Chapter 30

Probability

POINTS TO REMEMBER :

1. Probability : It is the concept which numerically measures the degree of certainty of the occurence of an event.

2. Experiment : An operation which can be produce some well define outcomes, is called an experiment.

3. Event : Each outcome of an experiment is called an event.

4. Trial : By a trial, we mean performing an experiment.

5. Empirical Probability : Suppose we perform an experiment and let n be the total number of trials. The empirical probability of the happening of an event E is defined as :

$$\therefore \ P(E) = \frac{\text{Number of trials in which the event happened}}{\text{Total number of trials}}$$

6. Classical Probability :

(*a*) Sample Space : In a random experiment, the set of all possible outcomes is called a sample space and it is denoted by S.

(*b*) Event : The collection of some or all outcomes in an experiment is called an event. If F is an event, then clearly $E \subseteq S$.

Probability of occurence of an Event (Classical Definition) : In an random experiment, let S be the sample space and E be the event, then $E \leq S$.

The probability of occurence of E, is defined as :

$$\therefore \ P(E) = \frac{\text{Number of distinct elements in E}}{\text{Number of distinct elements in S}} = \frac{n(E)}{n(S)}$$

Tossing a coin, throwing a die, drawing cards from a well-shuffled pack of 52 cards etc. are the examples of classical probability.

(*a*) Sure Event : The probability of a sure event is 1

(*b*) Impossible Event : The probability of an impossible event is 0.

(*c*) Complementary Event : Let E be an event and (not E) be an event which occurs only when E does not occur. The event (not E) is called the complementary event of E

$\therefore$ P(E) + P(not E) = 1

$\Rightarrow$ P(E) = 1 – P (not E).

Note : (*i*) P(not E) can also be written as P(E')

(*ii*) For any event E, we have $0 \leq P(E) \leq 1$

EXERCISE 30 (A)

Q. 1. A coin is tossed 200 times and it was found that head appears 72 times and tail appears 128 times.

If a coin is tossed at random what is the probability of getting (*i*) a head (*ii*) a tail ?

Sol. A coin is tossed 200 times.

Number of heads appear = 72

Number of tails appears = 128

(*i*) $P(E_1) = \dfrac{\text{Number of heads}}{\text{Total Number of trials}} = \dfrac{72}{200} = \dfrac{9}{25}$

(*ii*) $P(E_2) = \dfrac{\text{Number of tails}}{\text{Total number of trials}} = \dfrac{128}{200} = \dfrac{16}{25}$ Ans.

Q. 2. Two coins are tossed simultaneously 125 times and it was observed that both heads appeared 15 times. If two coins are tossed simultaneously at random. What is the probability of getting both heads?

Sol. Total number of times two coins tossed simultaneously = 125

Number of both heads appeared = 15

$\therefore$ $P(E) = \dfrac{\text{Number of heads}}{\text{Total number of trials}} = \dfrac{15}{125} = \dfrac{3}{25}$ Ans.

Q. 3. A die is thrown 260 times. Prime numbers appear on the upper face 39 times. If a die is thrown at random, what is the probability of getting a prime number ?

Sol. A die is thrown 260 times

Prime number on the faces of a die are 2, 3, 5

$\therefore$ Number of prime numbers which appears on the face = 39 times

$\therefore$ $P(E) = \dfrac{39}{260} = \dfrac{3}{20}$ Ans.

Q. 4. A survey of 650 men, showed that only 52 of them know English. Out of these men, if one is selected at random, what is the probability that the selected man knows English ?

Sol. Total number of men = 650

Number of men who know English = 52

$\therefore$ Probability will be

$P(E) = \dfrac{52}{650} = \dfrac{2}{25}$ Ans.

Q. 5. On the particular day, at a crossing in a city, the various types of 280 vehicles going past during a time-interval, were observed as under.

Type of vehicle	Two-wheelers	Three-wheelers	Four-wheelers
Frequency	91	63	126

Out of these vehicles, one is chosen at random. What is the probability that the chosen vehicle is
(*i*) a four-wheeler, (*ii*) a two-wheeler, (*iii*) a three-wheeler ?

Sol. Total number of wheelers = 91 + 63 + 126 = 280

(*i*) Now probability of four wheeler

$$P(E_1) = \frac{\text{Number of four - wheelers}}{\text{Total number of wheelers}} = \frac{126}{280} = \frac{9}{20}$$

(*ii*) Probability of two wheeler

$$P(E_2) = \frac{\text{Number of two wheelers}}{\text{Total number of wheelers}} = \frac{91}{280} \text{ Ans.}$$

(*iii*) Probability of three wheelers

$$P(E_3) = \frac{\text{Number of three wheelers}}{\text{Total number of wheelers}} = \frac{63}{280} = \frac{9}{40} \text{ Ans.}$$

Q. 6. The following table shows the blood-groups of 60 students of a class :

Blood group	A	B	O	AB
Number of students	16	12	23	9

One student of the class is chosen at random. What is the probability that the chosen student has blood groups.
(*i*) O (*ii*) AB (*iii*) A (*iv*) B ?

Sol. Number of total students in a class = 16 + 12 + 23 + 9 = 60

(*i*) Probability of students having blood group O

$$\therefore \quad P(E_1) = \frac{\text{Number of students having blood group O}}{\text{Total number of students}} = \frac{23}{60} \textbf{ Ans.}$$

(*ii*) Probability of students having blood group AB

$$P(E_2) = \frac{\text{Number of students having blood group AB}}{\text{Total number of students}} = \frac{9}{60} = \frac{3}{20} \textbf{ Ans.}$$

(*iii*) Probability of students having blood group A

$$P(E_3) = \frac{\text{Number of students having blood group A}}{\text{Total number of students}} = \frac{16}{60} = \frac{4}{15} \textbf{ Ans.}$$

(*iv*) Probability of students having blood group B

$$P(E_4) = \frac{\text{Number of students having blood group B}}{\text{Total number of students}} = \frac{12}{60} = \frac{1}{5} \text{ Ans.}$$

Exercise 30 (B)

Q. 1. An unbiased coin is tossed once.

(*i*) Describe the sample space S

(*ii*) Find the probability of getting a head

Sol. In a single toss of a coin, we get either Head (H) or tail (T)

(*i*) ∴ Sample space S = (H.T)

$\Rightarrow n(S) = 2$

(*ii*) Let E he the event of getting a head, then

E = (+1) $\Rightarrow n(E) = 1.$

$$P(E) = \frac{n(E)}{n(S)} = \frac{1}{2} \text{ Ans.}$$

Q. 2. Two unbiased coins are tossed simultaneously.

(*i*) Describe the sample space S.

(*ii*) Find the probability of getting

(*a*) two heads (*b*) at least one head

(*c*) at most one head

(*d*) exactly one head (*e*) no head

Sol. (*i*) In tossing two coins simultaneously. We get two heads (H.H) or one head and one tail (H.T), one tail and one head (T.H) or two tails (T.T).

∴ Sample space (S) = 4

(*ii*) (*a*) Probability of two heads = $P(E_1) = \frac{1}{4}$ (HH)

(*b*) Probability of at least one head $P(E_2) = \frac{3}{4}$ (HT. TH. HH)

(*c*) Probability of at the most one head = $P(E_3)$ $= \frac{3}{4}$ (HT. TH. TT)

(*d*) Probability of exactly one head = ?

$P(E_4) = \frac{2}{4} = \frac{1}{2}$ (HT.TH)

(*e*) Probability of no head = $P(E_5)$

$= \frac{1}{4}$ (TT) **Ans.**

Q. 3. Three unbiased coins are tossed simultaneously :

(*i*) Describe the sample space S.

(*ii*) Find the probability of getting :

(*a*) at most 2 heads

(*b*) at least 2 heads (*c*) exactly 2 heads

Sol. (*i*) Three unbiased coins are tossed simultaneously.

∴ We get (HHH), (HH.T), (HT.H), (HTT), (TTT), (THT), (TTH), (THH).

(*i*) ∴ Sample space S = 8

(*ii*) (*a*) Probability of getting at most two heads will be (HHT), (HTH), (HTT), (TTT), (THT), (TTH) and (THH) = 7

$$\therefore P(E_1) = \frac{7}{8}$$

(*b*) Probability of at least two heads will be (HHH), (HHT), (TH,H), (HTH) = 4

$$\therefore P(E_2) = \frac{4}{8} = \frac{1}{2}$$

(*c*) Probability of exactly two heads will be (HHT), (THH), (HTH) = 3

$$\therefore P(E_3) = \frac{3}{8} \text{ **Ans.**}$$

Q. 4. A dice is thrown once. What is the probability that the

(*i*) number is even

(*ii*) number is greater than 2 ? **(2009)**

Sol.

Dice is thrown once

Sample space = {1, 2, 3, 4, 5, 6}

(*i*) No. of ways in favour = 3

Total ways = 6

$$\therefore \text{Probability} = \frac{3}{6} = \frac{1}{2}$$

(*ii*) No. of ways in favour = 4

Total ways = 6

$\therefore$ Probability $= \frac{4}{6} = \frac{2}{3}$

Q. 5. A die is thrown once. Find the probability of getting :

(*i*) an odd number

(*ii*) a number less than 5.

(*iii*) a number greater than 4

(*iv*) a number less than 4

(*v*) a number other than 4

(*vi*) the number 6.

Sol. A die is thrown once only

$\because$ A die has 6 faces numbered as 1, 2, 3, 4, 5, 6

$n(S) = 6$

(*i*) Probability of an odd number which are 1, 3, 5 = $P(E_1) = \frac{3}{6} = \frac{1}{2}$

(*ii*) Probability of a number less than 5 which are 1, 2, 3, 4 = $P(E_2) = \frac{4}{6} = \frac{2}{3}$

(*iii*) Probability of a number greater than 4 which are 5, 6 = $P(E_3) = \frac{2}{6} = \frac{1}{3}$

(*iv*) Probability of a number less than 4 which are 1, 2, 3 = $P(E_4) = \frac{3}{6} = \frac{1}{2}$

(*v*) Probability of a number other than 4 which are 1, 2, 3, 5, 6 = $P(E_5) = \frac{5}{6}$

(*vi*) Probability of a number 6 = 1 = $P(E_6) = \frac{1}{6}$ Ans.

Q. 6. Two dice are thrown simultaneously. Find the probability of getting :

(*i*) 11 as the sum of the two numbers that turn up

(*ii*) a doublet of even numbers

(*iii*) a multiple of 3 as the sum of the two numbers that turn up

(*iv*) a total of at least 10.

Sol. On throwing two dice, the number of sample space S =

$n(S)$ will be (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) = 6 × 6 = 36

(*i*) 11 as sum will be (5, 6), (6, 5) = 2

$\therefore$ Probability $P(E_1) = \frac{2}{36} = \frac{1}{18}$

(*ii*) A doublet of even numbers are (2, 2), (4, 4), (6, 6) = 3

$\therefore$ Probability $P(E_2) = \frac{3}{36} = \frac{1}{12}$

(*iii*) A multiple of 3 as the sum of two numbers will be (1, 2), 1, 5), (2, 4), (3, 3), (3, 6), (4, 2), (4, 5), (5, 1), (5, 4), (6, 3) and (6, 6) = 11

$\therefore$ Probability $P(E_3) = \frac{11}{36}$

(*iv*) A total of at least 10 will be (4, 6), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6) = 6

$\therefore$ Probability $P(E_4) = \frac{6}{36} = \frac{1}{6}$ **Ans.**

Q. 7. A bag contains 8 red, 4 white and 3 black balls, one ball is drawn at random. What is the probability that the ball drawn is :

(*i*) white ? (*ii*) black ? (*iii*) black or white ? (*iv*) not black ? (*v*) neither red nor white ?

Sol. Number of total balls 4(S) = 8 red + 4 white + 3 black = 15 balls

One ball is drawn at random

(*i*) Probability of white ball $P(E_1) = \frac{4}{15}$

(*ii*) Probability of black ball $(PE_2) = \frac{3}{15} = \frac{1}{5}$

(*iii*) Probability of black or white ball $(PE_3) =$

$\frac{3+4}{15} = \frac{7}{15}$

iv) Probability of not black ball $P(E_4) =$

$\frac{8+4}{15} = \frac{12}{15} = \frac{4}{5}$

) Probability of neither red nor white ball =

$P(E_5) = \frac{3}{15} = \frac{1}{5}$ Ans.

Q. 8. A bag contains 6 black, 5 white and 9 green balls. One ball is drawn at random. What is the probability that the ball drawn is :

(*i*) black ? (*ii*) not green ?

(*iii*) either white or green ? (*iv*) neither white nor green ?

(*v*) white ?

Sol. Number of total balls $n(S)$ = 6 black + 5 white + 9 green = 20 balls

(*i*) Probability of a black ball $= P(E_1) = \frac{6}{20} = \frac{3}{10}$

(*ii*) Probability of not green $= P(E_2)$

$= \frac{6+5}{20} = \frac{11}{20}$

(*iii*) Probability of either white or green

$= P(E_3) = \frac{5+9}{20} = \frac{14}{20} = \frac{7}{10}$

(*iv*) Probability of neither white nor green

$= P(E_4) = \frac{6}{20} = \frac{3}{10}$

(*v*) Probability of white ball $= P(E_5) = \frac{5}{20} = \frac{1}{4}$

Ans.

Q. 9. It is given that a box of 150 electric bulbs contains 12 defective bulbs. One bulb is taken out at random from the box. What is the probability that the bulb drawn is

(*i*) defective (*ii*) not-defective

Sol. Total number of bulbs in a box = $n(S)$ = 150

Number of defective bulbs = 12

∴ Number of not defective bulbs = 150 – 12 = 138

(*i*) Probability of defective bulb $= P(E_1)$

$= \frac{12}{150} = \frac{2}{25}$

(*ii*) Probability of not-defective balls = P(not defective) $= P(E') = \frac{138}{150} = \frac{23}{25}$ Ans.

Q. 10. A box contains 16 cards bearing numbers 1, 2, 3, 4, ..., 15, 16 respectively. A card is drawn at random from the box. What is the probability that the number on the card is :

(*i*) an odd number ?

(*ii*) a prime number ?

(*iii*) a number divisible by 3 ?

(*iv*) a number not divisible by 4 ?

Sol. Total number of cards = 16

bearing number 1, 2, 3, 4, ... 15, 16

(*i*) Odd numbers are 1, 3, 5, 7, 9, 11, 13, 15 = 8

∴ Probability of odd numbers $= P(E_1)$

$= \frac{8}{16} = \frac{1}{2}$

(*ii*) Prime numbers are 2, 3, 5, 7, 11, 13 = 6

∴ Probability of prime numbers $P(E_2)$

$= \frac{6}{16} = \frac{3}{8}$

(*iii*) Numbers divisible by 3 are 3, 6, 9, 12, 15 = 5

∴ Probability of number divisible by 3 $= P(E_3)$

$= \frac{5}{16}$

(iv) Numbers not divisible by 4 are 1, 2, 3, 5, 6, 7, 9, 10, 11, 13, 14, 15 = 12

∴ Probability of number not divisible by 4 =

$P(E_4) = \frac{12}{16} = \frac{3}{4}$ Ans.

Q. 11. A box contains 15 balls bearing numbers 1, 2, 3, ..., 14, 15 respectively. A ball is drawn at random from the box. Find the probability that the number on the ball is:

(i) an even number

(ii) a number divisible by 5

(iii) tne number '6'

(iv) a number lying between 8 and 12

(v) a number greater than 9

(vi) a number less than 6

Sol. Total number of balls = 15

Bearing numbers 1, 2, 3, ..., 14, 15

(i) Even numbers are 2, 4, 6, 8, 10, 12, 14 = 7

∴ Probability of even number $P(E_1) = \frac{7}{15}$

(ii) Numbers divisible by 5 are 5, 10, 15 = 3

∴ Probability of number divisible by 5 = $P(E_2)$

$= \frac{3}{15} = \frac{1}{5}$

(iii) Number is '6' = 1

∴ Probability of number 6 = $P(E_3) = \frac{1}{15}$

(iv) Numbers lying between 8 and 12 are 9, 10, 11 = 3

∴ Probability of number between 8 and 12 =

$P(E_4) = \frac{3}{15} = \frac{1}{5}$

(v) Number greater than 9 are 10, 11, 12, 13, 14, 15 = 6

∴ Probability of number greater than 9 = $P(E_5)$

$= \frac{6}{15} = \frac{2}{5}$

(vi) Numbers less than 6 are 1, 2, 3, 4, 5 = 5

∴ Probability of number less than 6 = $P(E_6)$

$= \frac{5}{15} = \frac{1}{3}$ Ans.

Q. 12. In a class of 40 students, there are 16 boys and the rest are girls. From these students, one is selected at random. What is the probability that the selected student is a girl ?

Sol. Total number of students = 40

Number of boys = 16

∴ number of girls = 40 − 16 = 24

One girls is selected at random from total number of 40 students.

Here $n(S) = 40$ and probability = $\frac{n(E)}{n(S)}$

∴ Probability of girls = $\frac{24}{40} = \frac{3}{5}$ **Ans.**

Q. 13. One card is drawn at random from a well-shuffled deck of 52 cards. Find the probability of drawing :

(i) a king (ii) a '8' of spades (iii) a '10' of hearts (iv) a '5' of a black suit

(v) a '9' of a red suit. (vi) a red queen (vii) a diamond (viii) a black ace.

Sol. Number of cards = 52 with four suits and two colours : black and red. Each suit has 13 cards.

(i) Probability of a king = $P(E_1) = \frac{4}{52} = \frac{1}{13}$

(ii) Probability of '8' of spades = $P(E_2) = \frac{1}{52}$

(iii) Probability of '10' of hearts = $P(E_3) = \frac{1}{52}$

(iv) Probability of '5' of a black suit = $\frac{2}{52} = \frac{1}{26}$

(*v*) Probability of a '9' of a red suit = $\frac{2}{52} = \frac{1}{26}$

(*vi*) Probability of red queen = $\frac{2}{52} = \frac{1}{26}$

(*vii*) Probability of a diamond = $\frac{13}{52} = \frac{1}{4}$

(*viii*) Probability of a black ace = $\frac{2}{52} = \frac{1}{26}$ Ans.

Q. 14. A card is drawn at random from a well-shuffled deck of 52 cards. Find the probabilit that the card drawn is

(*i*) either a king or a queen

(*ii*) neither a king nor a queen.

Sol. A deck of 52 playing cards. their are 4 suits each of 13 cards.

(*i*) Number of cards either a king or queen = 4 + 4 = 8

∴ Probability of either a king or a queen = 8 = $P(E_1) = \frac{8}{52} = \frac{2}{13}$

(*ii*) Number of card which are neither a king nor a queen = 52 – 8 = 44

∴ Probability of neither a king nor a queen = P(neither a king nor a queen = $PE' = \frac{44}{52} = \frac{1}{1}$

Ans

Q. 15. If the probability of winning a game is 0.7 what is the probability of losing the game

Sol. Probability of winning a game = 0.7

∵ P(not winning a game) = 1 – 0.7 = 0.3

[∵ P(winning a game) + P(not winning a game) = 1} Ans.

Q. 16. Fill in the blanks :

(*i*) The probability of a sure event is ....

(*ii*) The probability of an impossible event is ....

(*iii*) If E is an event, then P(E) + P(not E) = ...

(*iv*) For any event E, we have 0 ≤ .... ≤ 1.

Sol. (*i*) The probability of a sure event is 1.

(*ii*) The probability of an impossible is **0**

(*iii*) If E is an event, then P(E) + P(not E) = **1.**

{∵ P(not E) is complement to P(E)}

(*iv*) For any event E, we have 0 ≤ **P(E)** ≤ 1.