UNIT – 4
GEOMETRY

CHAPTER 16

FUNDAMENTAL CONCEPTS

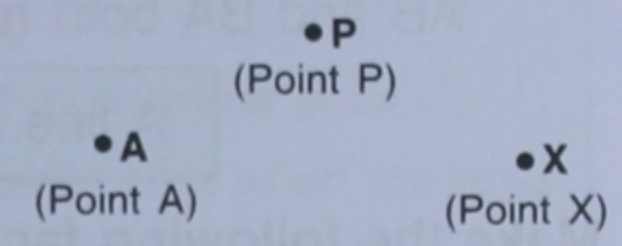
16.1 BASIC CONCEPT

Geometry *is the study of the position, shape, size and other properties of different figures.* Geometrical terms such as *point, line, plane, etc.*, carry the *basic ideas* for the *development of geometry.*

1. POINT :

A point is a mark of position. It has neither length nor width nor thickness; and occupies no space.

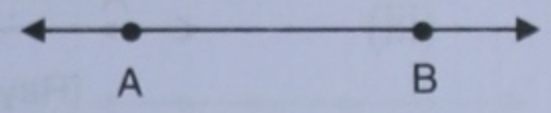
A point is represented by a dot. In general, it is denoted by a capital letter, such as A, P, X, etc. as shown alongside, and is read as '*point P*', '*point X*' and so on.



2. LINE :

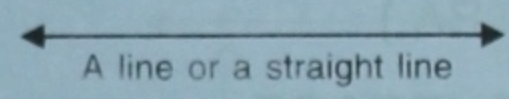
A line has only length. It has neither width nor thickness.

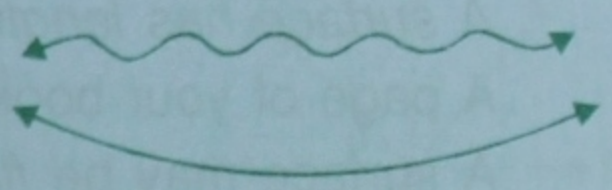
A line, as shown alongside, is represented by a straight mark with two arrow heads and is denoted by two capital letters.

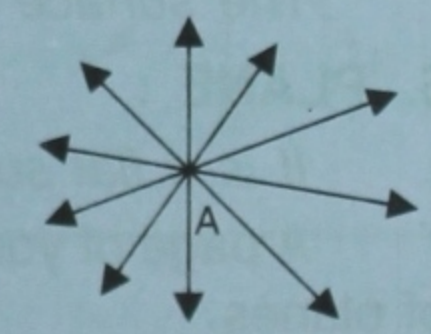


A line through points A and B is denoted as $\overleftrightarrow{AB}$ or $\overleftrightarrow{BA}$ and read as 'line AB' or 'line BA.'

1. *The two arrows of a line drawn in opposite directions indicate that the line has unlimited length, i.e. it can be extended up to any distance on either side.*
2. *A line may be straight or curved, but when we say 'a line', it means a straight line only.*
   The basic idea of a line is its straightness and that it extends infinitely in both directions.
3. *An unlimited number of lines can be drawn through a given fixed point.*
   The adjoining figure shows a fixed point A. It is clear from the figure that an unlimited number of lines can be drawn through A.
4. *One and only one line can be drawn through any two fixed points.*
   In the adjoining figure, A and B are two fixed points. It is clear from the figure that only one line can be drawn through the fixed points A and B.
5. *Every line has an infinite (uncountable) number of points in it.*



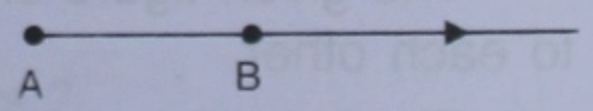




3. RAY :

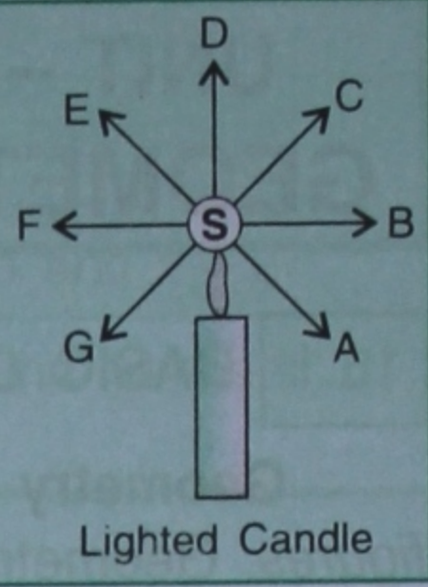
It is a line, i.e. a straight line, *that starts from a given fixed point and moves in the same direction.*

The adjoining figure shows a straight line that starts from a given fixed point A and moves through point B in the same direction. Therefore, it is a ray and is written as $\overrightarrow{AB}$.



1. *A ray has only one end* (fixed) *point, which is also known as its initial point.*
2. *A ray extends indefinitely in only one direction.*
3. *An unlimited number of rays can be drawn with the same initial point.*

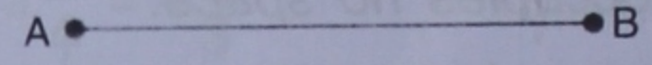
   The adjoining figure shows a point source of light ,S. SA, SB, SC, SD, etc., are rays of light starting from the source S. Clearly, there are a very large number of such rays each having the same initial point S.
4. *A ray is a part of a line.*



4. LINE SEGMENT :

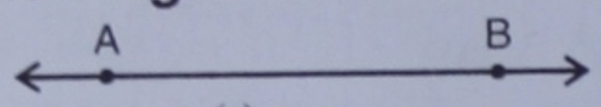
A line segment is a part of a straight line.

The adjoining figure shows a line segment AB, which has two end points A and B. AB and BA both represent the same line segment.

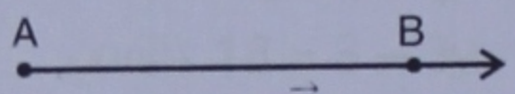


A line segment is a part of a line as well as of a ray.

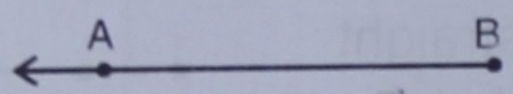
Make the following facts clear :

(i) 

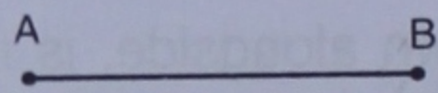
[Line AB, *i.e.* $\overleftrightarrow{AB}$ or line BA, *i.e.* $\overleftrightarrow{BA}$]

(ii) 

[Ray AB = $\overrightarrow{AB}$]

(iii) 

[Ray BA = $\overrightarrow{BA}$]

(iv) 

[Line segment AB (or BA)]

Clearly, line segment AB (or BA) is a part of both the rays AB ($\overrightarrow{AB}$) and BA ($\overrightarrow{BA}$). At the same time, the line segment AB (or BA) is a part of both the lines AB ($\overleftrightarrow{AB}$) and BA ($\overleftrightarrow{BA}$).

5. SURFACE :

A surface has length and width, but no thickness.

A page of your book, the outside of a box, a black-board, etc., represent surfaces.

A surface may be *flat* or *curved.*

The *surface of a wall* is *flat*, while the *surface of a cricket ball* is *curved.*

6. PLANE :

It is a flat surface. A plane has length and width, but no thickness.

A page of your book, the surface of a wall, the top of a table, etc., are some examples of planes.

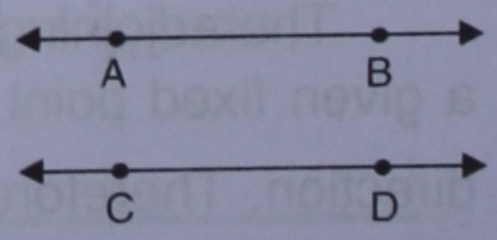
(i) *When a straight line is drawn through any two points on a plane, the line lies entirely within in the plane.*

(ii) *A plane is a surface that extends indefinitely in all directions.*

7. PARALLEL LINES :

Two straight lines are said to be ***parallel*** *to each other if they lie in the same plane and do not meet even when produced upto any extent on either side.*

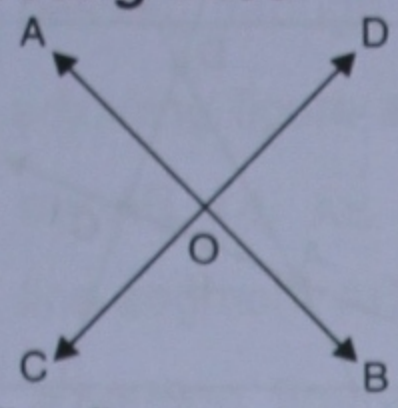
The given figure shows two lines AB and CD that are parallel to each other.

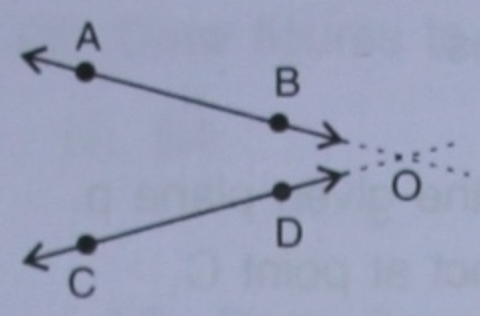


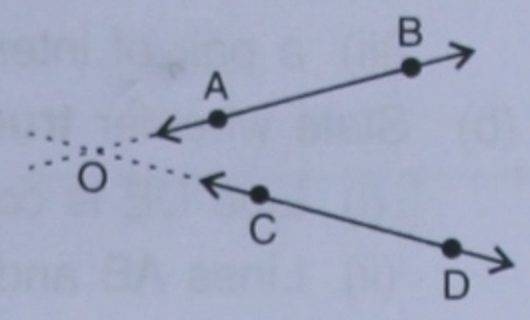
The line AB is parallel to the line CD is symbolically represented by writing AB // CD, read as *line AB is parallel to line CD.*

8. INTERSECTING LINES :

If two lines lie in the same plane and are not parallel to each other, they are called ***intersecting lines****.*







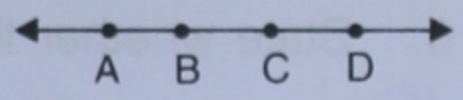
The figure given above shows two lines AB and CD that are not parallel. Such lines either intersect at point O or will intersect at point O if extended.

1. *The distance between two parallel lines is always the same whereas the distance between two non-parallel lines (lines that are not parallel) keeps changing.*
2. *Two different lines in a plane are either parallel or they intersect at only one point.*
3. *The definitions of parallel lines and intersecting lines are applicable only when the lines are in the same plane, i.e. when the lines are co-planar.*

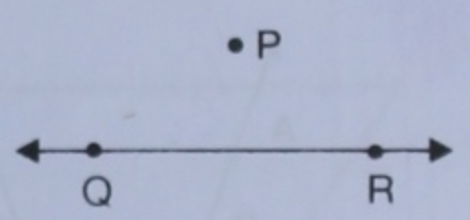
9. COLLINEAR POINTS :

If three or more points lie on the same straight line, then the points are called ***collinear points****.*

The given figure shows the collinear points A, B, C and D; as these points lie on the same line.



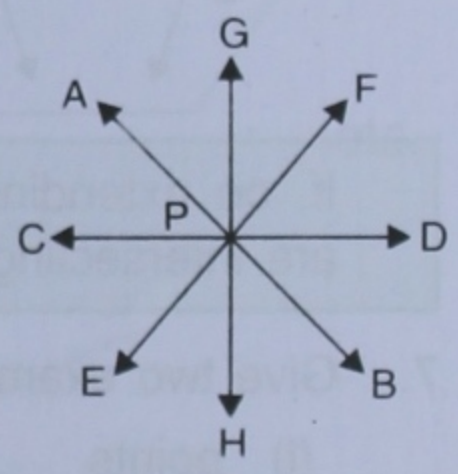
The adjoining figure shows three non-collinear points : P, Q and R. These points do not lie on the same straight line.



10. CONCURRENT LINES :

If three or more straight lines (in the same plane) pass through the same point, the lines are called ***concurrent lines and the point is called the point of concurrence****.*

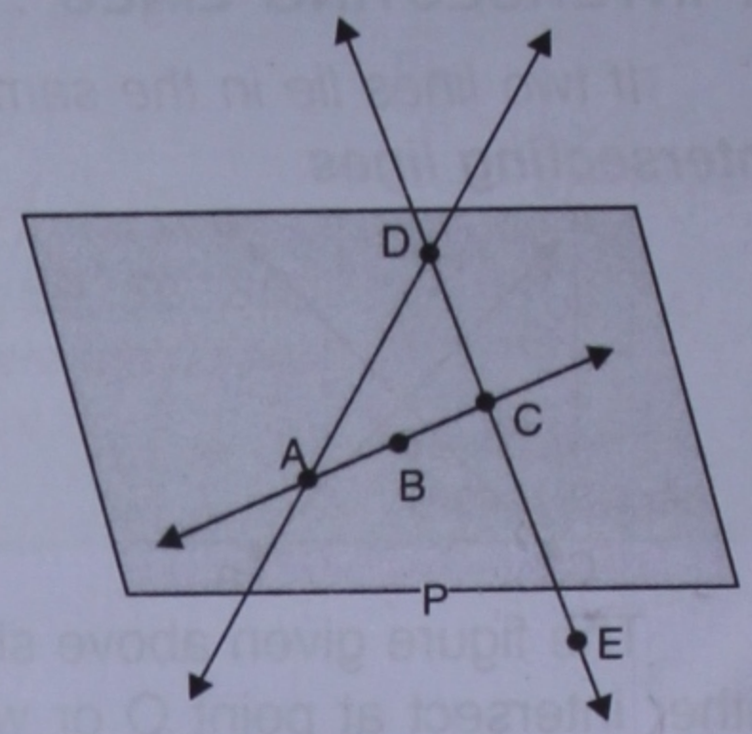
The given figure shows the concurrent lines AB, CD, EF and GH; all these lines are in the same plane and pass through the same point P. Clearly, *point P is the point of concurrence.*



EXERCISE 16(A)

1. State, **true** or **false**, if **false**, correct the statement :
   (i) A dot has width but no length.
   (ii) A ray has an infinite length only on one side of it.
   (iii) A line segment PQ is written as $\overleftrightarrow{PQ}$.
   (iv) $\overleftrightarrow{PQ}$ represents a straight line.
   (v) Three points are said to be collinear if they lie in the same plane.
   (vi) Three or more points, all lying in the same line, are called collinear points.
2. Write how many lines can be drawn through :
   (i) a given point ?
   (ii) two given fixed points ?
   (iii) three collinear points ?
   (iv) three non-collinear points ?

3. The shaded region of the given figure shows a plane :



(a) Name :

(i) three collinear points.

(ii) three non-collinear points.

(iii) a pair of intersecting lines.

(b) State whether **true** or **false** :

(i) Line DE is contained in the given plane p.

(ii) Lines AB and DE intersect at point C.

(iii) Points D, B and C are collinear.

(iv) Points D, B and E are collinear.

4. Correct the statement if it is wrong :

(i) A ray can be extended infinitely on either side.

(ii) A ray has a definite length.

(iii) A line segment has a definite length.

(iv) A line has two end points.

(v) A ray has only one end point.

5. State **true** or **false**, if **false**, give the correct statement :

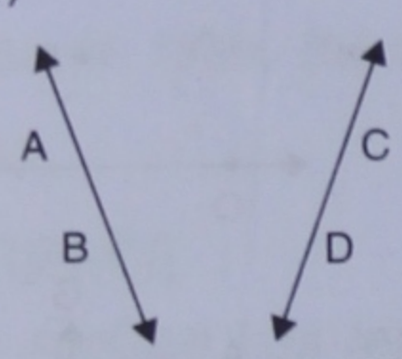
(i) A line has a countable number of points in it.

(ii) Only one line can pass through a given point.

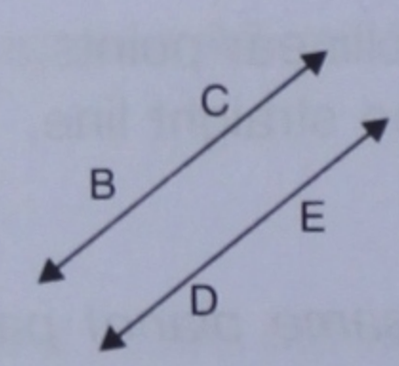
(iii) The intersection of two planes is a straight line.

6. State whether the following *pairs of lines* or *rays* appear to be *parallel* or *intersecting*.

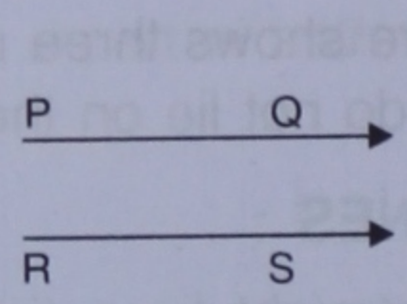
(i)



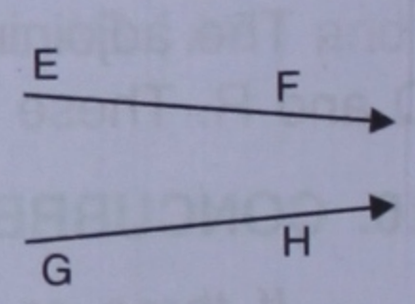
(ii)



(iii)



(iv)



If, on extending the two lines on either side, they intersect or appear to intersect, they are intersecting lines.

7. Give two examples, from your surroundings, for each of the following :

(i) points

(ii) line segments

(iii) plane surfaces

(iv) curved surfaces.

8. Under what condition will two straight lines in the same plane have :

(i) no point in common.

(ii) only one point in common.

(iii) an infinite number of points in common.

If possible, draw diagrams in support of your answer.

9. Mark two points A and B on a page of your exercise book. Mark a third point P such that :

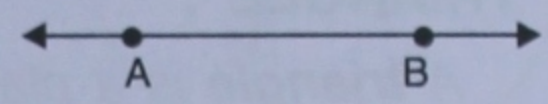
(i) P lies between A and B and the three points A, P and B are collinear.

(ii) P does not lie between A and B yet the three points are collinear.

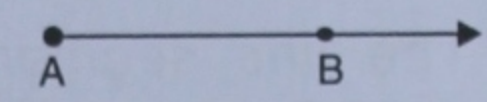
(iii) the three points do not lie in a line.

10. Mark two points P and Q on a piece of paper. How many lines can you draw :
    (i) passing through both the points P and Q ?
    (ii) passing through the point P ?
    (iii) passing through the point Q ?

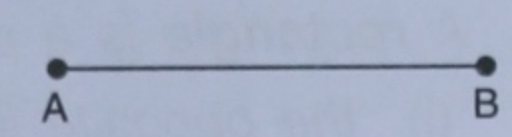
11. The adjoining figure shows a line AB. Draw figures to represent :
    (i) ray AB, *i.e.* $\overrightarrow{AB}$ (ii) $\overrightarrow{BA}$
    (iii) line segment AB.



12. The adjoining figure shows a ray AB. Draw figures to show :
    (i) ray BA, *i.e.* $\overrightarrow{BA}$ (ii) line AB
    (iii) line segment BA.



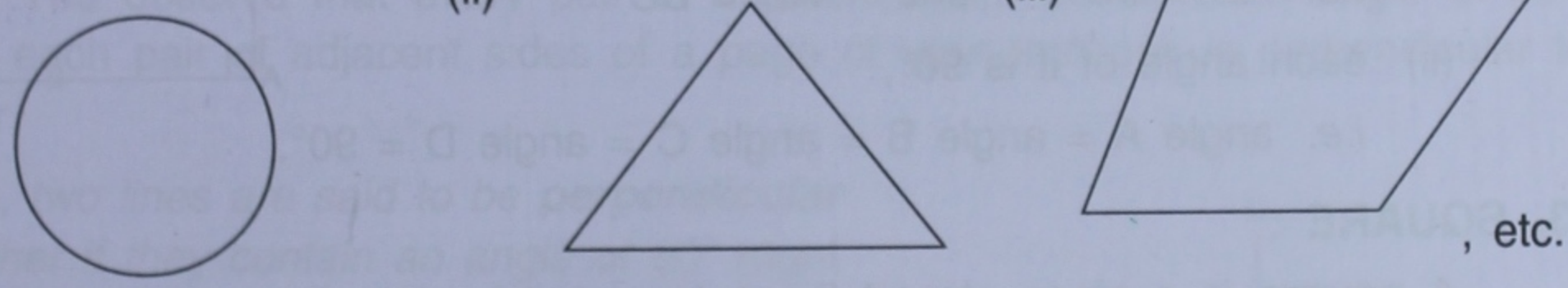
13. The adjoining figure shows a line segment AB. Draw figures to represent :
    (i) ray AB, *i.e.* $\overrightarrow{AB}$ (ii) line AB, *i.e.* $\overleftrightarrow{AB}$
    (iii) ray BA.



16.2 A PLANE CLOSED FIGURE

(a) The following are some closed figures drawn in a plane :

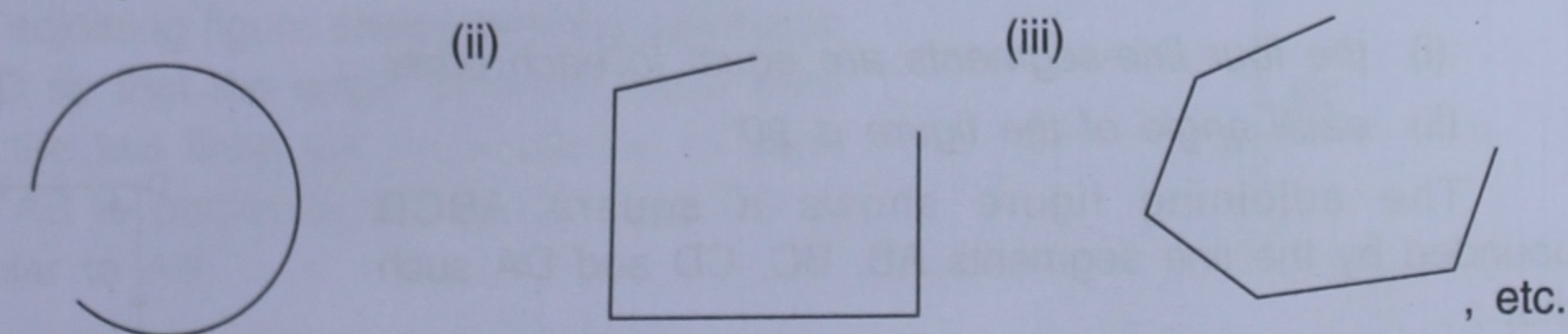
(i) (ii) (iii)



, etc.

(b) The following are some open figures drawn in a plane :

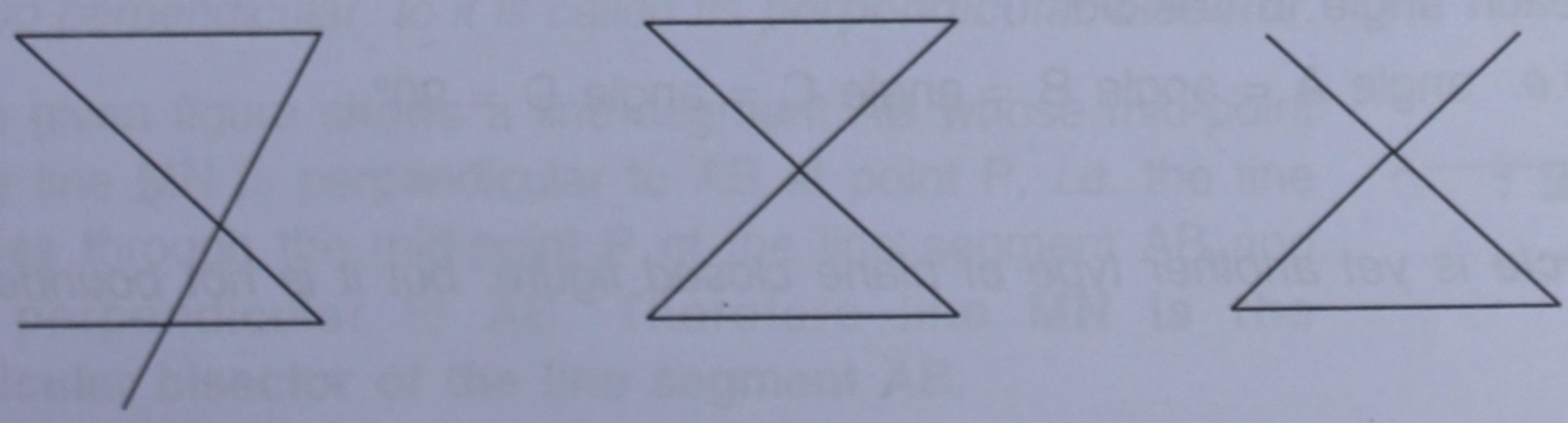
(i) (ii) (iii)



, etc.

(c) The following figures are in the same plane having intersecting line segments.

(i) (ii) (iii)



, etc.

What is the difference among the different sets of figures drawn above : for (a), for (b) and for (c) ?

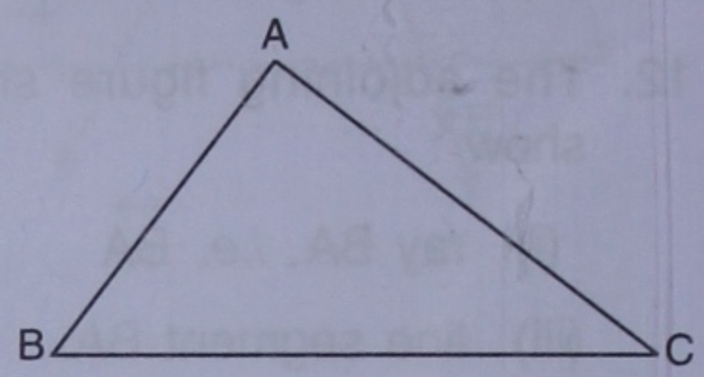
The figures in (a) drawn above, are *plane closed figures* because :

(i) they are bounded (enclosed) by continuous curves/lines and

(ii) the curves/lines in the figure do not intersect each other.

1. TRIANGLE :

*A **triangle** is a plane closed figure bounded by three line segments.*



The adjoining figure shows a triangle ABC bounded by the three line segments AB, BC and CA.

The *line segments* AB, BC and CA that form triangle ABC are called the sides of triangle ABC.

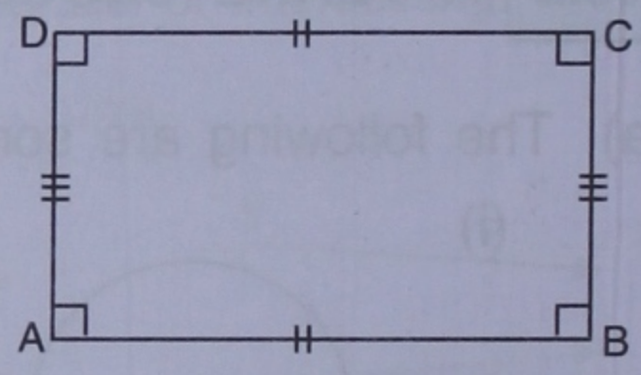
2. RECTANGLE :

*A **rectangle** is a plane closed figure bounded by four line segments such that :*

(i) *the opposite line segments (sides) are equal*

(ii) *each angle of the figure is 90°.*

The adjoining figure shows a rectangle ABCD bounded by the line segments AB, BC, CD and DA such that :



(i) the opposite line segments (sides) are equal.

i.e. AB = CD and AD = BC

(ii) each angle of it is 90°,

i.e. angle A = angle B = angle C = angle D = 90°.

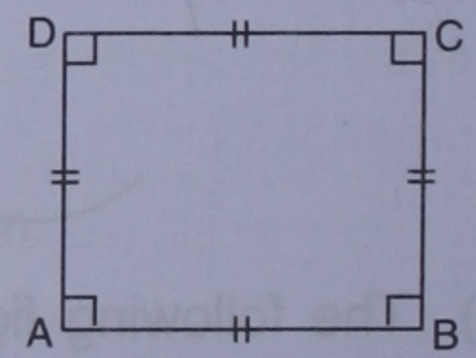
3. SQUARE :

*A **square** is a plane closed figure bounded by four line segments* (sides) *such that :*

(i) *the four line-segments are equal to each other*

(ii) *each angle of the figure is 90°.*

The adjoining figure shows a square ABCD bounded by the line segments AB, BC, CD and DA such that :



(i) the four line segments (sides) are equal,

i.e. AB = BC = CD = DA

(ii) each angle in it is 90°

i.e. angle A = angle B = angle C = angle D = 90°.

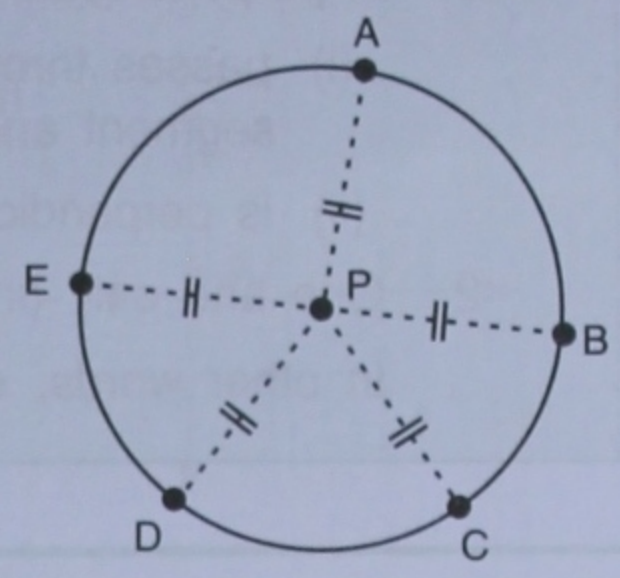
4. CIRCLE :

*A **circle** is yet another type of plane closed figure, but it is not bounded by line segments.*

For example :

1. A wheel of : a cycle, a car, a bus, etc.
2. The disc of the full moon, etc.

A circle is a closed smooth curve of which each point is equidistant from a fixed point inside it.



The adjoining figure shows a circle. P is a fixed point inside it and A, B, C, D, E, ..., etc. are some points on the circle.

Clearly, PA = PB = PC = PD = ...

The *fixed point* inside the circle is called the ***centre*** of the circle, whereas the *distance* of each *point* on the circle *from the fixed point* (centre) is called the ***radius*** of the circle.

∴ P = centre of the circle

and PA = PB = PC = ... = radius of the circle.

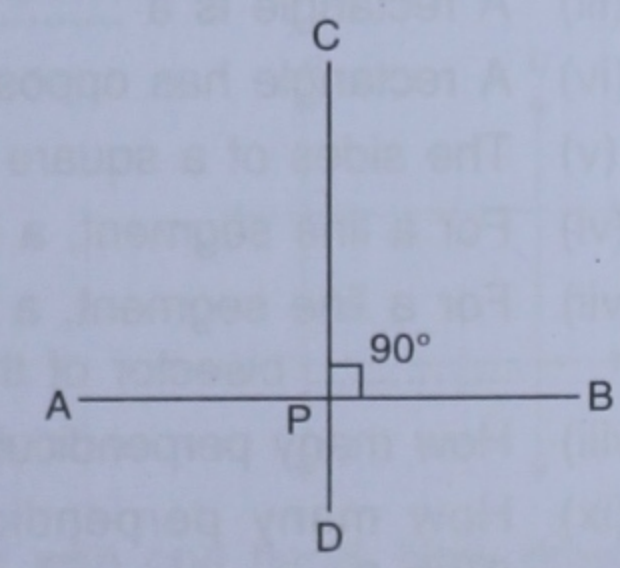
16.3 PERPENDICULAR LINE OF A LINE SEGMENT

Your geometry box has a pair of set-squares. Each set-square has three edges (sides). The two sides (edges) of each set square contain an angle of 90° hence these two sides are said to be *perpendicular* to each other.

Similarly, when you look at a page of your textbook carefully, what do you observe ? You observe that every pair of adjacent sides contains an angle of 90°. Therefore, each pair of adjacent sides of a page of your textbook is perpendicular to each other.

*Thus, two lines are said to be **perpendicular** to each other if they contain an angle of 90°* (right angle) *between them.*

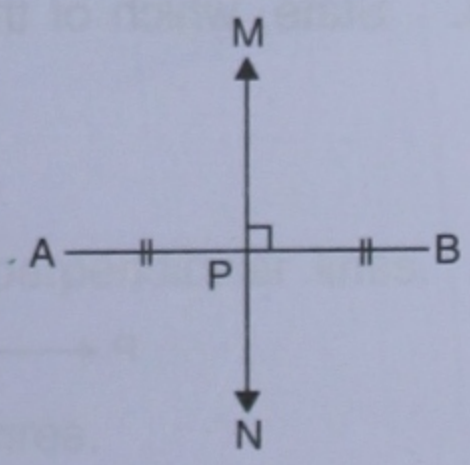
The adjoining figure shows two line segments AB and CD so that the angle between these lines is 90°; so the two lines are perpendicular to each other, *i.e.* AB is perpendicular to CD, and CD is perpendicular to AB.



16.4 PERPENDICULAR BISECTOR OF A LINE SEGMENT :

*For a line segment, the line that passes through its mid-point and is also perpendicular to it is called its **perpendicular bisector**.*

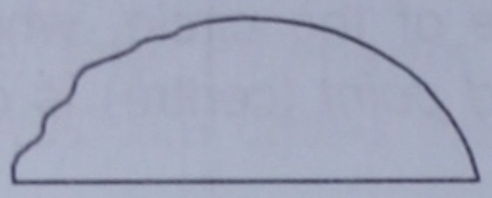
The given figure shows a line segment AB whose mid-point is P. The line MN is perpendicular to AB at point P, *i.e.* the line MN passes through the mid-point P of the line segment AB and is also perpendicular to AB. Therefore line **MN is the perpendicular bisector of the line segment AB**.

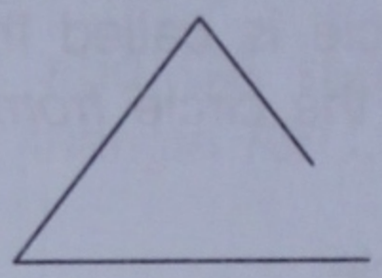


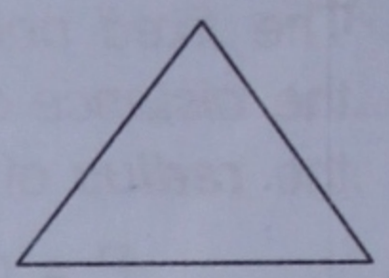
1. A perpendicular bisector of a line segment always :
   (i) passes through the mid-point of the line segment *i.e.* bisects the given line segment and
   (ii) is perpendicular to the given line segment.
2. One and only one perpendicular bisector can be drawn to a given line segment.
   In other words; *every line segment has a unique perpendicular bisector of it.*

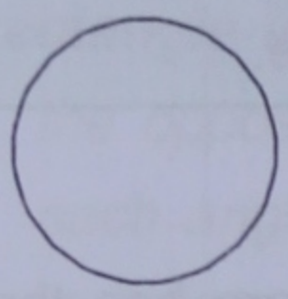
EXERCISE 16(B)

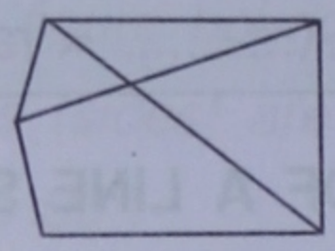
1. State, which of the following is a *plane closed figure* :

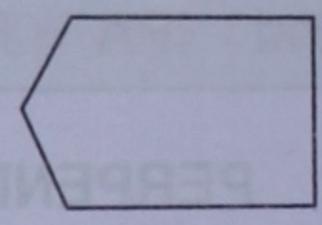
(i) 

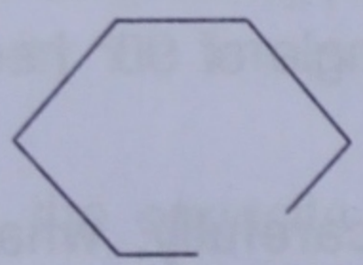
(ii) 

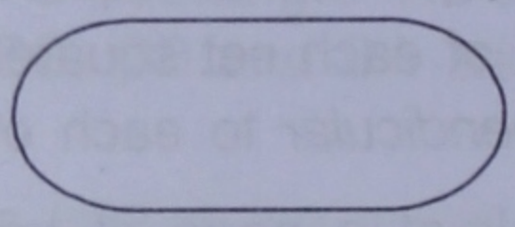
(iii) 

(iv) 

(v) 

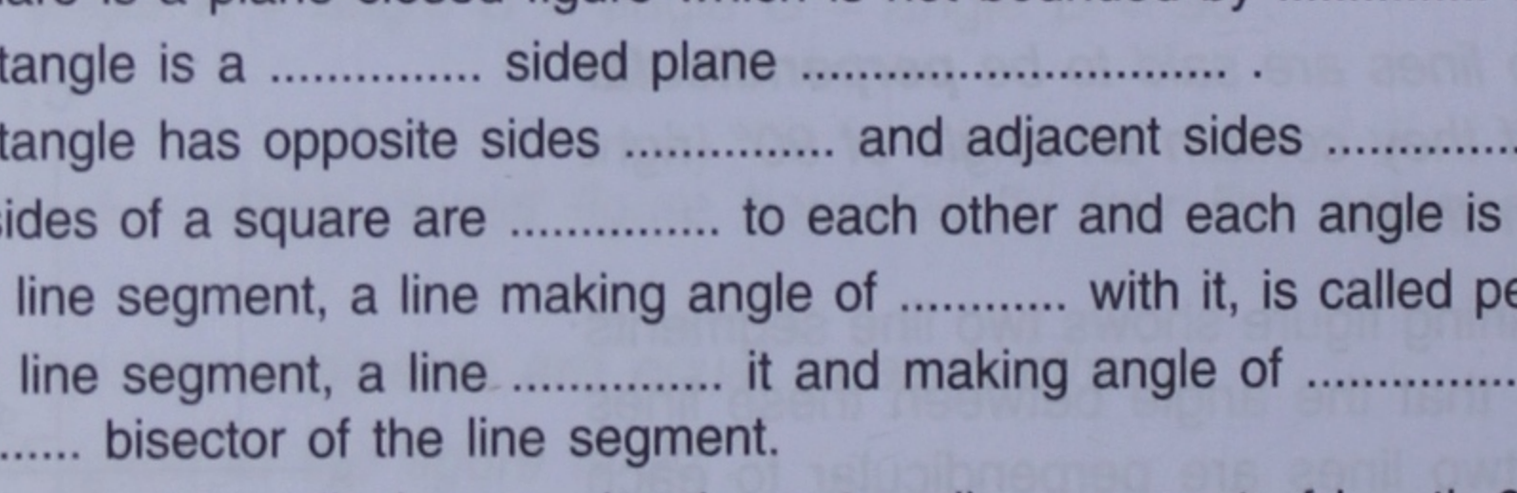
(vi) 

(vii) 

(viii) 

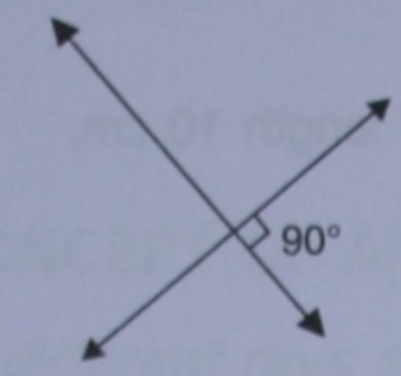
2. Fill in the blanks :
   (i) ..... is a three sided plane closed figure.
   (ii) A square is a plane closed figure which is not bounded by ..... .
   (iii) A rectangle is a ..... sided plane ..... .
   (iv) A rectangle has opposite sides ..... and adjacent sides ..... to each other.
   (v) The sides of a square are ..... to each other and each angle is .....
   (vi) For a line segment, a line making angle of ..... with it, is called perpendicular to it.
   (vii) For a line segment, a line ..... it and making angle of ..... with it, is called ..... bisector of the line segment.
   (viii) How many perpendiculars can be drawn to a line segment of length 6 cm ?
   (ix) How many perpendicular bisectors can be drawn to a line segment of length 6 cm ?
   (x) A perpendicular to a line segment will be its perpendicular bisector if it passes through the ..... of the given line segment.

3. State, which of the lines/line-segments are perpendicular to the line PQ :

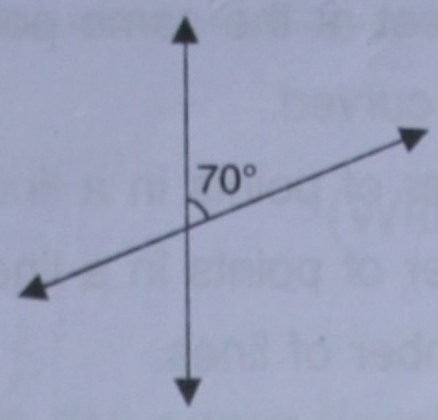


4. Which of the following figures shows two mutually perpendicular lines :

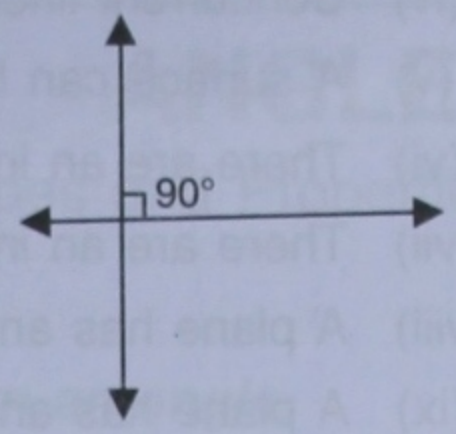
(i)



(ii)

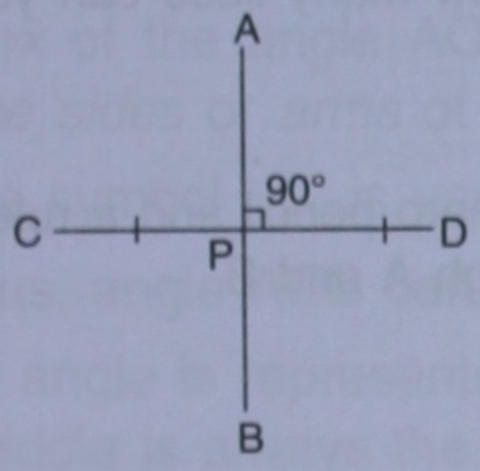


(iii)

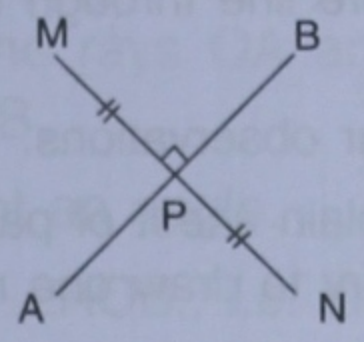


5. For each figure given below name the line segment that is perpendicular bisector of the other :

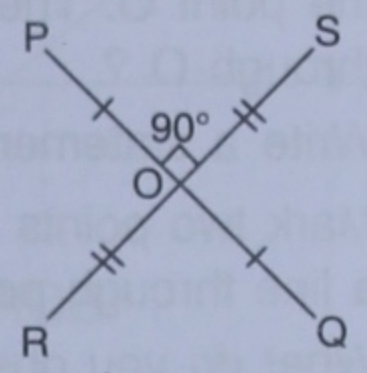
(i)



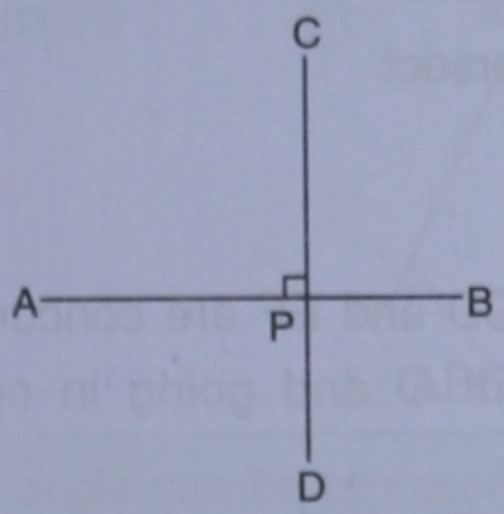
(ii)



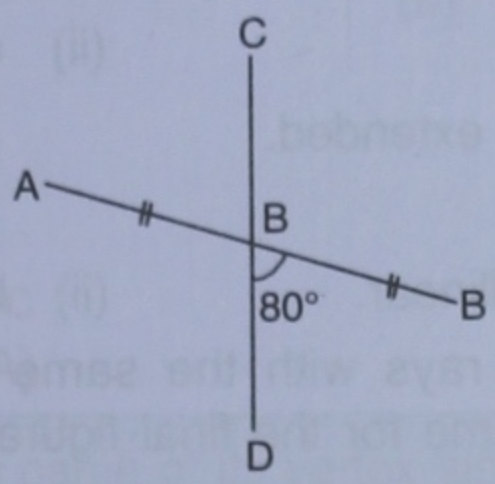
(iii)



(iv)

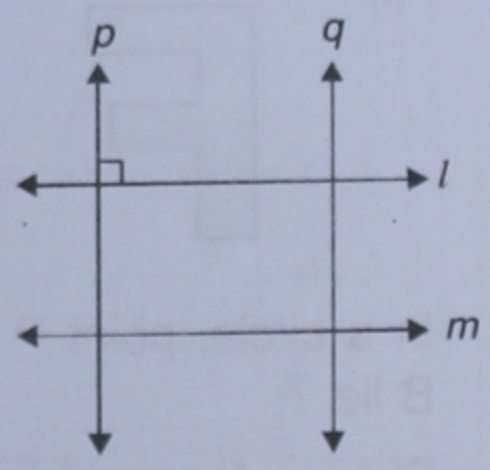


(v)



6. Name three objects from your surroundings that contain perpendicular edges.

7. Using the given figure, answer the following :



(i) Name the pairs of parallel lines.

(ii) Name the pairs of mutually perpendicular lines.

(iii) Is the line p parallel to the line l ?

(iv) Is the line q perpendicular to the line m ?

8. Place a scale (ruler) on a sheet of paper and hold it firmly with one hand. Now draw two line segments AB and CD along the longer edges of the scale. State whether segment AB is parallel to or perpendicular to segment CD.

9. Check your textbook :

(i) How many pairs of its edges are parallel to each other ?

(ii) How many pairs of its edges are perpendicular to each other ?

10. Give two examples from your surroundings for each of the following :

(i) intersecting lines (ii) parallel lines (iii) perpendicular lines.

11. State **true** or **false**, if **false**, give the correct statement :

(i) The maximum number of lines through three collinear points is three.

(ii) The maximum number of lines through three non-collinear points is three.

(iii) Two parallel lines always lie in the same plane.

(iv) Concurrent lines always meet at the same point.

(v) A surface can be plane or curved.

(vi) There are an infinite number of points in a line segment of length 10 cm.

(vii) There are an infinite number of points in a line.

(viii) A plane has an infinite number of lines.

(ix) A plane has an infinite number of points.

Revision Exercise (Chapter 16)

1. Mark a point O on a piece of paper. Using a sharp pencil and a ruler, draw a line through the point O. Then draw one more line through the point O. How many lines can you draw through O ?

   Write a statement regarding your observations.

2. Mark two points A and B on a plain sheet of paper. Using a sharp pencil and a ruler, draw a line through points A and B. Try to draw one more line through A and B.

   What do you observe ?

   Write a statement regarding your observations.

3. Draw two straight lines on a sheet of paper so that the lines drawn :

   (i) intersect each other (ii) do not intersect

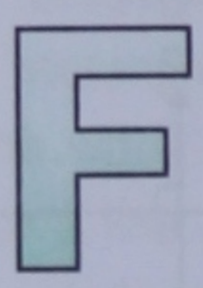
   (iii) appear to intersect when extended.

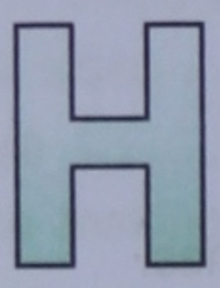
4. Draw a figure to show that :

   (i) points A, B and C are collinear. (ii) lines AB, CD and EF are concurrent

5. In your note-book, draw two rays with the same initial point O and going in opposite directions. Write the special name for the final figure obtained.

6. In each figure given below, write the the number of line segments used :

   (i) 

   (ii) 

   (iii) 

7. In a circle, point P is its centre and PA = PB = radius of the circle. Where do points A and B lie ?

8. Draw a four-sided closed figure in which :

   (i) all the sides are equal and each angle is 90°

   (ii) the opposite sides are equal and each angle is 90°

   Write the special names of each figure drawn.