UNIT – 1
NUMBERS

CHAPTER 1

NUMBER SYSTEM

1.1 INTRODUCING UNIT, NUMBER, NUMERAL AND NUMERATION

1. **In mathematics unit** (unity) **means a single thing**.
   For example, a pen, a boy, a metre, a day, etc.
2. **The number written before the name of a unit indicates** how many times that unit is taken.

For example :

(i) **Four pens** means a pen (unit/thing) is taken **4 times,** *i.e.* 4 pens are taken.

(ii) **Length = 3 m** means unit of length is **m** (metre), and it is taken **3 times**.

(iii) **Weight = 63 kg** means **unit** of weight used is **kg** (kilogramme), and it is taken 63 times.

1.2 NUMERAL AND NUMERATION

A numeral is a **symbol representing** a given **number** and **numeration represents** that **number in words**.

Number	Numeral	Numeration
3	3	three
15	15	fifteen
72	72	seventy-two
0	0	zero

1.3 HINDU-ARABIC SYSTEM OF NUMERATION

The Hindu-Arabic system of Numeration is in fact the *decimal system* that is in use all over the world. This system was developed by the ancient Hindu-Mathematicians in India and was carried to the West by the Arabs. For this reason, it is called **the Hindu-Arabic system of numeration**.

In Hindu-Arabic number system (also known as denary system); the ten **symbols** 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 are used to write a numeral (number). Each of these ten symbols is called a digit. Out of these digits :

(i) 0, 2, 4, 6 and 8 are even numerals.

(ii) 1, 3, 5, 7 and 9 are odd numerals.

1.4 PLACE VALUE (Local value) AND FACE VALUE (True value)

In a number, the **place value** of any digit, is the value of this digit according to its position in the number, whereas the **face value** of every digit used in the number is the digit itself.

Thus :

1. The **place value** of a digit depends upon the position it occupies in the number.
2. The face value of a digit is the digit itself.
3. Zero (0) is the only digit whose face value and place value are the same, irrespective of its position in the number. As a result, the face value of 0 is 0 and even the place value of 0 is 0.

For digit	Place value (*Local value*)	Face value (*True value*)
3 in 2305	300	3
0 in 907	0	0
7 in 472	70	7
5 in 1450	50	5
2 in 2000	2000	2
8 in 18605	8000	8

***Example* 1 :**

Write the place values of the two 6s (sixes) used in the number 36268 and find the sum of these two values.

***Solution* :**

In 36268, **one 6** occurs at **thousand's place**, so its **place value** = **6000** **(Ans.)**

The **other 6** occurs at **ten's place**, so its **place value** = **60** **(Ans.)**

The **sum of** these **two** **place values of 6** = 6000 + 60 = **6060** **(Ans.)**

***Example* 2 :**

Write the place values of the two 5s in 9,45,582 and find the difference of these place values.

***Solution* :**

In 9,45,582, **the first 5** occurs at thousand's place

$\Rightarrow$ **Its place value** = 5 thousand = **5000** **(Ans.)**

The **second 5** occurs at hundred's place

$\Rightarrow$ **Its place value** = 5 hundred = **500** **(Ans.)**

The **difference of the two place values of 5** = 5000 – 500 = **4500** **(Ans.)**

1. *A* **concrete number** *is a number which refers to a particular unit; such as : 8 metre, 12 kg, 18 km, 36 cm, etc.*
2. *An* **abstract number** *is a number which does not refer to any particular unit; such as : 8, 12, 18, 36, etc.*

EXERCISE 1(A)

1. Fill in the blanks :
   (i) In 20 kg, the unit is ....., which is taken ..... times.
   (ii) In 80 m, the unit is ....., which is taken ..... times.

(iii) If a unit cm (centimetre) is taken 5 times, the corresponding quantity is .....

(iv) If a unit km (kilometre) is taken 24 times, the corresponding quantity is .....

(v)

Number	Numeral	Numeration
.....53.....	.....	.....
.....	.....9.....	.....
....240....	.....	.....

2. Fill in the blanks :
   (i) **In 24,673,** the place value of 6 is ..... and the face value of 4 is .....
   (ii) **In 8,039,** the face value of 8 is ..... and the place value of 9 is .....
   (iii) **In 3,25,648,** the local value of 5 is ..... and the true value of 3 is .....
   (iv) **In 6,439,** the true value of 6 is ..... and the face value of 6 is .....

3. Find the difference between the place value and the face value of 9 in the number 3945.

4. In the number 40562,
   (i) the local value of 5 = ...
   (ii) the true value of 6 = ...
   (iii) the sum of the local value of 5 and the true value of 6 = ...

5. In the number 347825, write the difference between
   (i) the place value and the face value of 2.
   (ii) the place value and the face value of 5.

6. The number 978036 has six digits. Write the digit/digits used in this number that have the same place value and face value.

1.5 FORMING NUMBERS USING GIVEN DIGITS.

(a) Use digits 4 and 7 to form all possible two-digit numbers such that no digit is repeated.

Required numbers are **47** and **74**.

No digit is repeated.
Neither 44 nor 77 is allowed.

(b) Use digits 2, 5 and 8 to form all possible three-digit numbers such that no digit is repeated.

Required numbers are **258, 285, 528, 582, 825** and **852**.

Numbers of the type
225, 282, 222, 525, etc.
are not allowed

1.6 TO FORM THE SMALLEST AND THE GREATEST NUMBERS USING GIVEN DIGITS

(a) When the given digits include digit 0.

***Example* 3 :**

Form the smallest and the greatest 6-digit number using the digits 2, 0, 7, 8, 9 and 5 without repetition.

***Solution* :**

To obtain the smallest number :

The smallest digit, other than zero, is put at the extreme left, then comes zero, and then the remaining digits in ascending (increasing) order of value.

Since, out of the given digits 2, 0, 7, 8, 9 and 5, the smallest digit other than 0 is 2, write 2 at the extreme left, then write 0 and then the remaining digits (7, 8, 9 and 5) in ascending order of value, *i.e.*, 5, 7, 8 and 9.

Thus, the **required smallest number** is **205789.** **(Ans.)**

The number 025789 is not a 6-digit number; it is a 5-digit number.
A number cannot begin with the digit 0.

To obtain the greatest number :

Put the greatest digit at the extreme left, then put the remaining digits in descending order of their values, with 0 at the end.

Thus, the **required greatest number** is **987520**. **(Ans.)**

(b) When the given digits do not include digit 0 :

***Example* 4 :**

Form the smallest and the greatest 4-digit number using the digits 3, 8, 5 and 2 without repetition.

***Solution* :**

To obtain the smallest number :

Write the digit with smallest value at the extreme left, and then the remaining digits in ascending order of their values.

∴ **The required smallest number** = **2358** **(Ans.)**

To obtain the greatest number :

Write the digit with largest value at the extreme left, and then the remaining digits in descending order of value.

∴ **The required greatest number** = **8532** **(Ans.)**

***Example* 5 :**

(i) What is the smallest number of five digits ?

(ii) What is the greatest number of five digits ?

***Solution* :**

(i) To form the smallest number of five digits, place 1 (unity) at the extreme left, and then four zeros to the right of 1.

∴ The required **smallest number of five digits** = **10000** **(Ans.)**

(ii) In forming the greatest number of five digits, we should have the greatest digit, *i.e.* 9, in all places.

∴ The required **greatest number of five digits** = **99999** **(Ans.)**

1.7 SIMPLE DIVISIBILITY TESTS

(i) A number is divisible by 2 if its last (unit) digit is even or 0.

For example :

4, 16, 34, 80, 258, 3572, etc.

(ii) *A number is divisible by 3 if the sum of its digits is divisible by 3.*

For example :

Consider the numbers 36, 406 and 2367

Since the sum of the digits of the number 36 = 3 + 6 = 9, *which is divisible by 3,*

∴ **36 is divisible by 3**

Since the sum of the digits of the number 406 = 4 + 0 + 6 = 10, *which is not divisible by 3,*

∴ **406 is not divisible by 3**

Since the sum of the digits of the number 2367 = 2 + 3 + 6 + 7 = 18, *which is divisible by 3,*

∴ **2367 is divisible by 3**

(iii) *A number is divisible by 5 if its last* (unit) *digit is either 5 or 0.*

For example :

25, 85, 410, 2205, 550, etc.

(iv) *A number is divisible by 10 if its* unit *digit is zero.*

For example :

30, 70, 510, 900, 5000, etc.

***Example* 6 :**

How many three-digit numbers are there between 99 and 400 ?

***Solution* :**

Required three-digit numbers between 99 and 400

= Numbers from 100 to 399

= 100 + (101 to 200) + (201 to 300) + (301 to 399)

Thus, **the number of three-digit numbers between 99 and 400**

= 1 + 100 + 100 + 99 = **300** **(Ans.)**

EXERCISE 1 (B)

1. Form the greatest and the smallest 4-digit numbers using the given digits, without repetition

(i) 3, 7, 2 and 5 (ii) 6, 1, 4 and 9 (iii) 7, 0, 4 and 2

(iv) 1, 8, 5 and 3 (v) 9, 6, 0 and 7

2. Form the greatest and the smallest 3-digit numbers using any three different digits with the condition that digit 6 is always at the unit (one's) place.

The required **greatest number** is **986** and the required **smallest number** is **106**.

3. Form the greatest and the smallest 4-digit numbers using any four different digits, with the condition that digit 5 is always at ten's place.

4. Fill in the blanks :

   (i) The largest number of 5 digits is ..... and the smallest number of 6-digits is .....

   (ii) The difference between the smallest number of four digits and the largest number of three digits = ... − ..... = ...

   (iii) The sum (addition) of the smallest number of three digits and the largest number of two digits = ... + ... = ...

   (iv) If a number has an even number or zero at its unit's place, the number is always divisible by .....

   (v) A number is divisible by 3 if the sum of its digits is divisible by .....

   (vi) A number will be divisible by 5 if its unit's digit is .....

   (vii) On adding one to the largest five-digit number, we get ....., which is the smallest ..... digit number.

   (viii) On subtracting one from the smallest four-digit number, we get ....., which is the ..... three digit number.

5. Form the largest number with the digits 2, 3, 5, 9, 6 and 0 without repetition of digits.

6. Write the smallest and the greatest number of 4 digits without repetition of any digit.

7. Find the greatest and the smallest five-digit number with 8 in hundred's place and with all the digits different.

8. Find the sum of the largest and the smallest four-digit numbers.

9. Find the difference between the smallest and the greatest six-digit numbers.

10. Without making any actual division, find which of the following numbers are :
    9012, 4005, 2010, 504, 111111, 11112, 11121 and 99912

    (i) divisible by 2 (ii) divisible by 3

    (iii) divisible by 5 (iv) divisible by 10

11. (i) How many four-digit numbers are there between 999 and 3000 ?

    (ii) How many four-digit numbers are there between 99 and 3000 ?

12. How many four-digit numbers are there between 500 and 3000 ?

13. Write all the possible three-digit numbers using the digits 3, 6 and 8 if repetition of digits is allowed.

14. Write all the possible three-digit numbers using the digits 3, 6 and 8 if repetition of digits is not allowed.

15. From among the numbers 10, 12, 15, 21, 24, 26, 39, 48, 64 and 72, which are

    (i) divisible by 2. (ii) divisible by 3. (iii) divisible by 6.

Numbers divisible by both 2 and 3 are divisible by 6 also.

Since 12 is divisible by 2 as well as 3 therefore 12 is divisible by 6 too.

1.8 MORE ABOUT NUMBERS

1. Natural Numbers (N) :

N = { 1, 2, 3, ... }

We use 1, 2, 3, 4 ..... for counting purposes.

All such counting numbers are called **natural numbers.**

In natural numbers :

(i) The first and the smallest number is 1.

(ii) Any natural number can be obtained by adding 1 to its previous number.

(iii) There are infinite numbers, *i.e.* we cannot find the last (greatest) natural number.

2. Whole Numbers (W) :

The natural numbers (N) together with 0 (zero) form the set of **whole numbers.**

In this system :

W = { 0, 1, 2, 3, ... }

(i) The smallest number is zero.

(ii) Each number is 1 more than its previous number.

(iii) There are infinite numbers, *i.e.*, the largest whole number cannot be obtained.

3. Integers (Z or I) :

Group of numbers called integers contains the negatives of the natural numbers, zero and the natural numbers.

Z = { ....., –4, –3, –2, –1, 0, 1, 2, 3, 4, ...}

Zero is neither a negative number nor a positive number. It is a ***neutral*** *number.*

Integers have an infinite number of numbers towards the negative side as well as the positive side, *i.e.* the smallest and the largest integers cannot be obtained.

In this system, any number can be obtained by adding 1 to its previous number.

Also remember :

The difference between any two consecutive natural numbers, between any two consecutive whole numbers, and between any two consecutive integers, is always 1 (one).

4. Real Numbers (R) :

This system contains all the numbers : *negative, positive, zero* and **also** the *fractions.*

Integers contain the negatives of natural numbers, zero and the natural numbers; real numbers contain the fractions also.

For example :

(i) –10, –2, 3, 7, 0, etc. are **integers** and **also real numbers**.

(ii) –3.4, –2.5, $\frac{1}{2}$, 4.7, $5\frac{2}{3}$, 15.2, etc. are **real numbers** but **not integers**.

5. Even Natural Numbers (E) :

Group of all natural numbers that are **divisible by 2** is called the set of **even natural numbers.** ***Even natural numbers are also called multiples of 2.***

Thus, **E = { 2, 4, 6, 8, 10, 12, ... }**

1. There are infinite even natural numbers.
2. The difference between any two consecutive even natural numbers is always 2.

6. Odd Natural Numbers (O) :

The natural numbers that are **not divisible by 2** are called **odd natural numbers.**

Thus, **O = { 1, 3, 5, 7, 9, 11, ... }**

1. There are infinite odd natural numbers.
2. The difference between any two consecutive odd natural numbers is always 2.

7. Prime Natural Numbers (P) :

A natural number, that is **greater than 1 and divisible only by itself and by 1,** is called a **prime number.**

Example :

(i) 3 is a prime number as it is only divisible by 3 (itself) and 1.

(ii) 6 is not a prime number as it is divisible not only by itself (6) and 1, but also by 2 and 3.

(iii) 20 is not a prime number as it is divisible by 1, 2, 4, 5, 10 and 20.

Infact, every prime natural number is divisible by only two natural numbers. One of them is the number itself and the other is 1 (one).

Thus, **P = { 2, 3, 5, 7, 11, 13, 17, 19, ... }**

1. 2 is the smallest *prime number.*
2. There are infinite prime natural numbers.
3. Two (2) is the only prime number that is also an even number.

Revision Exercise (Chapter 1)

1. Fill in the blanks

(i) 5 kg contains unit ....., ... times.

(ii) 20 m contains unit ....., 20 .....

(iii) The number 15 in ..... is expressed in words as .....

(iv) In 347, the place value of 4 is ....., and its face value is .....

(v) In 8309, the place value of 8 is ....., and its local value is .....

(vi) In 6728, the face value of 7 is ....., and its true value is .....

(vii) In 429, the sum of the place value of 2 and its face value is .....

(viii) In 6724, the sum of the place value of 6 and the place value of 4 is .....

(ix) The 3-digit largest number is ....., and the 4-digit smallest number is .....

(x) The largest number of 4-digits without repetition of digits is ....., and the smallest 4-digit number (without repetition) is .....

2. Use the digits 3, 0, 5, 7, 8 and 4 to form the largest 4-digit number and the smallest 4-digit number without repetition of digits.

3. Consider the numbers : 14, 51, 105, 3290, 56298 and 2657

   (i) Which of these numbers is/are divisible by 2 ?

   (ii) Which of these numbers is/are divisible by 3 ?

   (iii) Which of these numbers is/are divisible by 10 ?

   (iv) Which of the given numbers is/are divisible by 6 ?

4. How many 3-digit numbers are there between 99 and 3000 ?

5. Form the greatest and the smallest 4-digit numbers using any one digit twice.

   (i) 4, 9 and 8 (ii) 8, 0 and 4

   (iii) 0, 3 and 9 (iv) 3, 1 and 7

(i) **Using digits 4, 9 and 8 :**
The required greatest 4-digit number = 9984 and the required smallest 4-digit number = 4489.

(ii) **Using digits 8, 0 and 4 :**
The required greatest 4-digit number = 8840 and the required smallest 4-digit number = 4008

6. Using each of the following digits (without repetition) form all possible numbers greater than 6000.

   (i) 3, 5 4 and 7 (ii) 4, 0, 8 and 2

   (iii) 3, 4, 6 and 9.