

## UNIT – 2

# ARITHMETIC PROBLEMS

## CHAPTER 8

# RATIO

(Including Proportions)

### 8.1 INTRODUCTION

Most of the time, we compare things, numbers, etc. (say,  $x$  and  $y$ ) by saying :

- |                                    |   |
|------------------------------------|---|
| (i) $x$ is <i>greater</i> than $y$ | (ii) $x$ is <i>less</i> than $y$        |
| (iii) $x$ is <i>double</i> of $y$  | (iv) $x$ is <i>one-third</i> of $y$     |
| (v) $\frac{x}{y} = \frac{4}{5}$    | (vi) $\frac{y}{x} = \frac{3}{2}$ , etc. |

The method of comparing two quantities (numbers, things, etc.) by dividing one quantity by the other, is called **ratio**.

Thus :  $\frac{x}{y} = \frac{4}{5}$  represents the ratio of  $x$  to  $y$ .

and,  $\frac{y}{x} = \frac{3}{2}$  represents the ratio of  $y$  to  $x$ .

### 8.2 RATIO

The relation of two quantities (both of the same kind and in the same unit) obtained on dividing one quantity by the other is called their **ratio**.

∴ The **ratio** of two quantities  $x$  to  $y$ , both of the same kind and in the same unit, is  $\frac{x}{y}$ , and is often written as  $x : y$  (read as  $x$  to  $y$  or  $x$  is to  $y$ ).

#### Meaning of the two quantities of the same kind and in the same unit :

- Both the quantities must of the same kind, means :** If one quantity is length, the other quantity must also be length; if quantity represents mass the other quantity must also be representing mass and so on.

**The ratio between unlike quantities has no meaning.**

For example, the ratio of length to mass has no meaning.

- Both the quantities must be in the same unit, means :** The two quantities must have the same unit of measurement.

For example, if the lengths of two objects are given to be 60 cm and 1.5 m; then before finding the ratio of one length to that of other, both of these lengths must either be converted into cm or into m.

#### Examples :

(i) The ratio of 5 kg to 15 kg =  $\frac{5 \text{ kg}}{15 \text{ kg}} = \frac{1}{3} = 1 : 3$

(ii) The ratio of 800 gm to 1.2 kg

$$= \frac{800 \text{ gm}}{1200 \text{ gm}} = \frac{2}{3} = 2 : 3$$

$$1.2 \text{ kg} = 1.2 \times 1000 \text{ gm} = 1200 \text{ gm}$$

(iii) The ratio of 2 m to 80 cm

$$= \frac{2 \text{ m}}{80 \text{ cm}} = \frac{200 \text{ cm}}{80 \text{ cm}} = \frac{5}{2} = 5 : 2$$

$$2 \text{ m} = 2 \times 100 \text{ cm} = 200 \text{ cm}$$

(iv) The ratio of  $1\frac{1}{2}$  years to 10 months

$$= \frac{18 \text{ months}}{10 \text{ months}} = \frac{9}{5} = 9 : 5$$

$$1\frac{1}{2} \text{ years} = \frac{3}{2} \times 12 \text{ months} = 18 \text{ months}$$

A ratio is a pure number and so has no unit.

1. The ratio of two numbers or quantities is denoted by the colon mark “ : ”.  
Thus, the ratio of two quantities  $p$  and  $q = p : q$
2. The ratio of two quantities of same kind and in the same unit is obtained on dividing one quantity by the other.

$$\text{Thus, the ratio of 20 kg to 80 kg} = \frac{20 \text{ kg}}{80 \text{ kg}} = \frac{1}{4} = 1 : 4$$

3. The **first term** of a ratio is called the **antecedent** and the **second term** is called the **consequent**.

In the ratio  $1 : 4$ , antecedent = 1 and consequent = 4.

4. A ratio must always be expressed in its lowest terms.
5. Whatever be the units of the terms of a ratio, the ratio has no unit. The ratio of 15 km and 20 km =  $\frac{15 \text{ km}}{20 \text{ km}} = \frac{3}{4} = 3 : 4$ . Here, the two quantities 15 km and 20 km have unit km, but their ratio  $3 : 4$  has no unit. On dividing, units cancel out.

6. The terms of a ratio are written in a definite order :

$$\text{The ratio of 5 kg and 8 kg} = \frac{5 \text{ kg}}{8 \text{ kg}} = \frac{5}{8} = 5 : 8 \text{ and}$$

$$\text{the ratio of 8 kg and 5 kg} = \frac{8 \text{ kg}}{5 \text{ kg}} = \frac{8}{5} = 8 : 5$$

**Remember :**  $5 : 8$  and  $8 : 5$  are not equal to each other.

### Example 1 :

Find the ratio of : (i) 60 to 48 (ii) 3.75 kg to 750 gm

### Solution :

$$\begin{aligned} \text{(i) Required ratio} &= \frac{60}{48} = \frac{5}{4} \\ &= 5 : 4 \end{aligned}$$

(Ans.)

(ii) Since  $3.75 \text{ kg} = 3.75 \times 1000 \text{ gm} = 3750 \text{ gm}$

$$\begin{aligned} \text{Required ratio} &= \frac{3.75 \text{ kg}}{750 \text{ gm}} \\ &= \frac{3750 \text{ kg}}{750 \text{ gm}} = \frac{5}{1} = 5 : 1 \end{aligned}$$

(Ans.)

### 8.3 CONVERTING INTO SIMPLE RATIO

#### Example 2 :

Express as simple ratio : (i)  $3\frac{1}{2} : 2\frac{1}{3}$       (ii)  $\frac{2}{3} : \frac{4}{5} : \frac{1}{2}$ .

#### Solution :

(i) Divide the first term of the ratio by its second term and then simplify.

$$\begin{aligned} \text{Given ratio} &= 3\frac{1}{2} : 2\frac{1}{3} = \frac{7}{2} : \frac{7}{3} \\ &= \frac{7}{2} \times \frac{3}{7} = \frac{3}{2} = \mathbf{3 : 2} \end{aligned} \quad (\text{Ans.})$$

#### Alternative method :

Multiply each terms of the ratio by the L.C.M. of their denominators and then simplify.

$$\begin{aligned} \therefore \text{Given ratio} &= \frac{7}{2} : \frac{7}{3} = \frac{7}{2} \times 6 : \frac{7}{3} \times 6 && \text{L.C.M. of 2 and 3 = 6} \\ &= 21 : 14 = \frac{21}{14} = \frac{3}{2} = \mathbf{3 : 2} \end{aligned} \quad (\text{Ans.})$$

$$\begin{aligned} \text{(ii) Given ratio} &= \frac{2}{3} : \frac{4}{5} : \frac{1}{2} = \frac{2}{3} \times 30 : \frac{4}{5} \times 30 : \frac{1}{2} \times 30 && \text{L.C.M. of 3, 5 and 2 = 30} \\ &= \mathbf{20 : 24 : 15} \end{aligned} \quad (\text{Ans.})$$

#### Example 3 :

The strength of a class is 50 with 30 boys and the remaining girls. Find the ratio of the number of boys to the number of girls in the class.

#### Solution :

$$\begin{aligned} &\text{Since the strength of the class} = 50 \\ \text{and} &\text{ the number of boys in the class} = 30 \\ \Rightarrow &\text{ The number of girls in the class} = 50 - 30 = 20 \\ \therefore &\text{ Required ratio} = \frac{\text{No. of boys in the class}}{\text{No. of girls in the class}} \\ &= \frac{30}{20} = \frac{3}{2} = \mathbf{3 : 2} \end{aligned} \quad (\text{Ans.})$$

#### Example 4 :

A man's monthly income is ₹ 15,000, out of which he spends ₹ 12,500 every month. Find the ratio of his :

- (i) savings to expenditure      (ii) expenditure to income  
(iii) income to savings

#### Solution :

$$\begin{aligned} \text{Since} &\text{ the monthly income of the man} = ₹ 15,000 \\ \text{And} &\text{ his monthly expenditure} = ₹ 12,500 \\ \Rightarrow &\text{ His savings per month} = ₹ 15,000 - ₹ 12,500 = ₹ 2,500 \end{aligned}$$



This means, if all the sweets are divided in  $1 + 3 = 4$  equal parts,

Then, **A gets** = One part out of the 4 equal parts made

$$= \frac{1}{4} \text{ of the total number of sweets}$$

$$= \frac{1}{4} \times 12 \text{ sweets} = \mathbf{3 \text{ sweets}} \quad (\text{Ans.})$$

And **B gets** = 3 parts out of the 4 equal parts made

$$= \frac{3}{4} \text{ of the total number of sweets}$$

$$= \frac{3}{4} \times 12 \text{ sweets} = \mathbf{9 \text{ sweets}} \quad (\text{Ans.})$$

Thus, if a whole quantity is divided into two parts in the ratio 3 : 4m

$$\therefore \text{The first part} = \frac{3}{7} \times \text{the whole quantity} \quad [\text{As, } 3 + 4 = 7]$$

$$\text{and the second part} = \frac{4}{7} \times \text{the whole quantity}$$

### Example 6 :

A pole of length 165 cm is divided into two parts such that their lengths are in the ratio 7 : 8. Find the length of each part of the pole.

#### Solution :

Here, 165 cm is to be divided into two lengths in the ratio 7 : 8 and  $7 + 8 = 15$ .

$$\therefore \text{Length of one (shorter) part} = \frac{7}{15} \times 165 \text{ cm} = \mathbf{77 \text{ cm}}$$

$$\text{and length of the other (longer) part} = \frac{8}{15} \times 165 \text{ cm} = \mathbf{88 \text{ cm}} \quad (\text{Ans.})$$

### Example 7 :

Divide 99 into three parts in the ratio 2 : 4 : 5.

#### Solution :

$$\text{Since } 2 + 4 + 5 = 11$$

$$\therefore \text{1st part} = \frac{2}{11} \times 99 = \mathbf{18}; \quad \text{2nd part} = \frac{4}{11} \times 99 = \mathbf{36}$$

$$\text{and } \text{3rd part} = \frac{5}{11} \times 99 = \mathbf{45} \quad (\text{Ans.})$$

### Example 8 :

Divide 268 into two parts in the ratio  $2\frac{1}{3} : 3\frac{1}{4}$ .

#### Solution :

$$\text{Given ratio} = 2\frac{1}{3} : 3\frac{1}{4} = \frac{7}{3} : \frac{13}{4} = \frac{7}{3} \times \frac{4}{13} = \frac{28}{39} = 28 : 39$$

$$\text{Since } 28 + 39 = 67$$

$$\therefore \text{1st part} = \frac{28}{67} \times 268 = 28 \times 4 = \mathbf{112} \quad (\text{Ans.})$$

$$\text{And } \text{2nd part} = \frac{39}{67} \times 268 = 39 \times 4 = \mathbf{156} \quad (\text{Ans.})$$

**Example 9 :**

The total weight of the mixture of two things  $A$  and  $B$  is 50 kg. If  $A$  and  $B$  are mixed in the ratio 3 : 7, find the quantity of  $B$  in the mixture.

**Solution :**

Since  $A$  and  $B$  are mixed in the ratio 3 : 7 and  $3 + 7 = 10$ ,

$$\therefore \text{Quantity of B in the mixture} = \frac{7}{10} \times 50 \text{ kg} = 35 \text{ kg} \quad (\text{Ans.})$$

**Example 10 :**

420 articles are divided among  $A$ ,  $B$  and  $C$ , such that  $A$  gets three times that of  $B$  and  $B$  gets five times that of  $C$ . Find the number of articles received by  $B$ .

**Solution :**

Let the number of articles  $C$  gets = 1

$$\Rightarrow \text{The number of articles that B gets} = \text{five times that of C} = 5 \times 1 = 5$$

$$\text{and the number of articles that A gets} = \text{three times that of B} = 3 \times 5 = 15$$

$$\therefore A : B : C = 15 : 5 : 1 \text{ and } 15 + 5 + 1 = 21$$

$$\Rightarrow \text{The no. of articles received by B} = \frac{5}{21} \times 420 = 100 \quad (\text{Ans.})$$

---

**EXERCISE 8(B)**


---

- ₹ 120 is to be divided between Hari and Gopi in the ratio 5 : 3. How much does each get ?
- Divide 72 in the ratio  $2\frac{1}{2} : 1\frac{1}{2}$ .
- Divide 81 into three parts in the ratio 2 : 3 : 4.
- Divide ₹ 10,400 among  $A$ ,  $B$  and  $C$  in the ratio  $1/2 : 1/3 : 1/4$ .
- A profit of ₹ 2,500 is to be shared among three persons in the ratio 6 : 9 : 10. How much does each person get ?
- The angles of a triangle are in the ratio 3 : 7 : 8. Find the greatest and the smallest angles.

The sum of the angles of a triangle is  $180^\circ$ .

- The sides of a triangle are in the ratio 3 : 2 : 4. If the perimeter of the triangle is 27 cm, find the length of each side.
  - An alloy of zinc and copper weighs  $12\frac{1}{2}$  kg. If, in the alloy, the ratio of zinc and copper is 1 : 4, find the weight of copper in it.
  - How will ₹ 31,500 be shared between  $A$ ,  $B$  and  $C$ , if  $A$  gets the double of what  $B$  gets, and  $B$  gets the double of what  $C$  gets ?
  - Mr. Gupta divides ₹ 81,000 among his three children, Ashok, Mohit and Geeta, in such a way that Ashok gets four times what Mohit gets and Mohit gets 2.5 times what Geeta gets. Find the share of each of them.
-

**8.5 PROPORTION**

Consider the following examples :

1. What is the ratio of the number of boys to the number of girls in a group of 8 boys and 12 girls ?

$$\text{The required ratio} = \frac{\text{Number of boys}}{\text{Number of girls}} = \frac{8}{12} = \frac{2 \times 4}{3 \times 4} = \frac{2}{3}$$

2. What is the ratio of ₹ 18 to ₹ 27 ?

$$\text{The required ratio} = \frac{18}{27} = \frac{2 \times 9}{3 \times 9} = \frac{2}{3}$$

It is observed in the examples given above that the ratios  $\frac{8}{12}$  and  $\frac{18}{27}$  are equal.

$$\text{i.e.} \quad \frac{8}{12} = \frac{18}{27} \quad \text{or} \quad 8 : 12 = 18 : 27$$

Such an equality of two ratios is called a **proportion** and read as :

**“8 is to 12 as 18 is to 27”.**

**Thus, a proportion is an expression which states that the two given ratios are equal.**

The numbers 8, 12, 18 and 27 that are used in the proportion are called its **terms**, i.e. **8** is the **first term**, **12** is the **second term**, **18** is the **third term** and **27** is the **fourth term** of the proportion **8 : 12 = 18 : 27**.

In general, the symbol for representing a proportion is “ : : ”.

**Example 11 :**

Check whether or not the two ratios form a proportion.

- (i) ₹ 6 : ₹ 8 and 12 kg : 16 kg                      (ii) 6 kg : 9 kg and 10 m : 16 m

**Solution :**

$$(i) \text{ Since } ₹ 6 : ₹ 8 = \frac{6}{8} = \frac{3}{4} \quad \text{and} \quad 12 \text{ kg} : 16 \text{ kg} = \frac{12}{16} = \frac{3}{4}$$

∴ **Ratios ₹ 6 : ₹ 8 and 12 kg : 16 kg are equal, they form a proportion.**

**(Ans.)**

$$(ii) \text{ Since, } 6 \text{ kg} : 9 \text{ kg} = \frac{6}{9} = \frac{2}{3} \quad \text{and} \quad 10 \text{ m} : 16 \text{ m} = \frac{10}{16} = \frac{5}{8}$$

∴ **Ratio 6 kg : 9 kg ≠ ratio 10 m : 16 m, they do not form a proportion.**

**(Ans.)**

1. In a proportion, the **first two terms** (quantities) must be of the same kind and of the same unit, whereas the **last two terms** (quantities) must also be of the same kind and of the same unit.

**[All the four quantities in a proportion may be of the same kind and the same unit.]**

2. In a proportion, the **first and the fourth terms** are called **extremes** whereas the **second and the third terms** are called **means**.

Thus, in  $8 : 12 = 18 : 27$ ; the terms **12** and **18** are the **means** and **8** and **27** are the **extremes**.

Also **Product of extremes = Product of means**

**Example 12 :**

Check whether or not the given ratios form a proportion :

(i)  $15 : 24$  and  $35 : 56$       (ii)  $2\frac{1}{4} : 5\frac{2}{5}$  and  $3\frac{1}{3} : 4\frac{1}{6}$ .

**Solution :**

(i) Product of extremes =  $15 \times 56 = 840$

and product of means =  $24 \times 35 = 840$

Since, product of extremes = product of means

$\Rightarrow$  **The given two ratios form a proportion**

**(Ans.)**

(ii)  $\therefore$  Product of extremes =  $2\frac{1}{4} \times 4\frac{1}{6} = \frac{9}{4} \times \frac{25}{6} = \frac{75}{8}$

and product of means =  $5\frac{2}{5} \times 3\frac{1}{3} = \frac{27}{5} \times \frac{10}{3} = \frac{18}{1}$

and since product of extremes  $\neq$  product of means

$\Rightarrow$  **The given two ratios do not form a proportion**

**(Ans.)****Example 13 :**

(i) The numbers 8, x, 9 and 36 are in proportion. Find x.

(ii) If  $x : 15 = 8 : 12$ , find x.

**Solution :**

(i) The numbers 8, x, 9 and 36 are in proportion

$$\Rightarrow 8 : x = 9 : 36$$

$$\Rightarrow x \times 9 = 8 \times 36$$

Product of means = Product of extremes

$$\Rightarrow x = \frac{8 \times 36}{9} = 32$$

**(Ans.)**

(ii)  $x : 15 = 8 : 12 \Rightarrow x \times 12 = 15 \times 8$

$$\Rightarrow x = \frac{15 \times 8}{12} = 10$$

**(Ans.)****Example 14 :**

The first, third and fourth terms of a proportion are 12, 8 and 14, respectively. Find the second term.

**Solution :**

Let the second term be x.

$$\therefore 12, x, 8 \text{ and } 14 \text{ are in proportion, i.e. } 12 : x = 8 : 14$$

$$\Rightarrow x \times 8 = 12 \times 14$$

$$\Rightarrow x = \frac{12 \times 14}{8} = 21$$

$\therefore$  **The second term of the proportion is 21.**

**(Ans.)**



**Example 15 :**

The ratio of the length and the width of a sheet of paper is 3 : 2. If the length is 12 cm, find the width.

**Solution :**

Let width =  $x$  cm

The ratio of length to width =  $12 : x$  [Given length = 12 cm]

According to the given statement,  $12 : x = 3 : 2$

$$\Rightarrow x \times 3 = 12 \times 2$$

$$\Rightarrow x = \frac{12 \times 2}{3} = 8$$

$\therefore$  Width = 8 cm (Ans.)

**EXERCISE 8(C)**

- In each of the following, check whether or not the given ratios form a proportion :
  - 8 : 16 and 12 : 15
  - 16 : 28 and 24 : 42
  - $12 \div 3$  and  $8 \div 2$
  - 25 : 40 and 20 : 32
  - $\frac{15}{18}$  and  $\frac{10}{12}$
  - $\frac{7}{8}$  and 14 : 16
- Find the value of  $x$  in each of the following proportions :
  - $x : 4 = 6 : 8$
  - $14 : x = 7 : 9$
  - $4 : 6 = x : 18$
  - $8 : 10 = x : 25$
  - $5 : 15 = 4 : x$
  - $16 : 24 = 6 : x$
- Find the value of  $x$  so that the given four numbers are in proportion :
  - $x, 6, 10$  and 15
  - $x, 4, 15$  and 30
  - 2,  $x, 10$  and 25
  - 4,  $x, 6$  and 18
  - 9, 12,  $x$  and 8
  - 4, 10, 36 and  $x$ .
- The first, second and the fourth terms of a proportion are 6, 18 and 75, respectively. Find its third term.
- Find the second term of the proportion whose first, third and fourth terms are 9, 8 and 24, respectively.
- Find the fourth term of the proportion whose first, second and third terms are 18, 27 and 32, respectively.
- The ratio of the length and the width of a school ground is 5 : 2. Find the length, if the width is 40 metres.
- The ratio of the sale of eggs on a Sunday and that of the whole week at a grocery shop was 2 : 9. If the total value of the sale of eggs in the same week was ₹ 360, find the value of the sale of eggs that Sunday.
- The ratio of copper and zinc in an alloy is 9 : 8. If the weight of zinc in the alloy is 9.6 kg, find the weight of copper in the alloy.
- The ratio of the number of girls to the number of boys in a school is 2 : 5. If the number of boys is 225, find :
  - the number of girls in the school.
  - the number of students in the school.

## Revision Exercise (Chapter 8)

1. A school has 1625 students, out of which 750 are girls. Find the ratio of the number of boys to the number of girls in the school.
2. Rohit earns ₹ 1,040 in 8 days and John earns ₹ 960 in 6 days. Find :
  - (i) one day's earnings of Rohit
  - (ii) one day's earnings of John.
  - (iii) the ratio of one day's earnings of Rohit and John.
3. A sample of an alloy consists of  $8\frac{3}{4}$  gm copper and  $1\frac{1}{2}$  gm silver. Find the ratio of the weights of :
  - (i) alloy to copper
  - (ii) copper to silver
  - (iii) silver to alloy.
4. Divide ₹ 1,450 between Richa and Sonu in the ratio 12 : 17.
5. Are the following numbers in proportion ?
  - (i) 32, 40, 48 and 60
  - (ii) 12, 15, 18 and 20
6. Find the value of  $x$  in each of the following such that the given numbers are in proportion.
  - (i) 14, 42,  $x$  and 75
  - (ii) 45, 135, 90 and  $x$
7. The costs of two articles are in the ratio 7 : 4. If the cost of the first article is ₹ 2,800, find the cost of the second article.
 

If the cost of the second article is ₹  $x$ ,  $\Rightarrow 7 : 4 = 2800 : x$
8. The ratio of the length and the width of a rectangular sheet of paper is 8 : 5. If the width of the sheet is 17.5 cm, find the length.
 

Let length =  $x$  cm  $\Rightarrow 8 : 5 = x : 17.5$
9. The ages of A and B are in the ratio 6 : 5. If A's age is 18 years, find the age of B.
10. A sum of ₹ 10,500 is divided among A, B and C in the ratio 5 : 6 : 4. Find the share of each.