Chapter 26

Heights and Distances

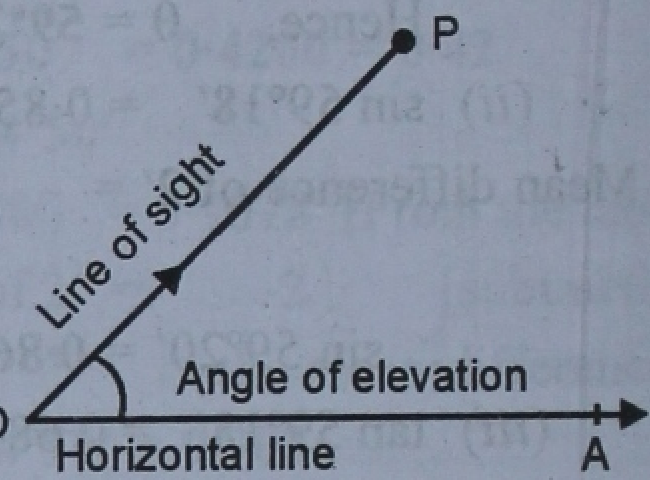
POINTS TO REMEMBER

1. **Line of Sight.** When the eye of a person at a point O looks at an object P, then the line OP is called the line of sight.

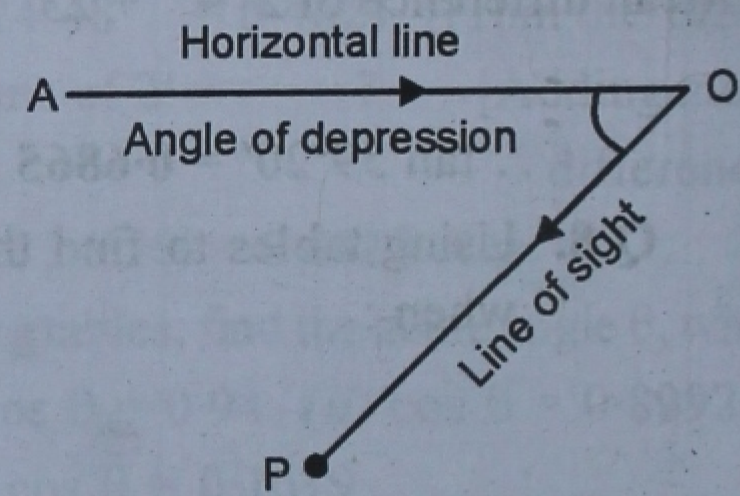
Angle of Elevation. Suppose a man from a point O, looks up at an object P, placed above the level of his eye. Then, the angle which the line of sight makes with the horizontal through O, is called the angle of elevation of P as seen from O.

Let OA be a horizontal line. Let a man at O on the level ground be looking up towards an object P, say an aeroplane or the top of a tower or the flag at the top of a building.

Then, $\angle AOP$ is the angle of elevation of P as seen from O. (see fig.)

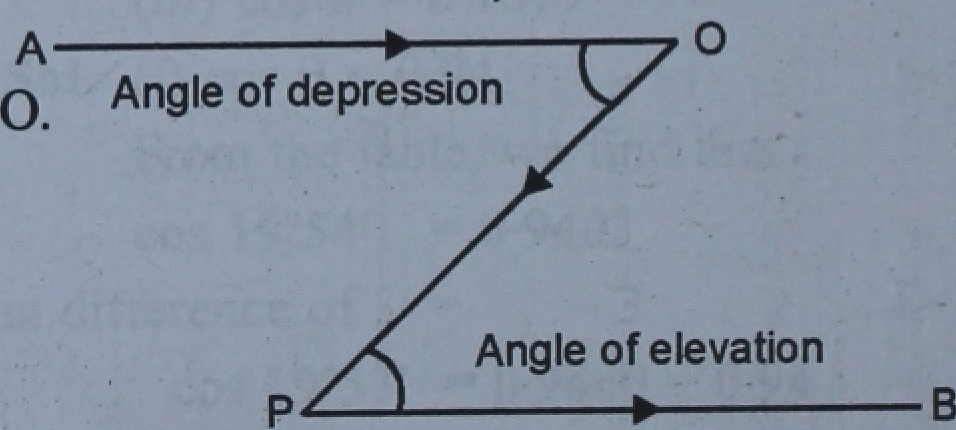


3. **Angle of Depression.** Suppose a man from a point O, looks down at an object P, placed below the level of his eye. Then, the angle which the line of sight makes with the horizontal through O, is called the angle of depression of P as seen from O.



Let AO be a horizontal line. Let a man at O, on the top of a tower be looking down towards an object P, say a ship in the sea.

Then, $\angle AOP$ is the angle of depression of P as seen from O.



An Important Remark :

Angle of depression of P as seen from O

= Angle of elevation of O as seen from P.

$\therefore \angle AOP = \angle BPO$ (Alternate angles, as AO ∥ PB)

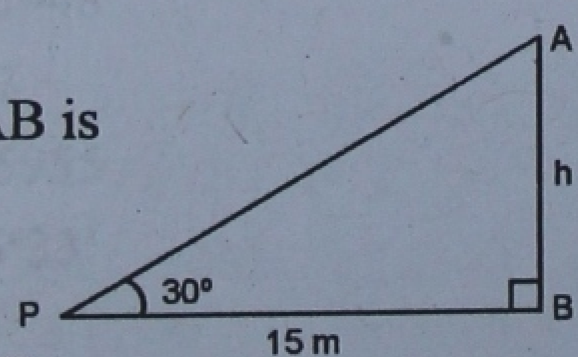
EXERCISE 26

Q.1. The angle of elevation of the top of a pole from a point on the level ground, 15 m away from the foot of the pole is 30°. Find the height of the pole. [Take $\sqrt{3}$ = 1·732]

Sol. Let P is the point and AB is the pole. Such that $\angle APB = 30°$

PB = 15 m

Let AB = height of the pole = h m



$$\therefore \tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{AB}{PB}$$

$$\Rightarrow \tan 30° = \frac{h}{15} \quad \Rightarrow h = 15 \times \tan 30°$$

$$\Rightarrow h = 15 \times \frac{1}{\sqrt{3}}$$

$$= \frac{15 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{15\sqrt{3}}{3} = 5\sqrt{3} \text{ m}$$

$= 5 \times 1·732 = 8·660 = 8·66$ m **Ans.**

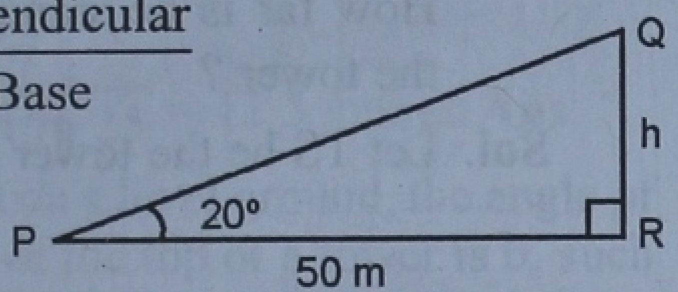
Q.2. (*i*) From a point on the level ground, at a distance of 50 m from the foot of a tower, the angle of elevation of the top of the tower is 20°. Find the height of the tower.

(*ii*) From the top of a cliff 92 *m* high, the angle of depression of a buoy is 20°. Calculate to the nearest metre, the distance of the buoy from the foot of the cliff. **(2005)**

Sol. Let P be the point on the ground and QR is the tower such that

$\angle QPR = 20°$, PR = 50 m and QR = h

Now, $\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$



$\tan 20° = \frac{QR}{PR}$

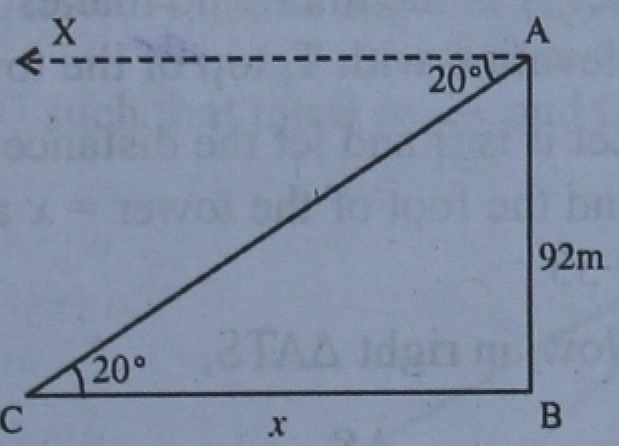
$\Rightarrow \tan 20° = \frac{h}{50} \Rightarrow 0{\cdot}3640 = \frac{h}{50}$

$\Rightarrow \quad h = 50 \times 0{\cdot}3640$

$= 18{\cdot}2000 = 18{\cdot}2$

Hence, height of the tower = 18·2 m **Ans.**

(*ii*) Let AB be cliff whose height is 92 m and C is buoy making depression angle of 20°.



$\therefore \angle ACB = 20°$

Let, CB = x m.

In right ΔABC,

$\cot\theta = \frac{BC}{AB} \Rightarrow \cot 20° = \frac{x}{92}$

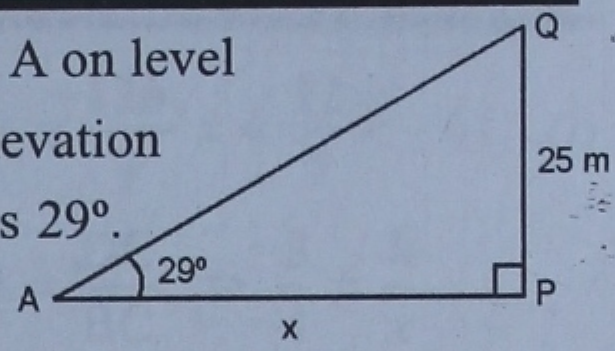
$\Rightarrow x = 92 \cot 20°$

$= 92 \times 2{\cdot}7475$ m

$= 252{\cdot}77$ m

Distance of buoy from foot of hill = 252.77m

Q.3. From a point A on level ground, the angle of elevation of the top of a tower is 29°. If the tower is 25 m high, find the distance of A from the foot of the tower, to the nearest metre,.



Sol. Let A be the point on the level ground and angle of elevation is 29°.

and let PQ is the tower such that PQ = 25 m.

Now, $\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$

$\Rightarrow \tan 29° = \frac{PQ}{AP} = \frac{25}{x} \Rightarrow 0{\cdot}5543 = \frac{25}{x}$

$\Rightarrow x = \frac{25}{0{\cdot}5543} \Rightarrow x = 45{\cdot}10$

Hence, distance from the foot of the tower = 45 m. **Ans.**

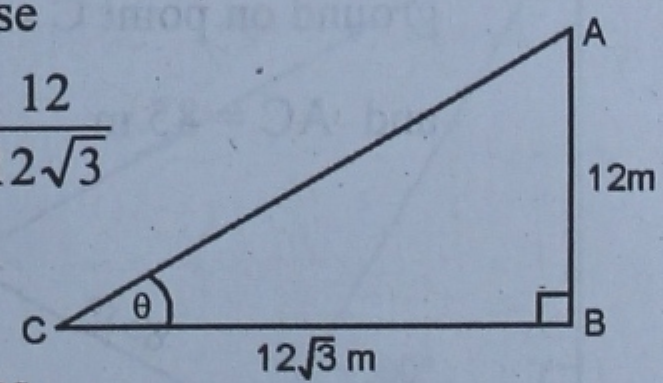
Q.4. A vertical pole is 12 m high and the length of its shadow is $12\sqrt{3}$ m. What is the angle of elevation of the sun ?

Sol. Let AB be the pole and BC be its shadow on the ground. Let angle of elevation be θ, then AB = 12 m,

BC = $12\sqrt{3}$ m and $\angle ACB = \theta$.

$\therefore \tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$

$\Rightarrow \tan\theta = \frac{AB}{BC} = \frac{12}{12\sqrt{3}}$

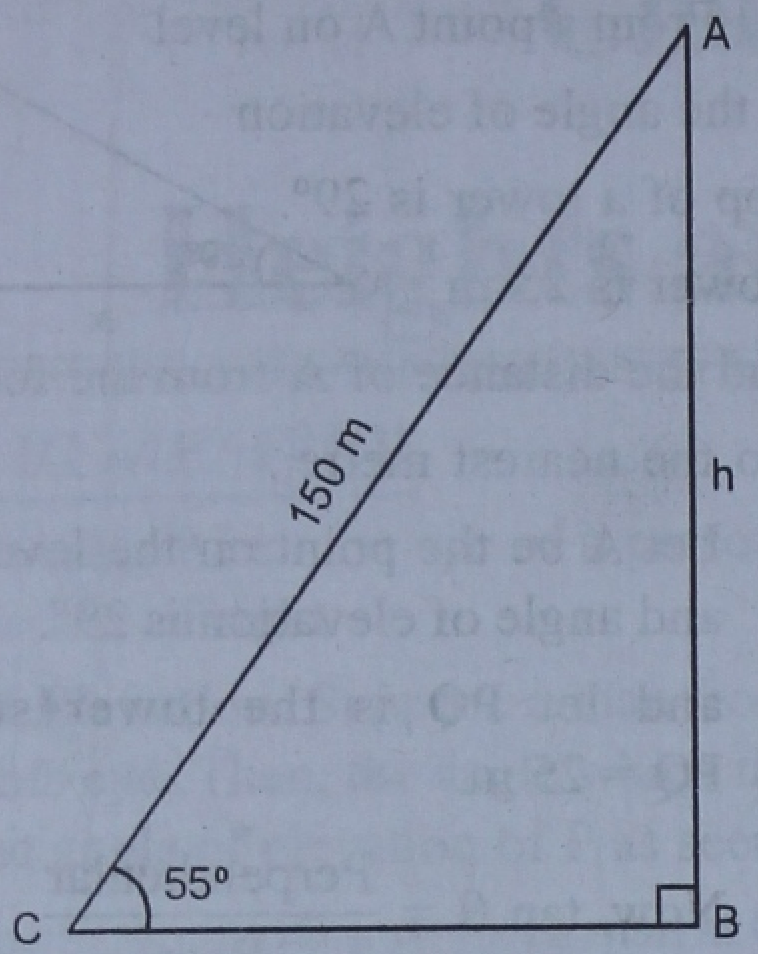


$= \frac{1}{\sqrt{3}} = \tan 30°$

$\therefore \quad \theta = 30°$ **Ans.**

Q.5. A kite is flying with a thread 150 m long. If the thread is assumed stretched straight and makes an angle of 55° with the horizontal, find the height of the kite above the ground.

Sol. Let A be the kite and AB is its height and AC be the thread which makes an angle of 55° with the ground (horizontal), then



AC = 150 m, AB = h m

Now, $\sin\theta = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}}$

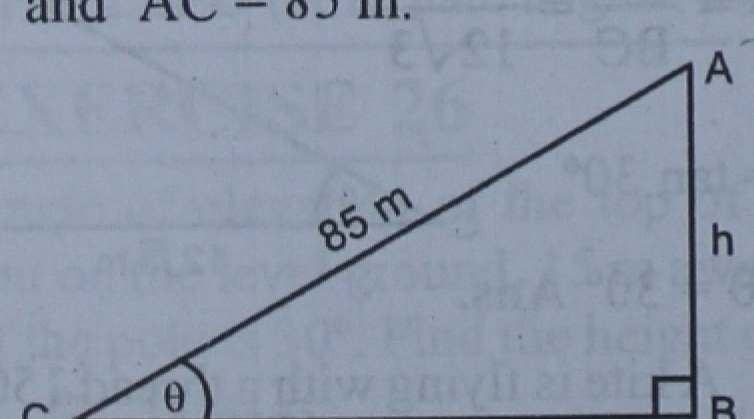
$\Rightarrow \sin 55^\circ = \dfrac{h}{150} \Rightarrow 0{\cdot}8192 = \dfrac{h}{150}$

$h = 150 \times 0{\cdot}8192 = 122{\cdot}88$

Hence, height of the kite = 122·88 m **Ans.**

Q. 6. The length of a string between a kite and a point on the ground is 85 m. If the string makes an angle θ with the level ground, such that $\tan\theta = \dfrac{15}{8}$, how high is the kite?

Sol. Let AB be the height of kite A and the string makes an angle of θ with the ground on point C such that $\tan\theta = \dfrac{15}{8}$ and AC = 85 m.



Now, $\sin\theta = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}}$

We are given, $\tan\theta = \dfrac{15}{8}$

$\therefore \sin\theta = \dfrac{15}{\sqrt{15^2 + 8^2}} = \dfrac{15}{\sqrt{225 + 64}}$

$= \dfrac{15}{\sqrt{289}} = \dfrac{15}{17}$

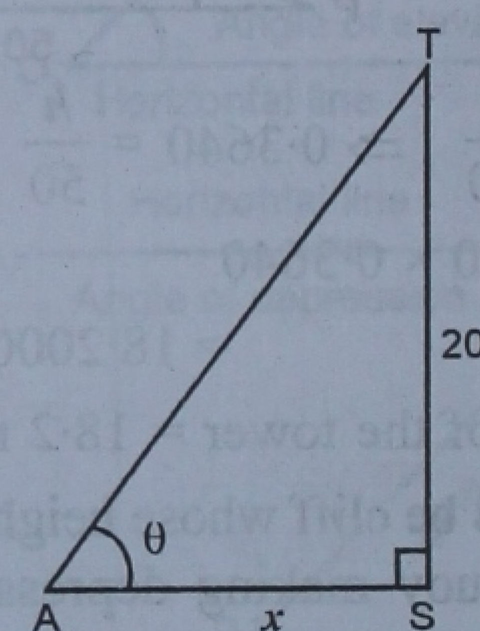
$\therefore \sin\theta = \dfrac{AB}{AC} \Rightarrow \dfrac{15}{17} = \dfrac{h}{85}$

$\Rightarrow \quad h = \dfrac{15 \times 85}{17} = 75$

Hence, height of kite = 75 m **Ans.**

Q. 7. A vertical tower is 20 m high. A man standing at some distance from the tower knows that the cosine of the angle of elevation of the top of the tower is 0·53. How far is he standing from the foot of the tower ? **(2001)**

Sol. Let TS be the tower then TS = 20 m



Let A is the man who makes an angle of elevation with T, top of the tower.

Let it is θ and let the distance of the man and the foot of the tower = x and $\cos\theta$ = 0·53

Now, in right ΔATS,

$\cos\theta = \dfrac{AS}{AT}$

$\Rightarrow 0{\cdot}53 = \dfrac{x}{AT}$

But, $AT = \sqrt{AS^2 + TS^2} = \sqrt{x^2 + 20^2}$

$= \sqrt{x^2 + 400}$

$\therefore \quad 0{\cdot}53 = \dfrac{x}{\sqrt{x^2 + 400}}$

Squaring on both sides,

$$(0{\cdot}53)^2 = \frac{x^2}{x^2+400}$$

$$\Rightarrow \quad 0{\cdot}2809 = \frac{x^2}{x^2+400}$$

$$x^2 = 0{\cdot}2809\, x^2 + 400 \times 0{\cdot}2809$$

$$\Rightarrow x^2 - 0{\cdot}2809\, x^2 = 112{\cdot}36$$

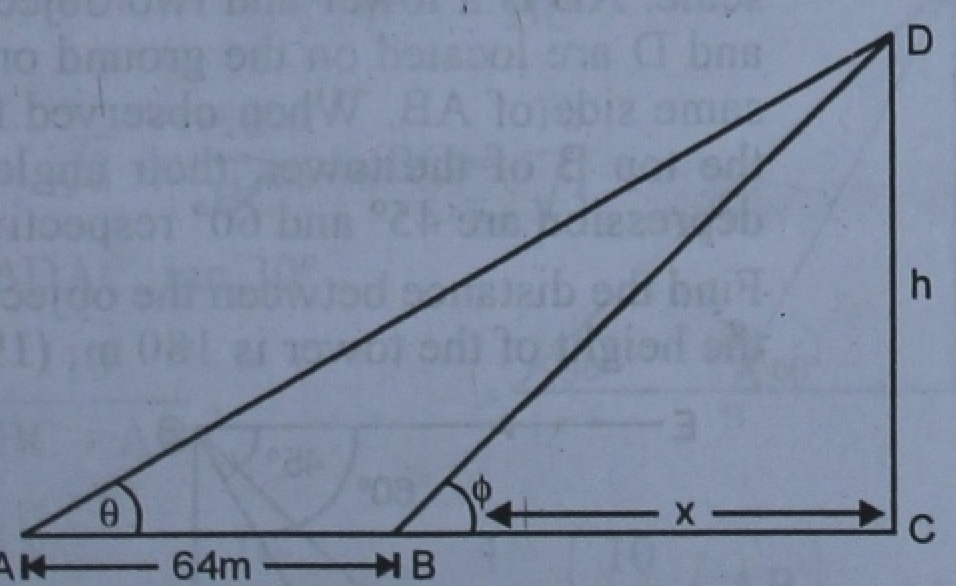
$$\Rightarrow 0{\cdot}7191\, x^2 = 112{\cdot}36$$

$$\Rightarrow x^2 = \frac{112{\cdot}36}{0{\cdot}7191} = 156{\cdot}25$$

$\therefore \quad x = \sqrt{156{\cdot}25} = 12{\cdot}5$ metres **Ans.**

Q. 8. At a point on a level ground, the angle of elevation of the top of a tower is θ, such that $\tan\theta = \frac{7}{12}$. On walking 64 m towards the tower, the angle of elevation is ϕ, where $\tan\phi = \frac{3}{4}$. Find the height of the tower.

Sol. Let CD be the tower and A is the point on the ground, the angle of elevation $= \theta$ and B is another point such that AB = 64 m and angle of elevation at B is θ such that $\tan\theta = \frac{7}{12}$ and $\tan\phi = \frac{3}{4}$



Let h be the height of tower and distance between B and C be x m

Now, $\tan\theta = \frac{DC}{AC} \Rightarrow \frac{7}{12} = \frac{h}{x+64}$

$\Rightarrow 7\,(x+64) = 12\,h$

$$\Rightarrow \quad x + 64 = \frac{12h}{7} \quad x = \frac{12}{7}h - 64 \quad ...(i)$$

and $\tan\theta = \frac{DC}{BC} \Rightarrow \frac{3}{4} = \frac{h}{x}$

$$\Rightarrow \quad x = \frac{4h}{3} \quad ...(ii)$$

From (i) and (ii),

$$\frac{12}{7}h - 64 = \frac{4}{3}h$$

$$\Rightarrow \frac{12}{7}h - \frac{4}{3}h = 64$$

$$\Rightarrow \frac{36h - 28h}{21} = 64 \quad \Rightarrow \frac{8}{21}h = 64$$

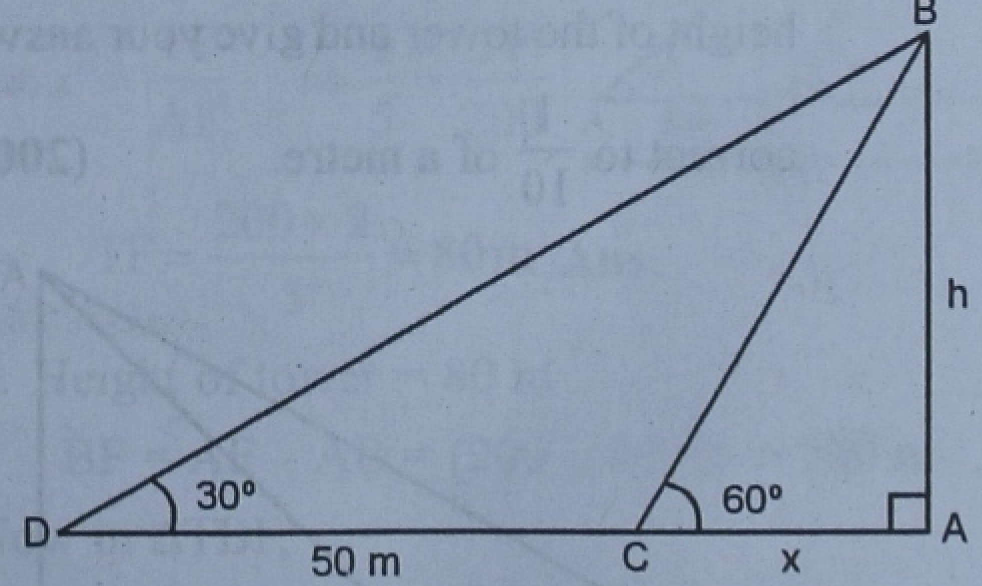
$$\Rightarrow \quad h = 64 \times \frac{21}{8} = 168$$

Hence, height of the tower = 168 m. **Ans.**

Q. 9. A man standing on the bank of a river observes that angle of elevation of a tree standing on the opposite bank is 60°. When he moves 50 m away from the bank, he finds the angle of elevation to be 30°. Calculate : **(2003)**

(i) the height of the tree, and

(ii) the width of the river.



Sol. Let C and D are the positions of the man and AB be the tree such that CD = 50 m. and angles of elevation at C and D be 60° and 30° respectively.

Let AB = h and AC = x

Now, $\tan 60° = \frac{AB}{AC} \Rightarrow \frac{h}{x}$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow \quad h = x\sqrt{3} \quad \ldots(i)$$

and $\tan 30^\circ = \frac{AB}{AD} = \frac{h}{50+x}$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{50+x}$$

$$\Rightarrow \quad h = \frac{50+x}{\sqrt{3}} \quad \ldots(ii)$$

From (*i*) and (*ii*),

$$\frac{50+x}{\sqrt{3}} = x\sqrt{3}$$

$$\Rightarrow 50 + x = 3x \Rightarrow 3x - x = 50$$

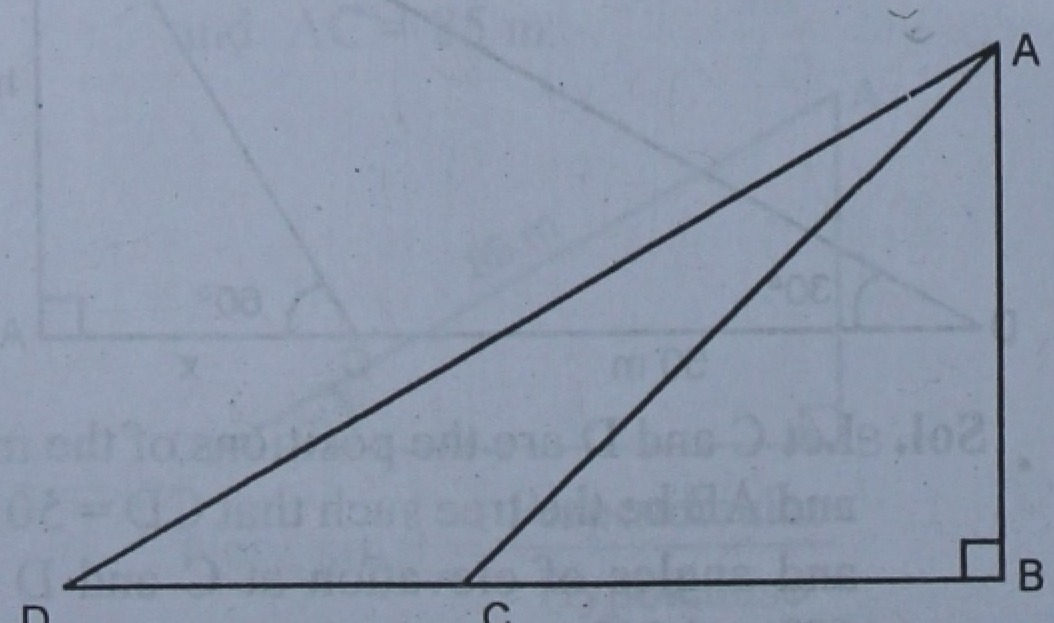
$$\Rightarrow 2x = 50 \Rightarrow x = \frac{50}{2} = 25$$

Substituting the value of x in (*i*),

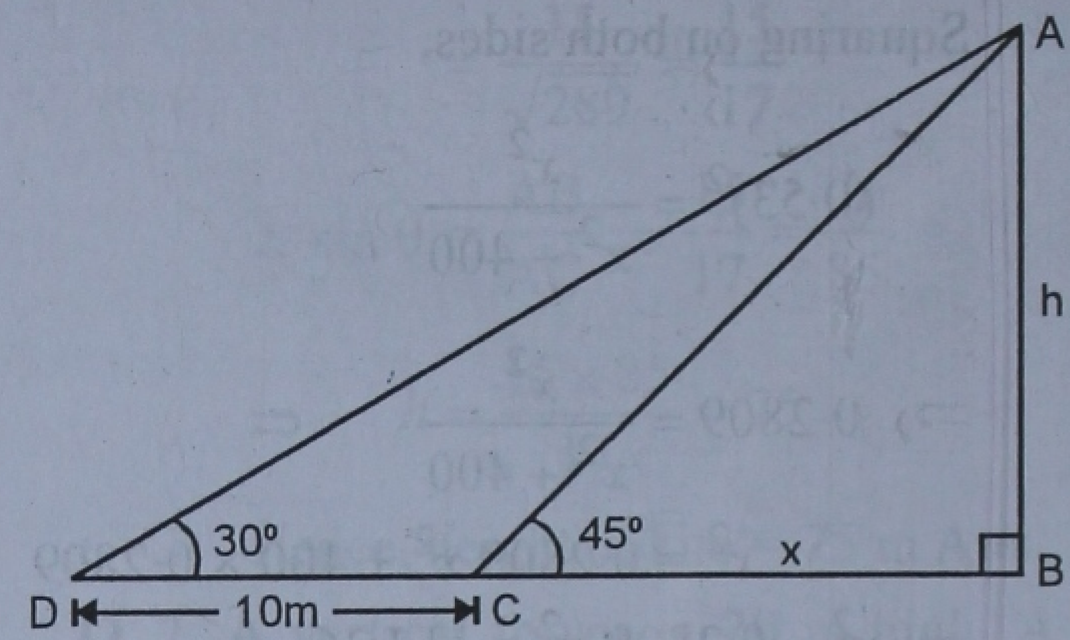
$$h = x\sqrt{3} = 25\sqrt{3} = 25\,(1{\cdot}732) = 43{\cdot}3$$

Hence, height of the tree = 43·3 m and width of the river = 25 m **Ans.**

Q. 10. The shadow of a vertical tower AB on level ground is increased by 10 m, when the altitude of the sun changes from 45° to 30°, as shown in the figure. Find the height of the tower and give your answer correct to $\frac{1}{10}$ of a metre. **(2002)**



Sol. Let AB be the tower, and AB = h and angles of elevations of the tower at C and D be 45° and 30° respectively and CD = 10 m. Let BC = x.



Now, $\tan 45^\circ = \frac{AB}{BC} = \frac{h}{x}$

$$\Rightarrow \quad 1 = \frac{h}{x} \Rightarrow x = h \quad \ldots(i)$$

and $\tan 30^\circ = \frac{AB}{BD} = \frac{h}{x+10}$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x+10}$$

$$\Rightarrow \quad \sqrt{3}h = x + 10$$

$$\Rightarrow \quad x = \sqrt{3}\,h - 10 \quad \ldots(ii)$$

From (*i*) and (*ii*),

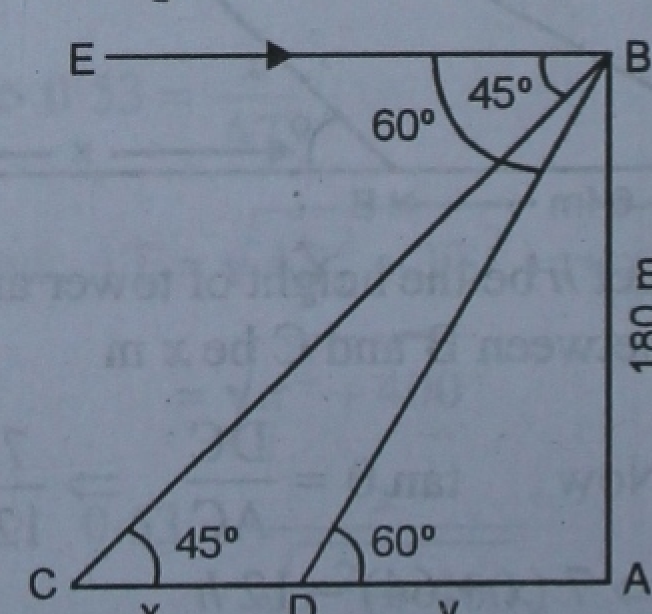
$$h = \sqrt{3}h - 10 \Rightarrow \sqrt{3}h - h = 10$$

$$(\sqrt{3} - 1)\,h = 10 \Rightarrow (1{\cdot}732 - 1)\,h = 10$$

$$\Rightarrow 0{\cdot}732\,h = 10 \Rightarrow h = \frac{10}{0{\cdot}732} = 13{\cdot}66$$

Hence, height of tower = 13·66 m **Ans.**

Q. 11. The adjoining figure is drawn, not to the scale. AB is a tower and two objects C and D are located on the ground on the same side of AB. When observed from the top B of the tower, their angles of depression are 45° and 60° respectively. Find the distance between the objects, if the height of the tower is 180 m. **(1998)**



Give your answer to the nearest metre.

Sol. In the figure, AB is the tower, C and D are two objects on the same side of the tower. Angle of depression with C and D from top of the tower are 45° and 60° respectively.

Let CD = x and AD = y, AB = 180 m

Now, $\tan 45^\circ = \frac{AB}{AC} = \frac{180}{x+y}$

$\Rightarrow \quad 1 = \frac{180}{x+y} \Rightarrow x + y = 180 \quad ...(i)$

$\tan\ 60^\circ = \frac{AB}{AD} = \frac{180^\circ}{y} \Rightarrow \sqrt{3} = \frac{180}{y}$

$\Rightarrow y = \frac{180 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{180\sqrt{3}}{3} = 60\sqrt{3} \quad ...(ii)$

From (i) and (ii),

$x + 60\sqrt{3} = 180$

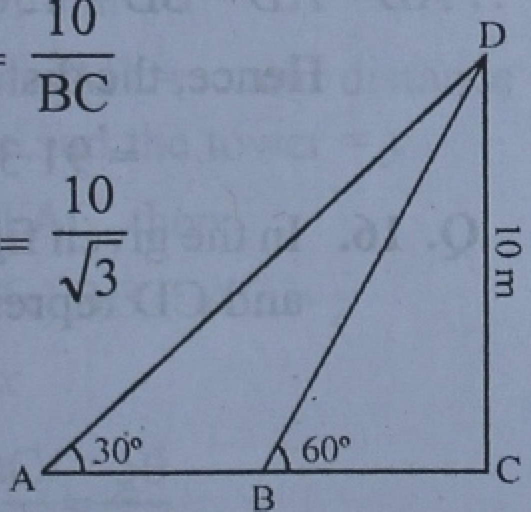
$\Rightarrow x = 180 - 60\sqrt{3} \Rightarrow x = 180 - (60 \times 1{\cdot}732)$

$= 180 - 103{\cdot}92 = 76{\cdot}08$

Hence, distance between two objects C and D = 76 m **Ans.**

12. From two points A and B on the same side of a building, the angles of elevation of the top of the building are 30° and 60° respectively. If the height of the building is 10 m, find the distance between A and B correct to two decimal places. **(2009)**

Sol. In ΔDBC, $\tan 60^\circ = \frac{10}{BC}$



$\Rightarrow \sqrt{3} = \frac{10}{BC} \Rightarrow BC = \frac{10}{\sqrt{3}}$

In ΔDAC, tan 30°

$= \frac{10}{BC + AB}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{10}{\frac{10}{\sqrt{3}} + AB} \Rightarrow \frac{1}{\sqrt{3}} \left(\frac{10}{\sqrt{3}} + AB\right) = 10$

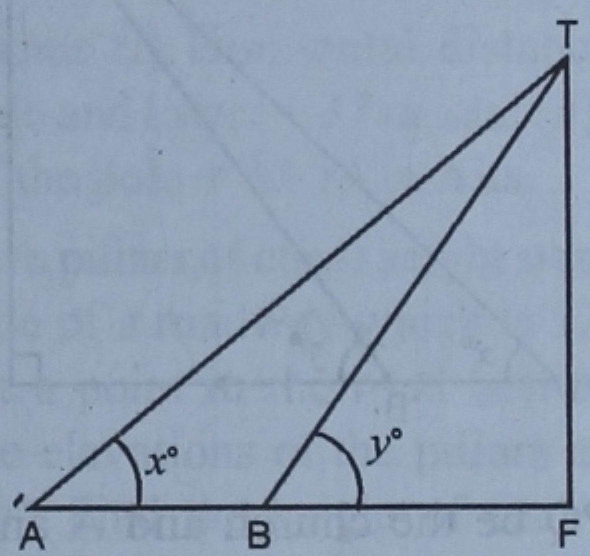
$\Rightarrow AB = 10\sqrt{3} - \frac{10}{\sqrt{3}} = \frac{30 - 10}{\sqrt{3}} = \frac{20}{\sqrt{3}} = \frac{20\sqrt{3}}{3}$

$= \frac{20 \times 1.732}{3} = 20 \times .577 = 11.540$ m

Q. 13. In the figure not drawn to scale, TF is a tower. The elevation of T from A is x°, where $\tan x = \frac{2}{5}$ and AF = 200 m. The elevation of T from B, where AB = 80 m is y°. Calculate :

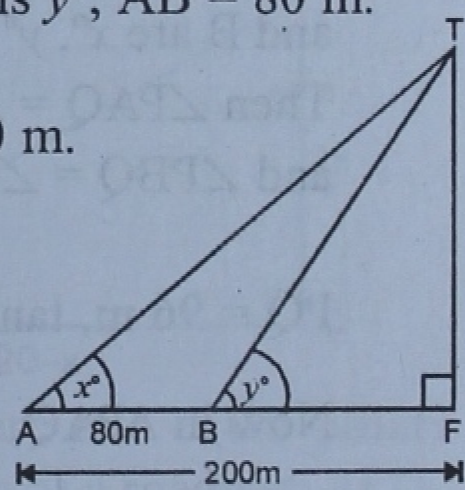
(i) The height of the tower TF ;

(ii) The angle y, correct to the nearest degree. **(1997)**



Sol. In the figure, TF is a tower, elevation of T from A is x° and from B is y°, AB = 80 m.

$\tan x = \frac{2}{5}$ and AF = 200 m.



In right ΔTAF,

$\tan x^\circ = \frac{TF}{AF} \quad \Rightarrow \frac{2}{5} = \frac{TF}{200}$

$\Rightarrow \quad TF = \frac{200 \times 2}{5} = 80\text{m}$ **Ans.**

∴ Height of tower = 80 m

BF = AF – AB = (200 – 80)m = 120 m

Now in ΔTBF,

$\tan y^\circ = \frac{TF}{BF} \Rightarrow \frac{80}{120} = \frac{2}{3} = 0{\cdot}6667$

From the table, we find that :

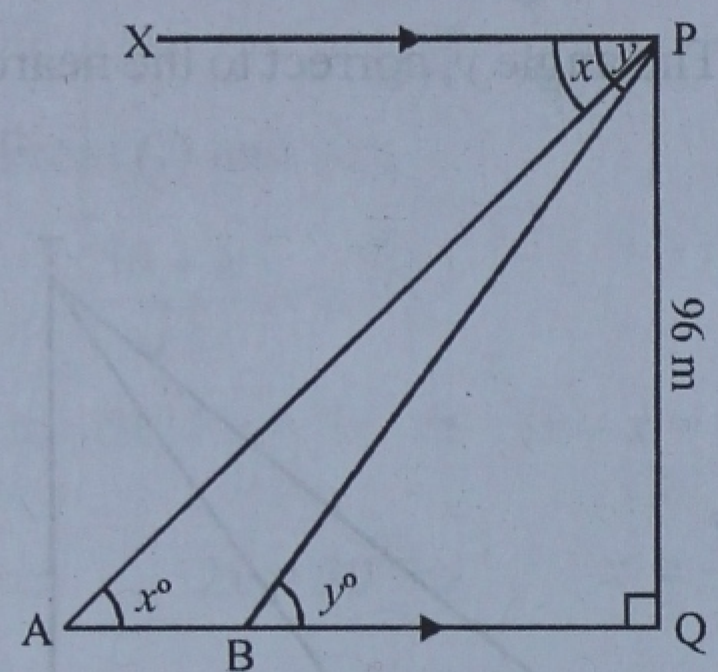
$\tan y = \tan 33^\circ 41'$

$\Rightarrow \quad y = 33^\circ 44'$ or 34° **Ans.**

(Correct to nearest degree)

Q. 14. From the top of a church spire 96 m high, the angles of depression of two vehicles on a road, at the same level as the base of the spire and on the same side of it are x^o and y^o, where $\tan x^o = \frac{1}{4}$ and $\tan y^o = \frac{1}{3}$. Calculate the distance between the vehicles. **(1994)**

Sol.



Let PQ be the church and A and B be two points on the ground level. The top P of the church makes angle of depression with A and B are x^o, y^o respectively

Then $\angle PAQ = \angle XPA = x^o$

and $\angle PBQ = \angle XPB = y^o$ {alternate angle}

PQ = 96 m, $\tan x^o = \frac{1}{4}$ and $\tan y^o = \frac{1}{7}$

Now in ΔPAQ,

$$\tan x^o = \frac{PQ}{AQ} \Rightarrow \frac{1}{4} = \frac{96}{AQ}$$

$\Rightarrow AQ = 96 \times 4 = 384$ m

Similarly in ΔPBQ

$$\tan y^o = \frac{PQ}{BQ} \Rightarrow \frac{1}{7} = \frac{96}{BQ}$$

$\Rightarrow BQ = 7 \times 96 = 672$ m

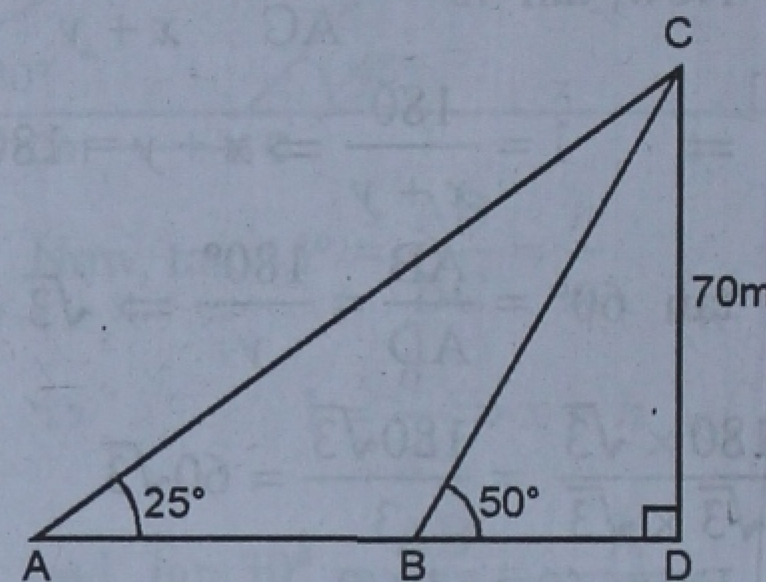
$\therefore$ Distance between A and B = AB = AQ – BQ

= 672 – 384 = 288 m

Q. 15. Two people standing on the same side of a tower in a straight line with it, measure the angles of elevation of the top of the tower as 25° and 50° respectively. If the height of the tower is 70 m, find the distance between the two people. **(2004)**

Sol. CD is the tower and CD = 70 m

A and B are two men standing on the same side of the tower making angles of elevation of 25° and 50° respectively.



Now, in right ΔBCD,

$$\tan\theta = \frac{CD}{BD} \Rightarrow \tan 50^o = \frac{70}{BD}$$

$$\Rightarrow BD = \frac{70}{\tan 50^o} = 70 \cot 50^o \quad \left(\because \frac{1}{\tan\theta} = \cot\theta\right)$$

$= 70\ (0{\cdot}8391) = 58{\cdot}737$ m

Similarly, in right ΔACD,

$$\cot 25^o = \frac{AD}{CD} = \frac{AD}{70} \Rightarrow AD = 70 \cot 25^o$$

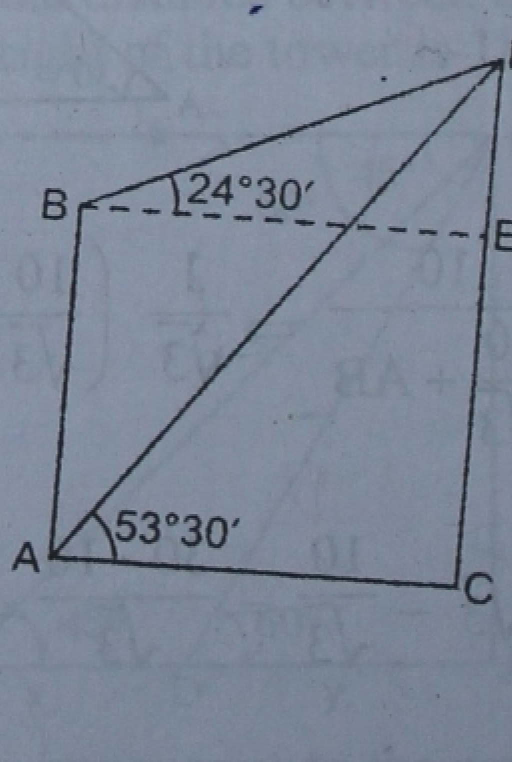
$= 70 \times 2{\cdot}1445 = 150{\cdot}115$

$\therefore AB = AD - BD = 150{\cdot}115 - 58{\cdot}737 = 91{\cdot}378$m

Hence, the distance between two persons

$= 91{\cdot}378$ m $= 91{\cdot}38$ m **Ans.**

Q. 16. In the given figure, AB represents a pole and CD represents a 50 m high tower,

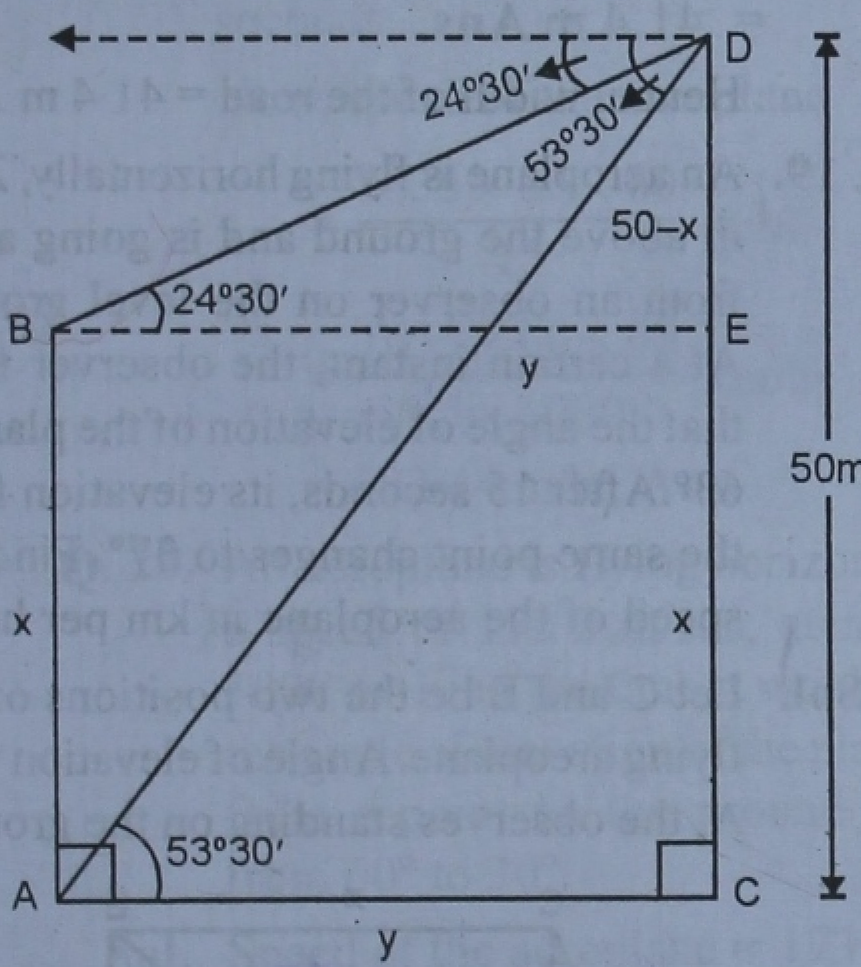


both of which are standing on the same horizontal plane. From the top of the tower, the angles of depression of the top and the foot of the pole are 24°30′ and 53°30′ respectively. Calculate :

(*i*) the horizontal distance between the pole and the tower,

(*ii*) the height of the pole.

Sol. AB is the pole and CD is the tower. The angle of depression from the top of the tower to the top and foot of the pole are 24°30′ and 53°30′ respectively.



Length of DC = 50 m

Let height of pole AB = x and distance between the pole and the tower = y

Now, draw BE || AC, then

CE = AB = x, BE = AC = y

and ED = 50 – x

$$\tan 53^\circ 30' = \frac{DC}{AC} = \frac{50}{y}$$

$$\Rightarrow 1{\cdot}3514 = \frac{50}{y} \quad \text{(From the table)}$$

$$\Rightarrow y = \frac{50}{1{\cdot}3514} \quad ...(i)$$

$$\text{and } \tan 24^\circ 30' = \frac{DE}{BE}$$

$$\Rightarrow 0{\cdot}4557 = \frac{50-x}{y} \quad \text{(From the table)}$$

$$\Rightarrow 0{\cdot}4557\, y = 50 - x$$

$$x = 50 - 0{\cdot}4557\, y \quad ...(ii)$$

$$= 50 - 0{\cdot}4557 \times \frac{50}{1{\cdot}3514} \quad \text{[from (i)]}$$

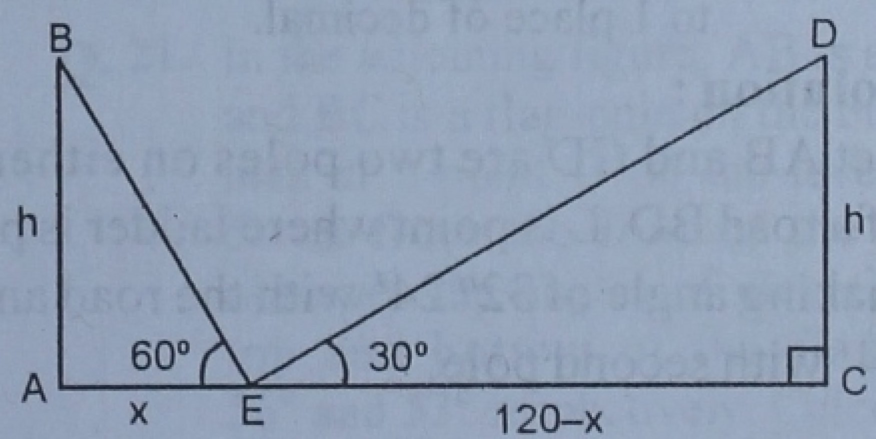
$$= 50 - 16{\cdot}86 = 33{\cdot}14$$

$$\text{and } y = \frac{50}{1{\cdot}3514} = 36{\cdot}998$$

$$= 37{\cdot}0 \quad \text{(approx)}$$

Hence (*i*) Horizontal distance between pole and tower = 37 m. and (*ii*) the height of the pole = 33·14 m **Ans.**

Q. 17. Two pillars of equal height stand on either side of a roadway which is 120 m wide. At a point in the road between pillars, the elevations of the pillars are 60° and 30°. Find the height of each pillar and the position of the point.



Sol. Let AB and CD are two equal pillars stand on either side of a roadway AC.

Then AC = 120 m.

Angle of elevation of each pillar to a point E on the road are 60° and 30° respectively.

Let AB = CD = h

AE = x, then EC = 120 – x

$$\text{Now, } \tan 60^\circ = \frac{AB}{AE} \Rightarrow \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow h = \sqrt{3}\, x \quad ...(i)$$

$$\text{and } \tan 30^\circ = \frac{CD}{EC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{120-x}$$

$\Rightarrow \quad h = \frac{120 - x}{\sqrt{3}} \quad \Rightarrow \sqrt{3}h = 120 - x$

$\Rightarrow \sqrt{3}h = 120 - \frac{h}{\sqrt{3}}$ [Value of x from (i)]

$\Rightarrow \quad 3h = 120\sqrt{3} - h \Rightarrow 3h + h = 120\sqrt{3}$

$\Rightarrow \quad 4h = 120\sqrt{3} \quad \Rightarrow \quad h = \frac{120\sqrt{3}}{4} = 30\sqrt{3}$

$\Rightarrow \quad h = 30\ (1{\cdot}732) = 51{\cdot}96$

Substituting the value of h in (i),

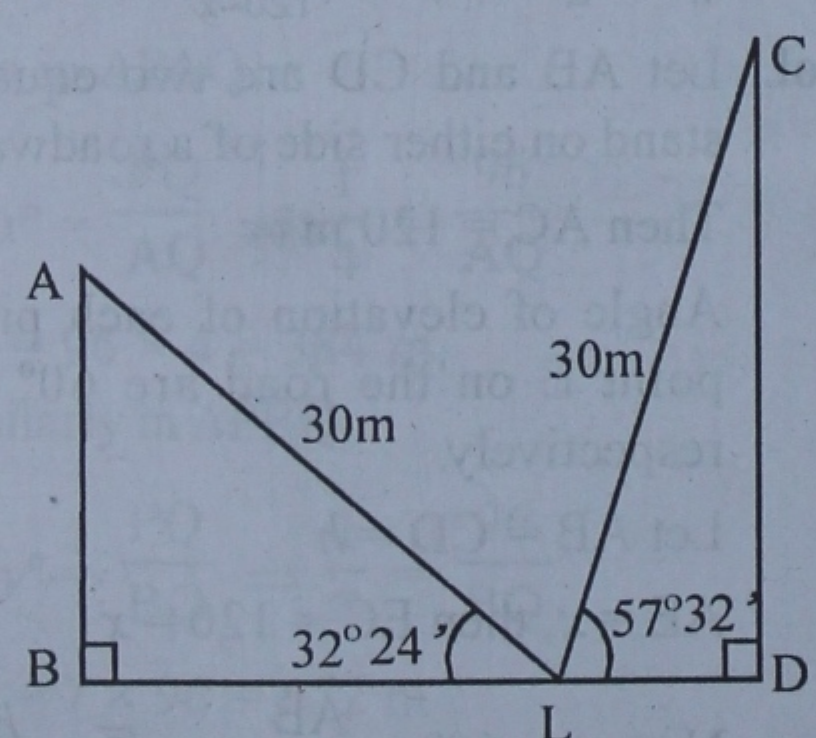
$30\sqrt{3} = \sqrt{3}x \Rightarrow x = 30$

Hence, height of each pillar = 51·96 m. and the position of the point is 30 m from the pillar with elevation 60°. **Ans.**

Q. 18. Two vertical poles are on either side of a road. A 30 m long ladder is placed between the two poles. When the ladder rests against one pole, it makes an angle of 32° 24' with the pole and when it is turned to rest against another pole, it makes an angle of 57° 32' with the road. Calculate the width of the road, correct to 1 place of decimal.

Solution :

Let AB and CD are two poles on either side of a road BD. L is point where ladder is placed making angle of 32° 24′ with the road and 57° 24′ with second pole.



Length of ladder AL = CL = 30 m.

In right Δ ABL.

$\text{Cos}\theta = \frac{BL}{AL} \quad \Rightarrow \quad \text{Cos } 32°24' = \frac{BL}{30}$

$BL = 30 \cos 32°24' = 30 \times 0.8443$
$= 25.3290 = 25.329$ m

Similarly in right CLD,

$\therefore \text{Cos } 57°32' = \frac{LD}{CL} = \frac{LD}{30}$

$\Rightarrow \quad 0.5378 = \frac{LD}{30}$

$\Rightarrow \quad LD = 30 \times 0.5378$

$\Rightarrow \quad LD = 16.1340 = 16.134$

$\therefore \quad BD = BL + LD$

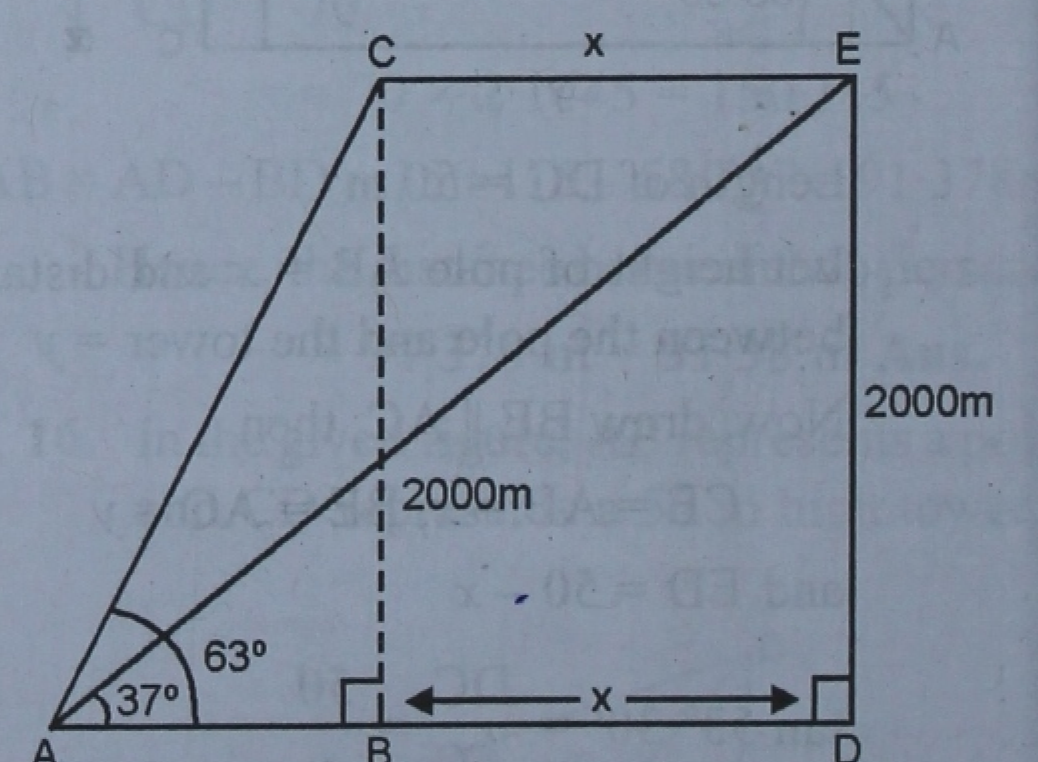
$= 25.329 + 16.134 = 41.463$ m

$= 41.4$ m **Ans.**

Hence, width of the road = 41·4 m **Ans.**

Q. 19. An aeroplane is flying horizontally, 2000 m above the ground and is going away from an observer on the level ground. At a certain instant, the observer finds that the angle of elevation of the plane is 63°. After 15 seconds, its elevation from the same point changes to 37°. Find the speed of the aeroplane in km per hour.

Sol. Let C and E be the two positions of the flying areoplane. Angle of elevation with A, the observes standing on the ground.



CB = ED = 2000 m

Let CE = BD = x m

Now, $\tan 63° = \frac{CB}{AB} = \frac{2000}{AB}$

$\Rightarrow \quad 1{\cdot}9626 = \frac{2000}{AB}$ (From the table)

$$\Rightarrow \quad AB = \frac{2000}{1.9626} = 1019.0 \text{ m}$$

and $\tan 37^\circ = \frac{ED}{AD} = \frac{2000}{AD}$

$$\Rightarrow 0.7536 = \frac{2000}{AD} \quad \text{(From the table)}$$

$$\Rightarrow \quad AD = \frac{2000}{0.7536} = 2654 \text{ m.}$$

$\therefore x = CE = BD = AD - AB$

$= 2654 - 1019 = 1635$ m.

$\therefore$ The aeroplane fly 1635 m in 15 seconds.

Now, speed of the areoplane

$$= \frac{1635 \times 60 \times 60}{15} \text{ m/hr}$$

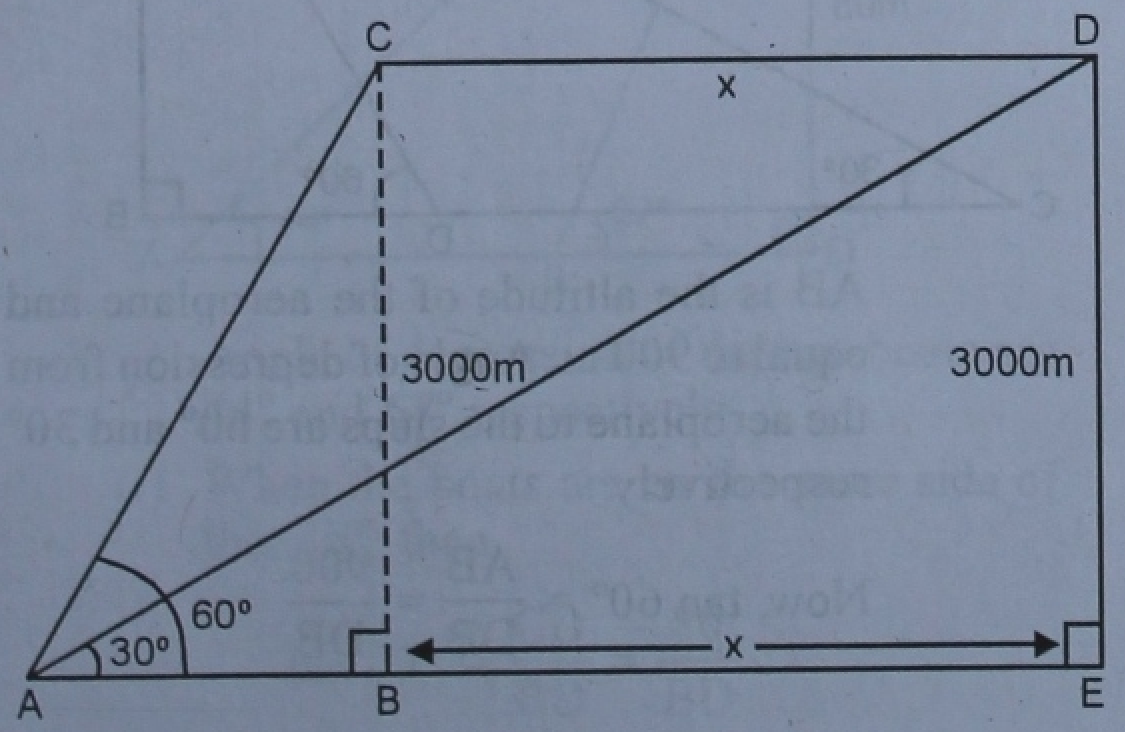
$$= \frac{1635 \times 60 \times 60}{15 \times 1000} \text{ km/hr}$$

$= 392.4$ km/hr. **Ans.**

Q. 20. An aeroplane is flying horizontally with a speed of 173·2 m/sec, at a height of 3000 m. Find the time it would take for the angle of elevation of the plane as seen from a point on the ground to change from 60° to 30°.

Sol. Speed of the aeroplane = 173·2 m/sec.

Let C and D be the two positions of the aeroplane and angle of elevation at C and D are 60° and 30° respectively. A is the point on the ground and CB and DE are the height of the aeroplane at C and D positions, then CB = DE = 3000 m.



Now, $\tan 30^\circ = \frac{DE}{AE} = \frac{3000}{AE}$ (in Δ AED)

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{3000}{AE} \Rightarrow AE = 3000\sqrt{3} \text{ m}$$

$= 3000 (1.732) = 5196$ m

and $\tan 60^\circ = \frac{CB}{AB} = \frac{3000}{AB}$ (in Δ ABC)

$$\Rightarrow \sqrt{3} = \frac{3000}{AB} \Rightarrow AB = \frac{3000}{\sqrt{3}}$$

$$\Rightarrow AB = \frac{3000 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{3000\sqrt{3}}{3}$$

$\Rightarrow AB = 1000\sqrt{3}$

$= 1000 (1.732) = 1732$ m

$\therefore \quad x = CD = BE = AE - AB$

$= 5196 - 1732 = 3464$ m.

Now, speed of the aeroplane = 173·2 m/sec.

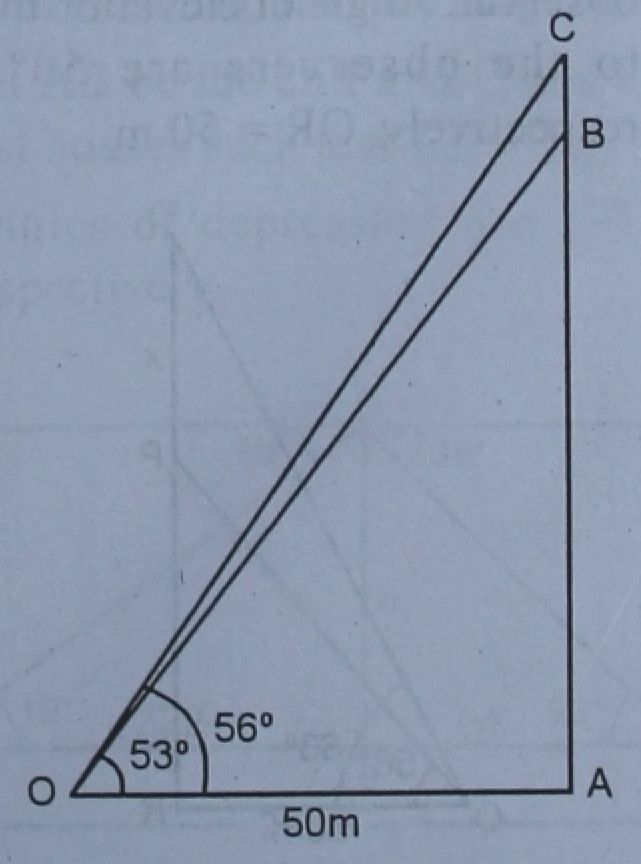
$\therefore$ Time taken to fly from C to D.

$$= \frac{3464 \times 10}{1732} = 20 \text{ seconds. Ans.}$$

Q. 21. In the adjoining figure, AB is a building and BC is a flag-pole on the building. A man at a point O on the level ground, 50 m from the foot of the building observes the angles of elevation of the top and bottom of the flag-pole as 56° and 53° respectively. Calculate :

(*i*) the height of the building,

(*ii*) the height of the flag-pole.



Sol. In the figure,

AB is the building and BC is a flag-pole.

A man at a point O on the level ground 50 m from the foot of the building.

Angles of elevation are 56° and 53° of the top and bottom of the flag-pole.

Let AB = h and BC = x, AC = $h + x$

OA = 50 m

(*i*) Now, $\tan 53° = \frac{AB}{OA} = \frac{h}{50}$

$h = 50 \times \tan 53°$

$= 50 \times 1{\cdot}3270 = 66{\cdot}35$ m.

(*ii*) and $\tan 56° = \frac{AC}{OA} = \frac{h+x}{50}$

$\Rightarrow 1{\cdot}4826 = \frac{h+x}{50}$

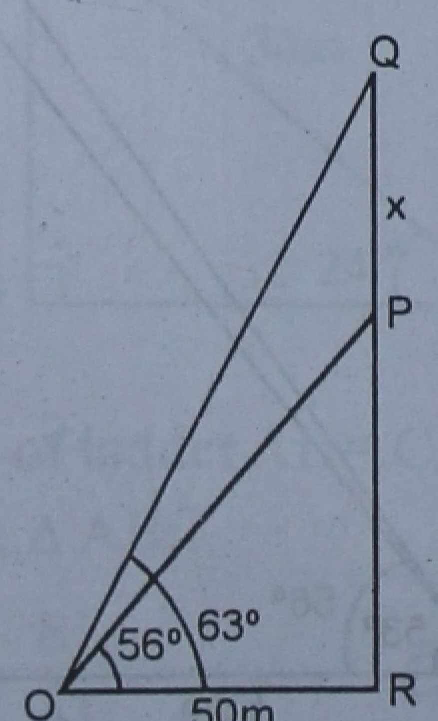
$\Rightarrow \quad h + x = 50 \times 1{\cdot}4826 = 74{\cdot}13$

$\therefore \quad x = 74{\cdot}13 - 66{\cdot}35 = 7{\cdot}78$

Hence (*i*) the height of the building = 66·35 m and (*ii*) the height of the flag pole = 7·28 m **Ans.**

Q. 22. Two climbers are at points P and Q on a vertical cliff face. To an observer O, 50 m from the foot of the cliff, on the level ground, P is at an elevation of 56° and Q of 63°. What is the distance between the climbers ?

Sol. P and Q are two climber and O is the observer. Angle of elevation the climbers to the observers are 56° and 63° respectively. OR = 50 m.



Now, In right ΔPOR

$\tan \theta = \frac{PR}{OR} \Rightarrow \tan 60° = \frac{PR}{50}$

$\Rightarrow 1{\cdot}4826 = \frac{PR}{50}$

$\therefore$ PR = 1·4826 × 50 = 74·13 m

and in right ΔQOR,

$\tan 63° = \frac{QR}{OR} = \frac{QR}{50}$

$\Rightarrow 1{\cdot}9626 = \frac{QR}{50}$ (From the table)

$\Rightarrow \quad QR = 1{\cdot}9626 \times 50$

$\Rightarrow \quad QR = 98{\cdot}13$ m

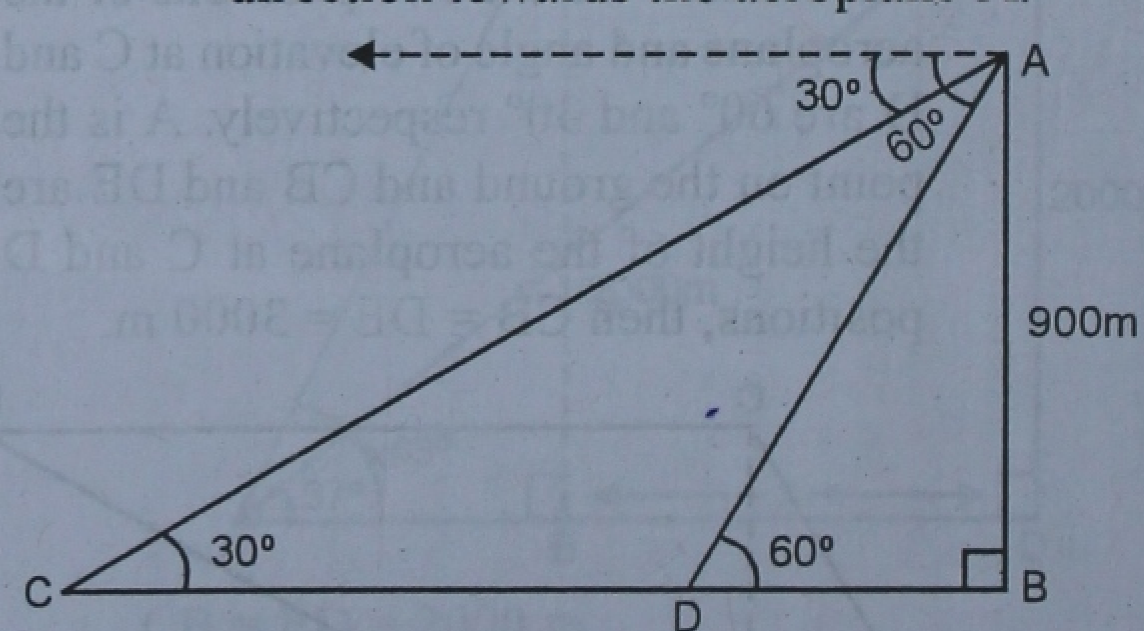
$\therefore$ PQ = QR − PR = 98·13 − 74·13

= 24 m **Ans.**

Hence, distance between the two climbers = 24 m **Ans.**

Q. 23. An aeroplane at an altitude of 900 m finds that two ships are sailing towards it in the same direction. The angles of depression of the ships as observed from the plane are 60° and 30° respectively. Find the distance between the ships.

Sol. C and D are two ships sailing in the same direction towards the aeroplane A.



AB is the altitude of the aeroplane and equal to 900 m. Angle of depression from the aeroplane to the ships are 60° and 30° respectively.

Now, $\tan 60° = \frac{AB}{DB} = \frac{900}{DB}$

$\Rightarrow \sqrt{3} = \dfrac{900}{DB}$

$\Rightarrow DB = \dfrac{900}{\sqrt{3}} = \dfrac{900 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$

$\Rightarrow AB = \dfrac{900\sqrt{3}}{3} = 300\sqrt{3}$

$= 300\ (1{\cdot}732) = 519{\cdot}6$ m

and $\tan 30^\circ = \dfrac{AB}{CB} = \dfrac{900}{CB}$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{900}{CB}$

$\Rightarrow CB = 900\sqrt{3} = 900\ (1{\cdot}732)$

$\Rightarrow CB = 1558{\cdot}8$ m

$\therefore CD = CB - DB = 1558{\cdot}8 - 519{\cdot}6$

$= 1039{\cdot}2$ m **Ans.**

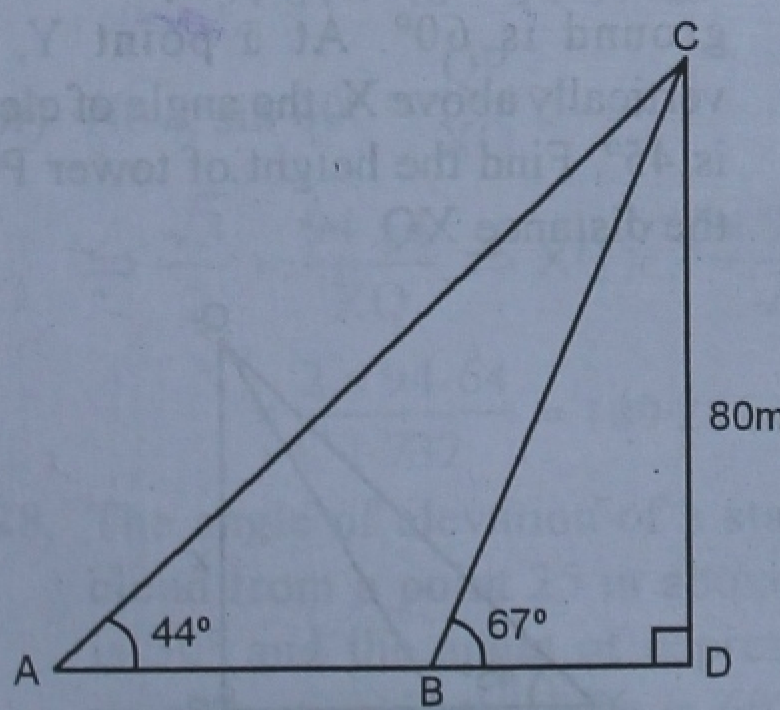
Q. 24. From the top of a cliff 80 m high, the angles of depression of two boats are 44° and 67° respectively. Find the distance between the boats, when the boats are :

(*i*) on the same side of the cliff,

(*ii*) on the opposite sides of the cliff.

Sol. Let A and B are two boats, and CD is the cliff.

$\therefore$ CD = 80 m.



Angle of depression of the two boats are 44° and 67° respectively

(*i*) When the boats are on the same side of the cliff, then

$\tan 67^\circ = \dfrac{CD}{BD} = \dfrac{80}{BD}$

$\Rightarrow 2{\cdot}3559 = \dfrac{80}{BD}$

$\Rightarrow BD = \dfrac{80}{2{\cdot}3559} = 33{\cdot}96$ m

and $\tan 44^\circ = \dfrac{CD}{AD} = \dfrac{80}{AD}$

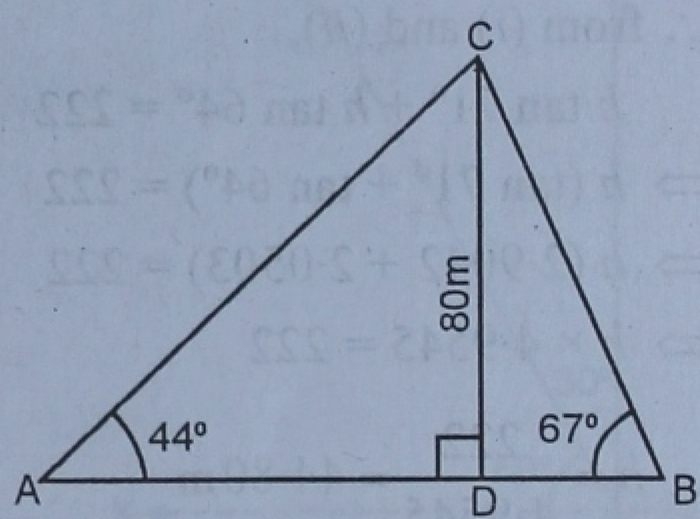
$\Rightarrow 0.9657 = \dfrac{80}{AD}$

$\Rightarrow AD = \dfrac{80}{0{\cdot}9657} = 82{\cdot}84$ m

$\therefore AB = AD - BD$

$= 82{\cdot}84 - 33{\cdot}96 = 48{\cdot}88$ m

(*ii*) When the boats are in the opposite directions, then



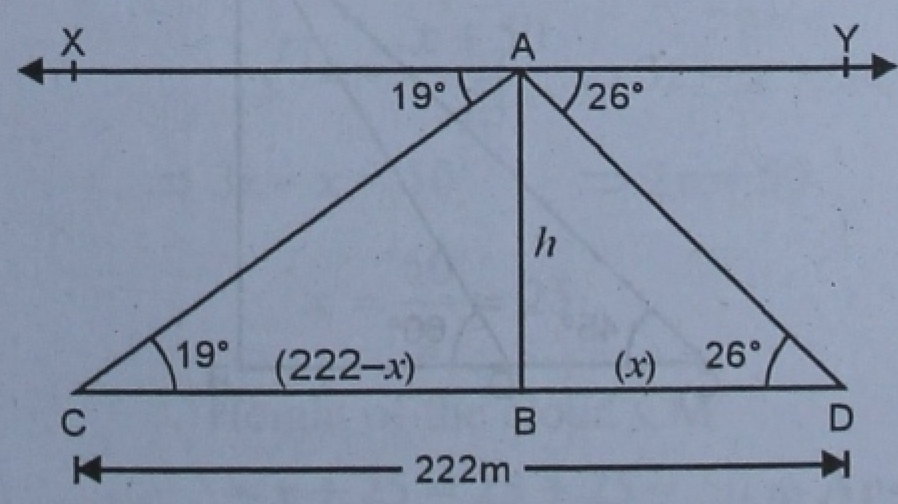
$AB = AD + BD = 82{\cdot}84 + 33{\cdot}96$

$= 116{\cdot}8$ m **Ans.**

Q. 25. A boy standing on a vertical cliff in a jungle observes two rest houses in line with him on opposite sides deep in the jungle below. If their angles of depression are 19° and 26° and the distance between them is 222 m, find the height of the cliff.

Sol. Let AB be the cliff and C and D are two rest houses such that CD = 222 m

Angles of depression are 19° and 26° respectively.



Let height of cliff AB = h and let BD = x, then BC = $(222 - x)$m.

Now in right Δ ABC,

$$\cot\theta = \frac{BC}{AB} \Rightarrow \cot 19^\circ = \frac{BC}{h}$$

$\Rightarrow$ BC = $h \cot 19^\circ = h \cot (90^\circ - 71^\circ)$

$= h \tan 71^\circ$ ...(*i*)

Again in right Δ ABD,

$$\cot 26^\circ = \frac{BD}{h} \Rightarrow BD = h \cot 26^\circ$$

$\Rightarrow$ BD = $h \cot (90^\circ - 64^\circ)$

$= h \tan 64^\circ$ ...(*ii*)

But, BC + BD = CD = 222 m

∴ from (*i*) and (*ii*),

$h \tan 71^\circ + h \tan 64^\circ = 222$

$\Rightarrow h (\tan 71^\circ + \tan 64^\circ) = 222$

$\Rightarrow h (2{\cdot}9042 + 2{\cdot}0503) = 222$

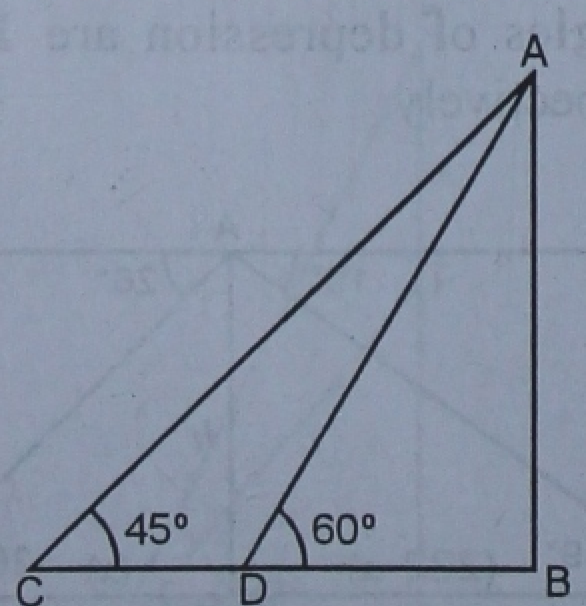
$\Rightarrow h \times 4{\cdot}9545 = 222$

$$\therefore h = \frac{222}{4{\cdot}9545} = 44{\cdot}80 \text{ m}$$

Hence, height of the cliff = 44·80 m **Ans.**

Q. 26. A man in a boat rowing away from a light house 150 m high, takes 1·5 minutes to change the angle of elevation of the top of the lighthouse from 60° to 45°. Find the speed of the boat.

Sol. C and D are the two positions of the boat and AB is the light house. The angles of elevation of the top of the lighthouse are 60° and 45° respectively.



Now, $\tan 60^\circ = \frac{AB}{DB} = \frac{150}{DB}$

$$\Rightarrow \sqrt{3} = \frac{150}{DB}$$

$$\Rightarrow DB = \frac{150}{\sqrt{3}} = \frac{150\sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{150\sqrt{3}}{3}$$

$DB = 50\sqrt{3} = 50\,(1{\cdot}732) = 86{\cdot}6$ m.

and $\tan 45^\circ = \frac{AB}{CB} = \frac{150}{CB}$

$$\Rightarrow 1 = \frac{150}{CB} \Rightarrow CB = 150 \text{ m.}$$

∴ Distance between C and D

$= 150 - 86{\cdot}6 = 63{\cdot}4$ m

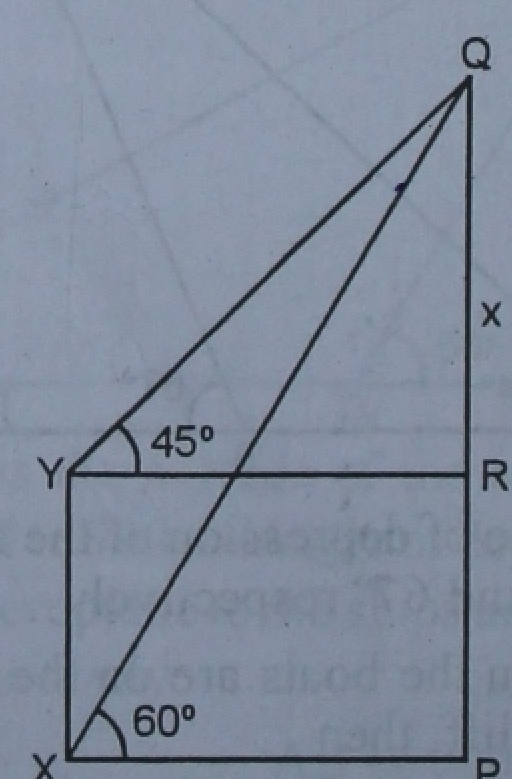
Now, distance of 63·4 m is covered in 1·5 minutes.

$$\therefore \text{Speed of the boat} = \frac{63{\cdot}4}{1{\cdot}5} \times 60$$

$$= \frac{634 \times 60}{15} = 2536 \text{ m/hr.}$$

$$= \frac{2536}{60 \times 60} \text{ m/sec} = 0{\cdot}7 \text{ m/sec.}$$ **Ans.**

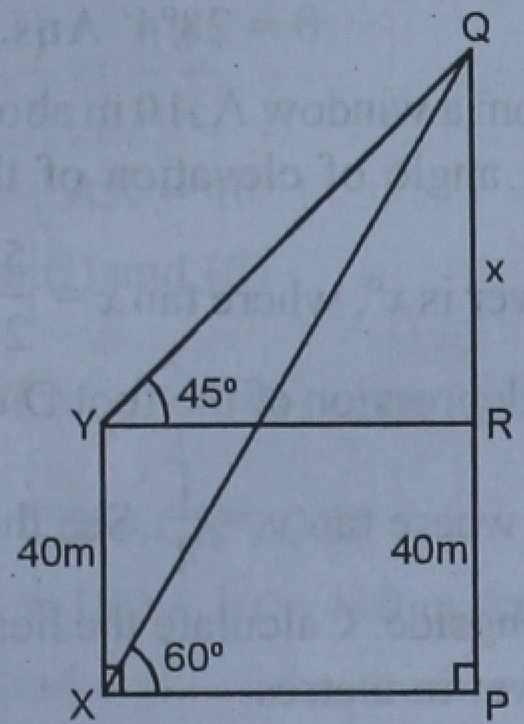
Q. 27. The angle of elevation of the top Q of a vertical tower PQ from a point X on the ground is 60°. At a point Y, 40 m vertically above X, the angle of elevation is 45°. Find the height of tower PQ and the distance XQ.



Sol. $\tan 45^\circ = \frac{QR}{YR} = \frac{x}{YR}$

$\Rightarrow \quad 1 = \frac{x}{YR} \quad \Rightarrow YR = x$

$XP = YR = x.$



Now, $\tan 60^\circ = \frac{QP}{XP} = \frac{x+40}{x}$

$\Rightarrow \sqrt{3} = \frac{x+40}{x} \quad \Rightarrow x\sqrt{3} = x + 40$

$\Rightarrow x\sqrt{3} - x = 40 \Rightarrow 1{\cdot}732x - x = 40$

$\Rightarrow \quad 0{\cdot}732x = 40 \Rightarrow x = \frac{40}{0{\cdot}732} = 54{\cdot}64$

$\therefore$ Height of tower $PQ = x + 40$

$= 54{\cdot}64 + 40 = 94{\cdot}64$ m.

(*ii*) Now, $\sin 60^\circ = \frac{QP}{XQ}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{94{\cdot}64}{XQ} \Rightarrow XQ = \frac{2 \times 94{\cdot}64}{\sqrt{3}}$

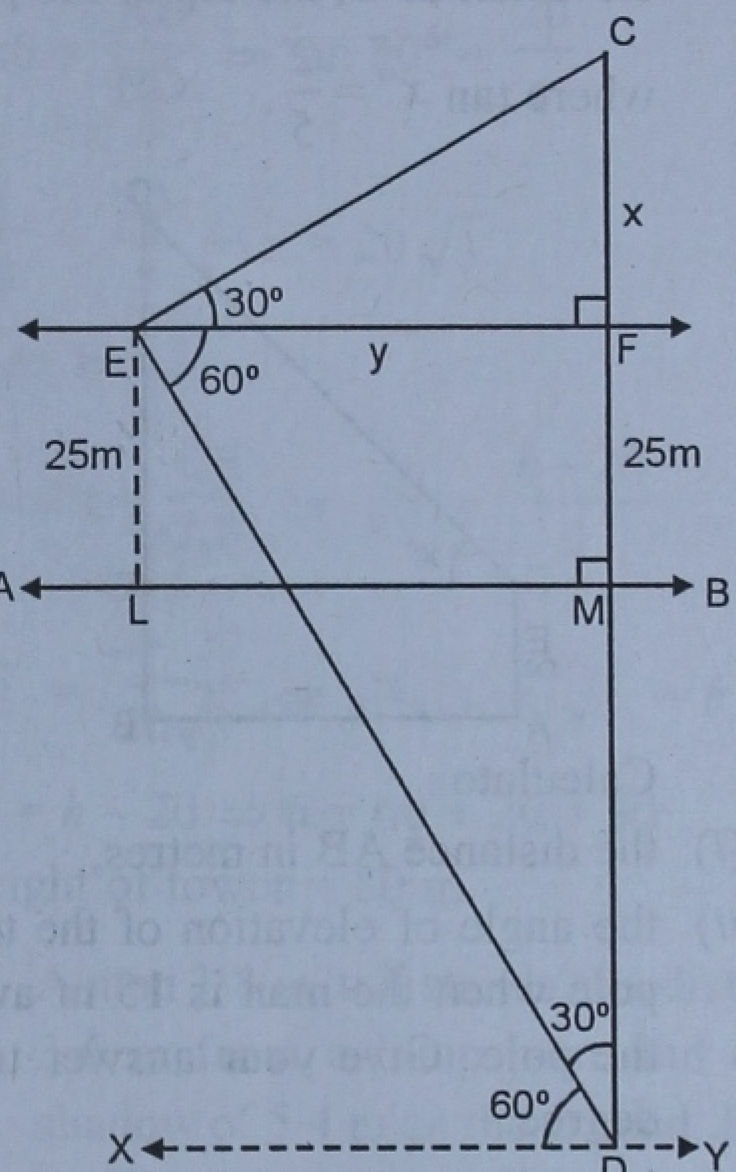
$= \frac{2 \times 94{\cdot}64}{1{\cdot}732} = 109{\cdot}28$ m **Ans.**

Q. 28. The angle of elevation of a stationary cloud from a point 25 m above a lake is 30° and the angle of depression of its reflection in the lake is 60°. What is the height of the cloud above the lake-level?

Sol. Let AB be the level of the lake and EF ∥ AB at a distance of 25 m. C is the cloud and BD is its reflection in the lake. From E, the angle of elevation of the cloud C is 30° and angle of depression of the reflection is 60°. Let CF = x and EF = y.

$\therefore \quad MD = CM = x + 25$

The height of the cloud = $CM = x + 25$ m



Now, in right ΔCEF,

$\tan 30^\circ = \frac{x}{y} \Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{y}$

$\Rightarrow \quad y = \sqrt{3}x$ ...(*i*)

Again, in right ΔEFD,

$\tan 60^\circ = \frac{FD}{EF} = \frac{25+25+x}{y} = \frac{x+50}{y}$

$\Rightarrow \sqrt{3} = \frac{x+50}{y} \Rightarrow y = \frac{x+50}{\sqrt{3}}$ ...(*ii*)

From (*i*) and (*ii*),

$\sqrt{3}x = \frac{x+50}{\sqrt{3}} \Rightarrow 3x = x + 50$

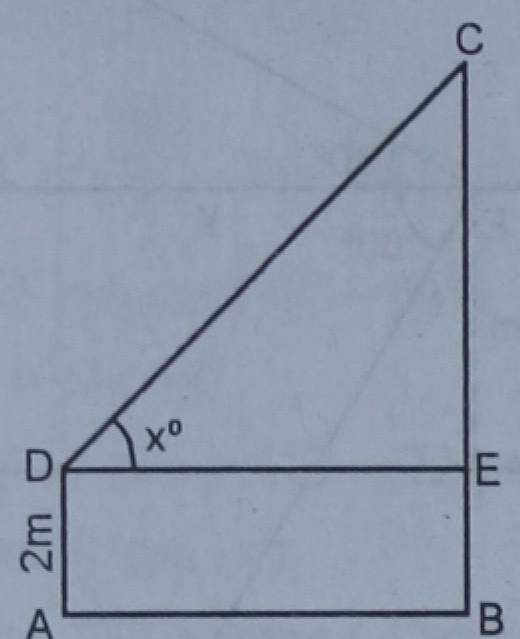
$\Rightarrow 3x - x = 50 \quad \Rightarrow 2x = 50$

$\therefore \quad x = \frac{50}{2} = 25$

$\therefore$ Height of the cloud CM

$= x + 25 = 25 + 25 = 50$ m **Ans.**

Q. 29. In the adjoining figure, a man stands on the ground at a point A, which is on the same horizontal plane as B, the foot of the vertical pole BC. The height of the pole is 10 m. The man's eye is 2 m above the ground. He observes the angle of elevation of C, the top of the pole as x^o, where $\tan x^o = \frac{2}{5}$.



Calculate :

(*i*) the distance AB in metres,

(*ii*) the angle of elevation of the top of the pole when the man is 15 m away from the pole. Give your answer to nearest degree.

Sol. Let BC be the pole and D is the man's eye 2 m above the ground. x is the angle of elevation of C. The top of the pole and

$\tan x^o = \frac{2}{5}$

BC = 10 m, AD = 2 m

AB = DE and BE = AD = 2 m

$\therefore$ EC = BC – BE = 10 – 2 = 8 m

(*i*) Now AB = DE,

then, $\tan x^o = \frac{CE}{DE} \Rightarrow \frac{2}{5} = \frac{8}{DE}$

$\Rightarrow DE = \frac{5 \times 8}{2} = 20$ m

$\therefore$ AB = 20 m.

(*ii*) If AB = 15 m, and CE = 8 m and θ be the angle of elevation of C at D, then

$\tan \theta = \frac{CE}{DE} = \frac{8}{AB} = \frac{8}{15} = 0{\cdot}5333$

From the table, we find that

$\tan 28^o = 0{\cdot}5317$

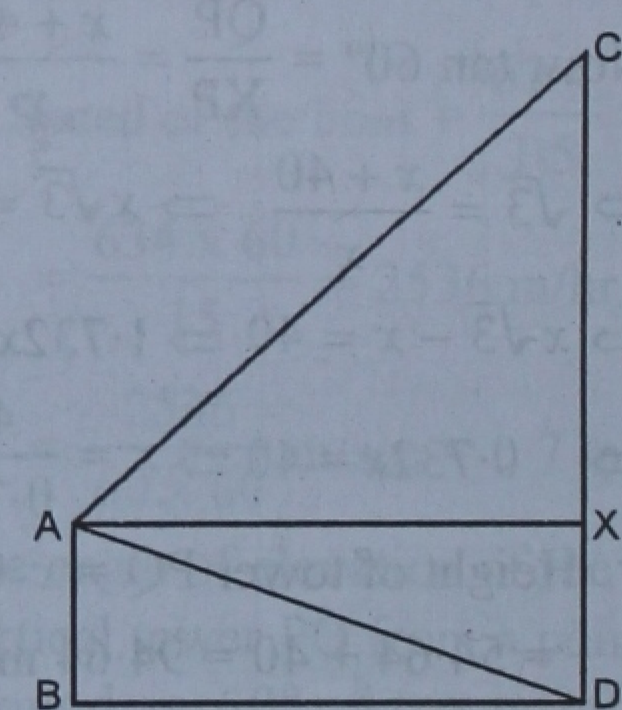
Mean difference 64′ = 15

(Adding the mean differences)

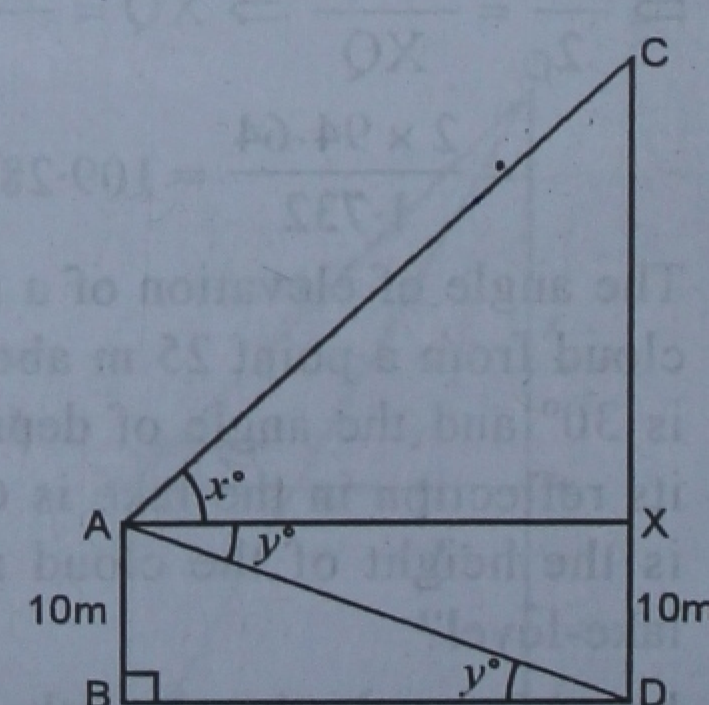
$\therefore \tan 28^o\,4' = {\cdot}5332 = \tan \theta$

$\theta = 28^o 4'$ **Ans.**

Q. 30. From a window A, 10 m above the ground, the angle of elevation of the top C of a tower is x^o, where $\tan x = \frac{5}{2}$ and the angle of depression of the foot D of the tower is y^o, where $\tan y = \frac{1}{4}$. See the figure given alongside. Calculate the height CD of the tower in metres. **(2000)**



Sol. In right ΔAXC, CD is tower, AB is window



$\tan x^o = \frac{CX}{AX} \Rightarrow \frac{5}{2}$

$\Rightarrow \quad CX = \frac{5}{2} AX$

Similarly, in ΔABD

$$\tan y^o = \frac{AB}{BD} = \frac{AB}{AX}$$

$$\Rightarrow \frac{1}{4} = \frac{10}{AX}$$

$\Rightarrow \quad AX = 40 \quad .... (ii)$

From (*i*) and (*ii*)

$$CX = \frac{5}{2} \times 40 = 100 \text{ m}$$

$\therefore \quad CD = CX + XD$

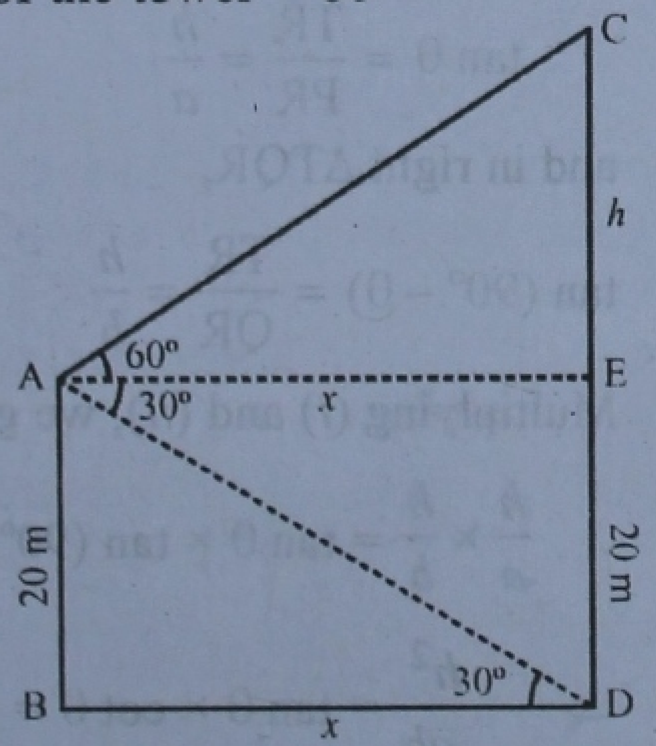
$= 100 + 10 = 110$ m **Ans.**

Hence height of the tower = 110 m.

Q. 31. A vertical pole and a vertical tower are on the same level ground. From the top of the pole the angle of elevation of the top of the tower is 60° and the angle of depression of the foot of the tower is 30°. Find the height of the tower if the height of the pole is 20 m.

Sol. Let AB be the vertical pole and CD be the tower on the same level ground

Angle of elevation from the top of the pole to top of the tower = 60°



Angle of depression of the foot of the tower is 30°

Height of Pole AB = 20 m

Let height of tower = *h* m and BD = *x* m

Draw AE ∥ BD, then

ED = AB = 20 m and CE = $h - 20$ and AE = BD

In right angle ΔABD,

$$\tan\theta = \frac{AB}{BD} \Rightarrow \tan 30^o = \frac{20}{x}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{20}{x} \Rightarrow x = 20\sqrt{3}$$

In ACE

$$\tan 60^o = \frac{CE}{AE} \Rightarrow \sqrt{3} = \frac{h-20}{x}$$

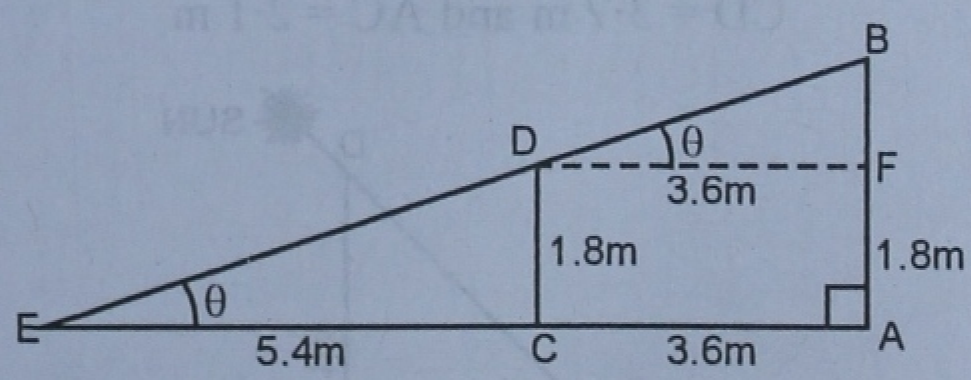
$$\Rightarrow \sqrt{3} = \frac{h-20}{20\sqrt{3}} \Rightarrow 20\sqrt{3} \times \sqrt{3} = h - 20$$

$$\Rightarrow 60 = h - 20 \Rightarrow h = 60 + 20 = 80$$

$\therefore$ Height of tower = 80 m

Q. 32. A man 1·8 m tall stands at a distance of 3·6 m from a lamp post and casts a shadow of 5·4 m on the ground. Find the height of the lamp post.

Sol. Let AB be the lamp post and CD be the man and let CE be the shadow of CD. Let θ be the angle of elevation.



Now, CD = 1·8 m

CE = 5·4 m

and CA = 3·6 m

DF = CA = 3·6 m

and AF = CD = 1·8 m

In ΔCDE,

$$\tan\theta = \frac{DC}{EC} = \frac{1\cdot8}{5\cdot4} = \frac{1}{3}$$

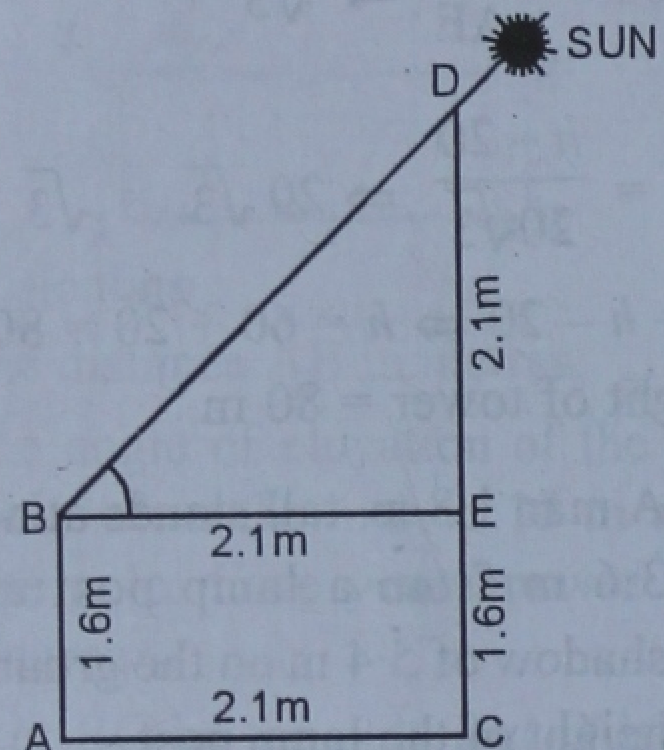
In ΔBDF, $\tan\theta = \frac{BF}{DF} \Rightarrow \frac{1}{3} = \frac{BF}{3 \cdot 6}$

$\Rightarrow BF = \frac{3 \cdot 6}{3} = 1 \cdot 2$ m

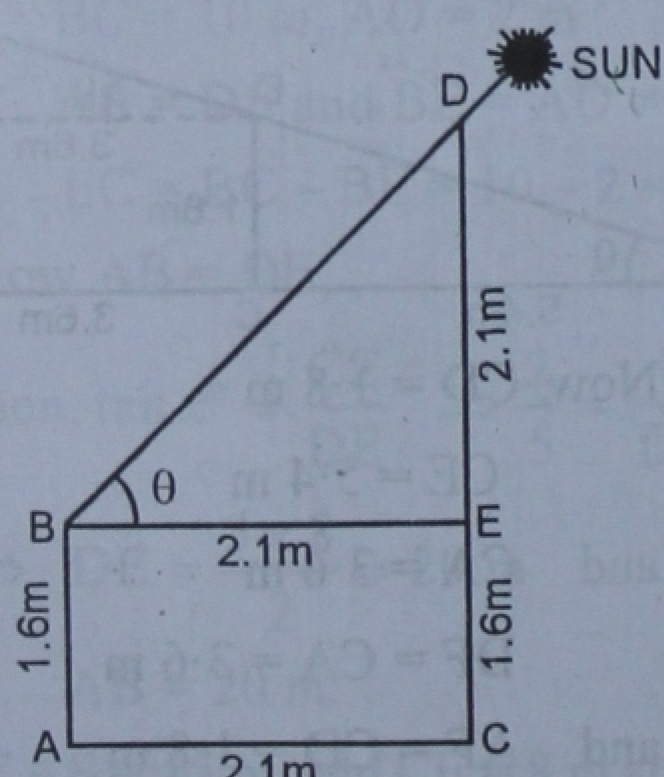
$\therefore AB = AF + BF$

$= 1 \cdot 8 + 1 \cdot 2 = 3$ m **Ans.**

Q. 33. A boy 1·6 m tall can just see the sun over a wall 3·7 m high, which is 2·1 m away from him. Find the angle of elevation of the sun.



Sol. Let AB be the boy and CD be the wall AB = 1·6 m, CD = 3·7 m and AC = 2·1 m



BE ∥ AC is drawn, then

EC = AB = 1·6 m

and ED = CD – CE = (3·7 – 1·6) m

= 2·1 m

BE = AC = 2·1 m

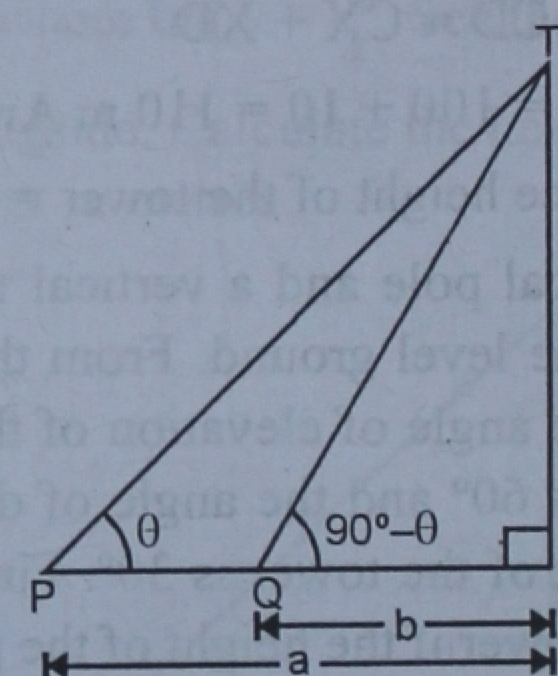
Now in ΔBED,

$\tan\theta = \frac{DE}{BE} = \frac{2 \cdot 1}{2 \cdot 1} = 1 = \tan 45^\circ$

$\therefore \quad \theta = 45^\circ$ **Ans.**

Q. 34. The angles of elevation of the top of a tower from two points on the ground at distances a metres and b metres from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is $\sqrt{ab}$ metres.

Sol. Let the angle of elevation from the top of the tower TR be θ and (90° – θ) with P and Q respectively.



Then, PR = a, QR = b

Let TR = h

Now, in right ΔTPR,

$\tan\theta = \frac{TR}{PR} = \frac{h}{a}$ ...(*i*)

and in right ΔTQR,

$\tan(90^\circ - \theta) = \frac{TR}{QR} = \frac{h}{b}$ ...(*ii*)

Multiplying (*i*) and (*ii*), we get

$\frac{h}{a} \times \frac{h}{b} = \tan\theta \times \tan(90^\circ - \theta)$

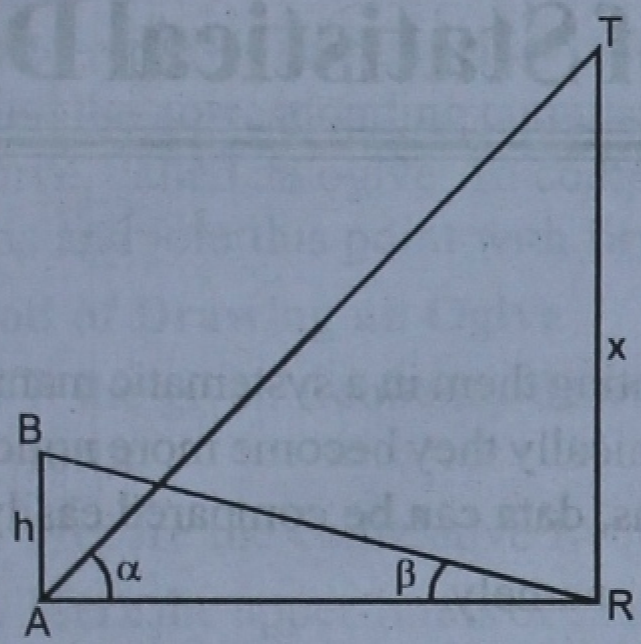
$\Rightarrow \frac{h^2}{ab} = \tan\theta \times \cot\theta = 1$

$\therefore h^2 = ab \Rightarrow h = \sqrt{ab}$

Hence, height of the tower = $\sqrt{ab}$ m

Hence proved.

Q. 35. A tower subtends an angle α at a point on the same level as the foot of the tower and at a second point h metres above the first, the depression of the foot of the tower is β. Show that the height of the tower is ($h \tan \alpha \cot \beta$).



Sol. Let TR be the tower and A is a point such that angle of elevation of T with A is α. B is another point h m above A

i.e. AB = h m and angle of depression of the foot of the tower is β.

Let TR = x, Then

In ΔATR, $\tan \alpha = \frac{TR}{AR} = \frac{x}{AR}$

$$\therefore \quad AR = \frac{x}{\tan \alpha} \quad ...(i)$$

In ΔABR, $\tan \beta = \frac{AB}{AR} = \frac{h}{AR}$

$$\Rightarrow \quad AR = \frac{h}{\tan \beta} = h \cot \beta \quad ...(ii)$$

From (i) and (ii),

$$\frac{x}{\tan \alpha} = h \cot \beta$$

$$\Rightarrow \quad x = h \tan \alpha \cot \beta$$

Hence, height of tower = $h \tan \alpha \cot \beta$

Hence proved.