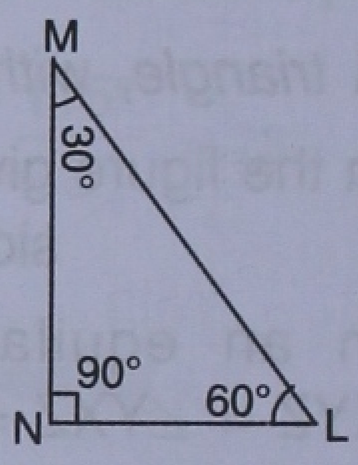
(i)



(ii)

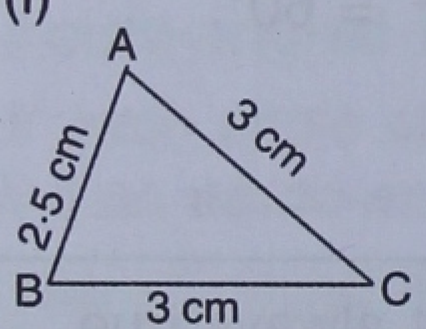


(iii)



9. Classify the following triangles according to side :

(i)



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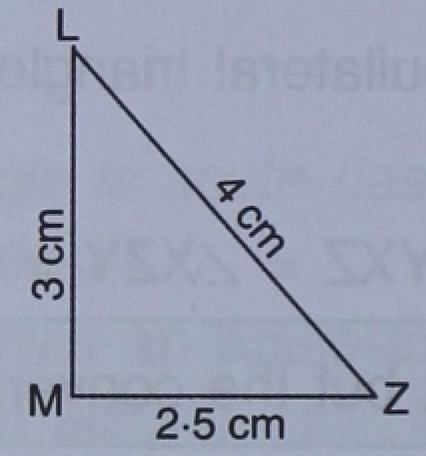

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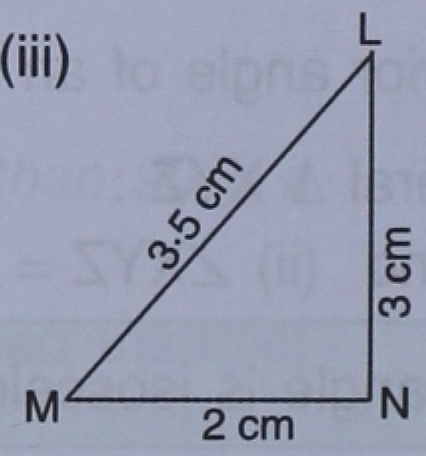

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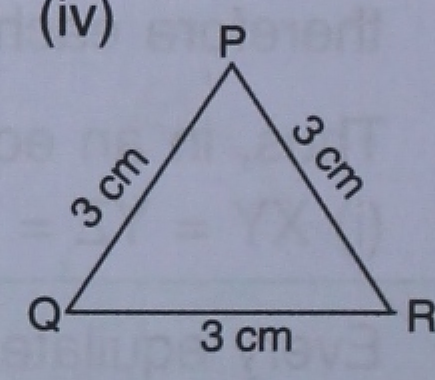


20.9 CONSTRUCTION OF TRIANGLES :

Here, we shall be constructing a triangle when any one of the following three conditions is given :

1. The lengths of the three sides.
2. The lengths of two sides and the angle included between these two sides.
3. Any two angles and the included side *i.e.* the side common to both the angles.

Construction 1 :

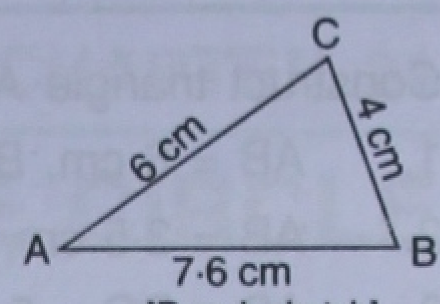
When the lengths of three sides are given.

Example 2 :

Construct a Δ ABC such that CB = 4 cm, AC = 6 cm and AB = 7·6 cm

Steps :

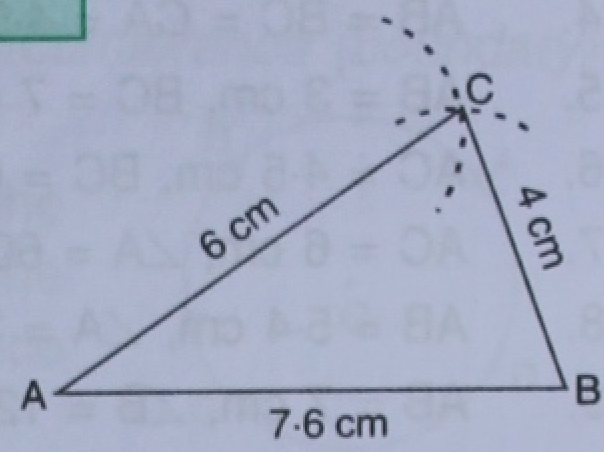
1. Draw a rough sketch of the triangle, as shown alongside.
2. Draw one of the sides, say, AB = 7·6 cm.



[Rough sketch]

> *We can draw any side first, but usually we start with the longest side.*

3. Using compass and taking A as centre, draw an arc of radius 6 cm.
4. With B as centre, draw an arc of radius 4 cm, that cuts the first arc at point C.
5. Join AC and BC.



The triangle ABC so obtained is the required triangle.

Construction 2 :

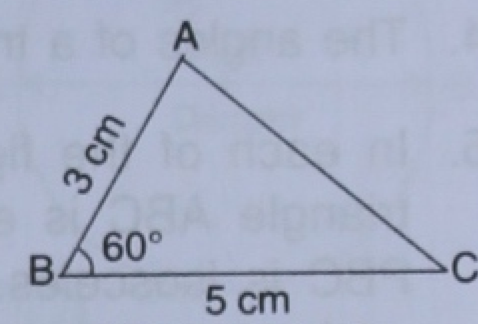
When two sides and the included angle (i.e. the angle formed between the two given sides) are given.

Example 3 :

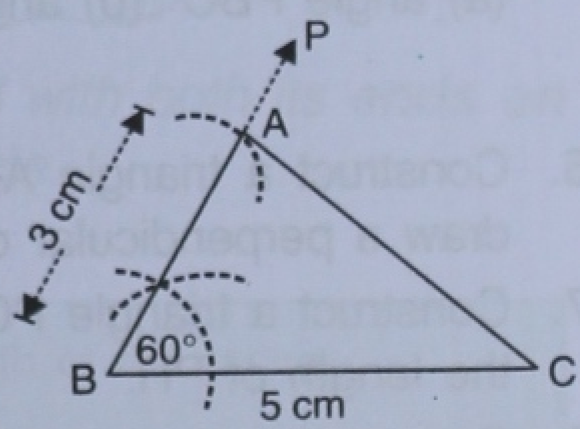
Construct a Δ ABC given AB = 3 cm, BC = 5 cm and $\angle ABC = 60°$.

Steps :

1. Draw a rough sketch of the triangle as shown alongside.
2. Draw BC = 5 cm.
3. With the help of compass, construct $\angle PBC = 60°$.
4. With B as centre, draw an arc of 3 cm length which cuts BP at point A.
   $\therefore$ BA = 3 cm
5. Join A and C.



[Rough sketch]



Clearly, the triangle ABC so obtained is the required triangle.

Construction 3 :

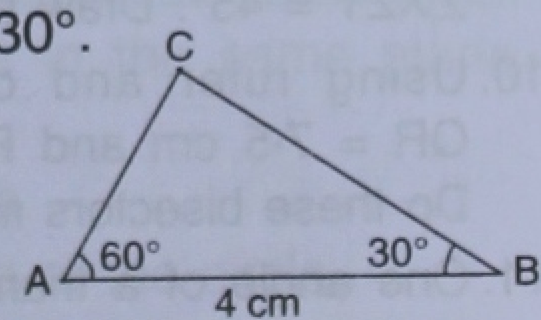
When two angles and included side are given.

Example 4 :

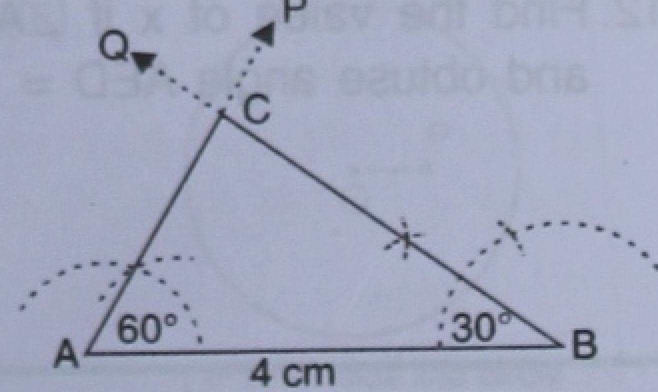
Construct Δ ABC when AB = 4 cm, $\angle A = 60°$ and $\angle B = 30°$.

Steps :

1. Draw AB = 4 cm.
2. At A, draw AP making an angle of 60° with AB, *i.e.* draw angle PAB = 60°.
3. At B, draw BQ making an angle of 30° with AB, *i.e.* $\angle QBA = 30°$.
4. AP and BQ intersect each other at point C.



[Rough sketch]



The triangle ABC so obtained is the required triangle.

EXERCISE 20(B)

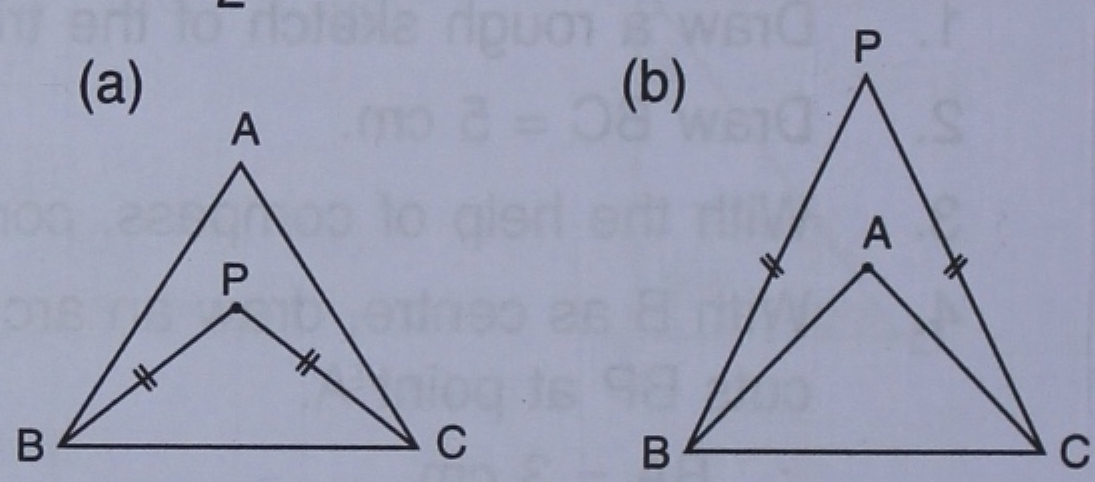
Construct triangle ABC when :

1. AB = 6 cm, BC = 8 cm and AC = 4 cm.
2. AB = 3·5 cm, AC = 4·8 cm and BC = 5·2 cm.
3. AB = BC = 5 cm and AC = 3 cm. Measure angles A and C. Is $\angle A = \angle C$?
4. AB = BC = CA = 4·5 cm. Measure all the angles of the triangle. Are they equal ?
5. AB = 3 cm, BC = 7 cm and $\angle B = 90°$.
6. AC = 4·5 cm, BC = 6 cm and $\angle C = 60°$.
7. AC = 6 cm, $\angle A = 60°$ and $\angle C = 45°$. Measure AB and BC.
8. AB = 5·4 cm, $\angle A = 30°$ and $\angle B = 90°$. Measure $\angle C$ and side BC.
9. AB = 7 cm, $\angle B = 120°$ and $\angle A = 30°$. Measure AC and BC.
10. BC = 3 cm, AC = 4 cm and AB = 5 cm. Measure angle ACB. Give a special name to this triangle.

Revision Exercise (Chapter 20)

1. If each of the two equal angles of an isosceles triangle is 68°, find the third angle.
2. One of the angles of a triangle is 110°, the two other angles are equal. Find their value.
3. The angles of a triangle are in the ratio 3 : 5 : 7. Find each angle.
4. The angles of a triangle are $(2x - 30°)$, $(3x - 40°)$ and $(\frac{5}{2}x + 10°)$. Find the value of x.
5. In each of the figures given alongside, triangle ABC is equilateral and triangle PBC is isosceles. If PBA = 20°, find in each case :

   (a) angle PBC (b) angle BPC.



6. Construct a triangle ABC given AB = 6 cm, BC = 5 cm and CA = 5·6 cm. From vertex A draw a perpendicular on to side BC. Measure the length of this perpendicular.
7. Construct a triangle PQR given PQ = 6 cm, $\angle P = 60°$ and $\angle Q = 30°$. Measure angle R and the length of PR.
8. Construct a triangle ABC given BC = 5 cm, AC = 6 cm and $\angle C = 75°$. Draw the bisector of the interior angle at A. Let this bisector meet BC at P; measure BP.
9. Using ruler and compass only, construct a triangle XYZ given YZ = 7 cm, $\angle XYZ = 60°$ and $\angle XZY = 45°$. Draw the bisectors of angles X and Y.
10. Using ruler and compass only, construct a triangle PQR given PQ = 5·5 cm, QR = 7·5 cm and RP = 6 cm. Draw the bisectors of the interior angles at P, Q and R. Do these bisectors meet at the same point ?
11. One angle of a triangle is 80° and the other two are in the ratio 3 : 2. Find the unknown angles of the triangle.
12. Find the value of x if $\angle A = 32°$, $\angle B = 55°$ and obtuse angle AED = 115°.