Unit 6
Trigonometry

Chapter 25
Trigonometrical Identities

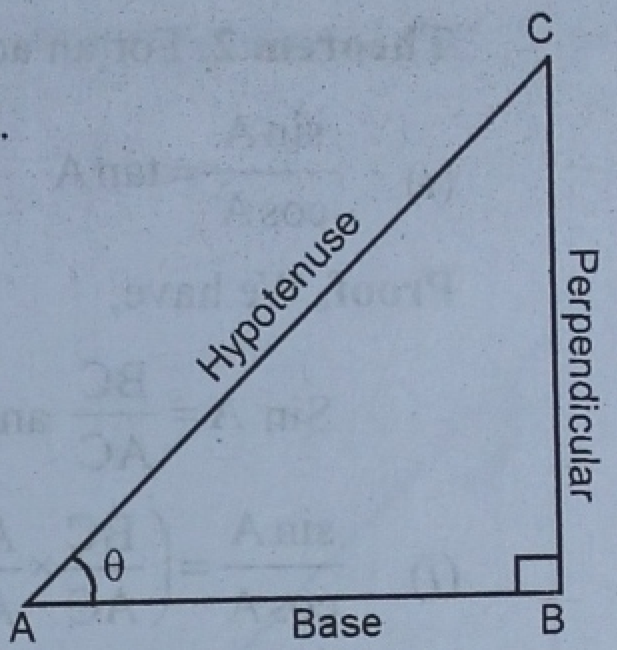
POINTS TO REMEMBER

1. Trigonometrical Ratios or T-Ratios.

Let A be an acute angle of a right ΔABC in which $\angle B = 90^\circ$.

Then, Base = AB, Perpendicular = BC and Hypotenuse = AC.

For acute $\angle A = \theta$, we define



(*i*) sine $\theta = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}} \Rightarrow \sin\theta = \dfrac{BC}{AC}$.

(*ii*) cosine $\theta = \dfrac{\text{Base}}{\text{Hypotenuse}} \Rightarrow \cos\theta = \dfrac{AB}{AC}$.

(*iii*) tangent $\theta = \dfrac{\text{Perpendicular}}{\text{Base}} \Rightarrow \tan\theta = \dfrac{BC}{AB}$.

(*iv*) cotangent $\theta = \dfrac{\text{Base}}{\text{Perpendicular}} \Rightarrow \cot\theta = \dfrac{AB}{BC}$.

(*v*) secant $\theta = \dfrac{\text{Hypotenuse}}{\text{Base}} \Rightarrow \sec\theta = \dfrac{AC}{AB}$.

(*vi*) cosecant $\theta = \dfrac{\text{Hypotenuse}}{\text{Perpendicular}} \Rightarrow \text{cosec}\,\theta = \dfrac{AC}{BC}$.

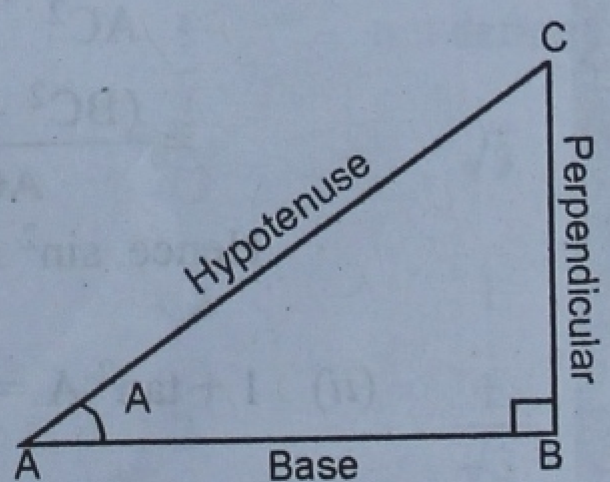
2. (A) Relations Between T-Ratios (Theorems)

Theorem 1. For an acute angle A, prove that :

(*i*) $\text{cosec } A = \dfrac{1}{\sin A}$ (*ii*) $\sec A = \dfrac{1}{\cos A}$

(*iii*) $\cot A = \dfrac{1}{\tan A}$

Proof. We have,



(*i*) $\text{cosec } A = \dfrac{AC}{BC}$ and $\sin A = \dfrac{BC}{AC}$.

$\therefore$ $\text{cosec } A = \dfrac{1}{\sin A}$.

(*ii*) $\sec A = \frac{AC}{AB}$ and $\cos A = \frac{AB}{AC}$.

$\therefore \quad \sec A = \frac{1}{\cos A}$.

(*iii*) $\cot A = \frac{AB}{BC}$ and $\tan A = \frac{BC}{AB}$.

$\therefore \quad \cot A = \frac{1}{\tan A}$

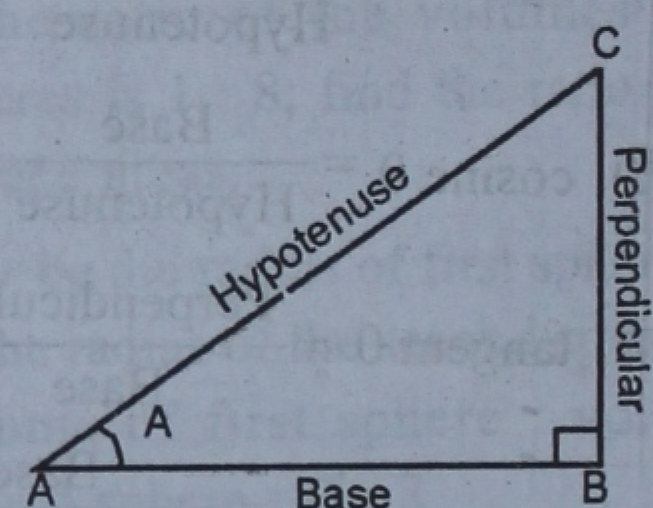
(B) Quotient Relations

Theorem 2. For an acute angle A, prove that :

(*i*) $\frac{\sin A}{\cos A} = \tan A$ (*ii*) $\frac{\cos A}{\sin A} = \cot A$ (*iii*) $\tan A.\cot A = 1$

Proof. We have,

$$\text{Sin } A = \frac{BC}{AC} \text{ and } \cos A = \frac{AB}{AC}.$$



(*i*) $\frac{\sin A}{\cos A} = \left(\frac{BC}{AC} \times \frac{AC}{AB}\right) = \frac{BC}{AB} = \tan A.$

(*ii*) $\frac{\cos A}{\sin A} = \left(\frac{AB}{AC} \times \frac{AC}{BC}\right) = \frac{AB}{BC} = \cot A.$

(*iii*) $\tan A.\cot A = \left(\frac{BC}{AB} \times \frac{AB}{BC}\right) = 1.$

(C) Square Relations

Theorem 3. For an acute angle A. prove that :

(*i*) $\sin^2 A + \cos^2 A = 1$ (*ii*) $1 + \tan^2 A = \sec^2 A$ (*iii*) $1 + \cot^2 A = \text{cosec}^2 A$

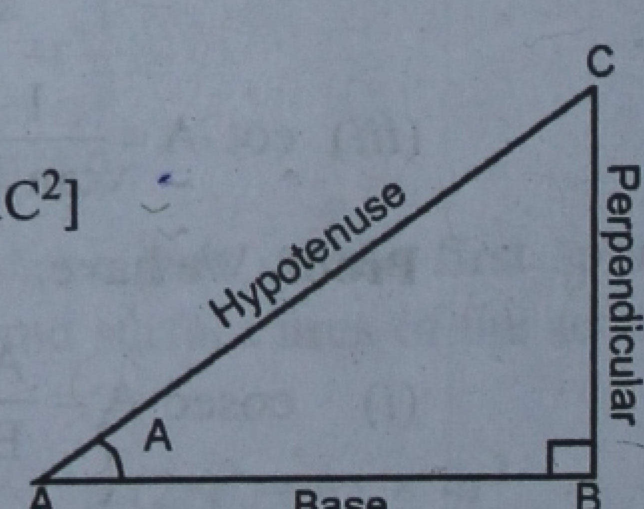
Proof. We have,

(*i*) $\sin^2 A + \cos^2 A = \left(\frac{BC}{AC}\right)^2 + \left(\frac{AB}{AC}\right)^2$

$$= \frac{BC^2}{AC^2} + \frac{AB^2}{AC^2}$$

$$= \frac{(BC^2 + AB^2)}{AC^2} = \frac{AC^2}{AC^2} = 1 \qquad [\because BC^2 + AB^2 = AC^2]$$

Hence, $\sin^2 A + \cos^2 A = 1$.



(*ii*) $1 + \tan^2 A = 1 + \left(\frac{BC}{AB}\right)^2 = 1 + \frac{BC^2}{AB^2}$

$$= \frac{(AB^2 + BC^2)}{AB^2} = \frac{AC^2}{AB^2} \qquad [\because AB^2 + BC^2 = AC^2]$$

$$= \left(\frac{AC}{AB}\right)^2 = \sec^2 A.$$

Hence, $1 + \tan^2 A = \sec^2 A$.

(*iii*) $1 + \cot^2 A = 1 + \left(\frac{AB}{BC}\right)^2 = 1 + \frac{AB^2}{BC^2}$

$$= \frac{(BC^2 + AB^2)}{BC^2} = \frac{AC^2}{BC^2} = \left(\frac{AC}{BC}\right)^2 = \text{cosec}^2 A$$

Hence, $1 + \cot^2 A = \text{cosec}^2 A$.

3. Trigonometrical Ratios of Complementary Angles

Complementary Angles

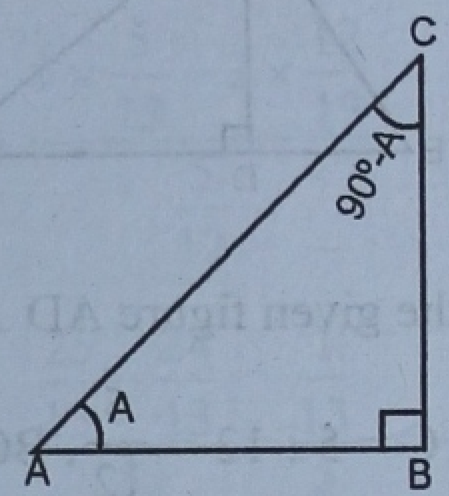
Two angles are said to be complementary, if the sum of their measures is 90°. Thus, A and (90° – A) are complementary angles.

4. T-Ratios of Complementary Angles

Consider a right triangle ABC in which ∠BAC = 90°.

∴ ∠ACB = (90° – A).

Thus, we have :



(*i*) $\sin(90° - A) = \frac{AB}{AC} = \cos A$;

(*ii*) $\cos(90° - A) = \frac{BC}{AC} = \sin A$;

(*iii*) $\tan(90° - A) = \frac{AB}{BC} = \cot A$;

(*iv*) $\text{cosec}(90° - A) \frac{1}{\sin(90° - A)} = \frac{1}{\cos A} = \sec A$;

(*v*) $\sec(90° - A) = \frac{1}{\cos(90° - A)} = \frac{1}{\sin A} = \text{cosec } A$;

(*vi*) $\cot(90° - A) = \frac{1}{\tan(90° - A)} = \frac{1}{\cot A} = \tan A$.

USING TRIGONOMETRIC TABLES

Table Showing T-Ratios of 0°, 30°, 45°, 60° and 90°

A	sin A	cos A	tan A	cosec A	sec A	cot A
0°	0	1	0	not defined	1	not defined
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$	$\sqrt{3}$
45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2	$\frac{1}{\sqrt{3}}$
90°	1	0	not defined	1	not defined	0

READING OF TRIGONOMETRIC TABLES

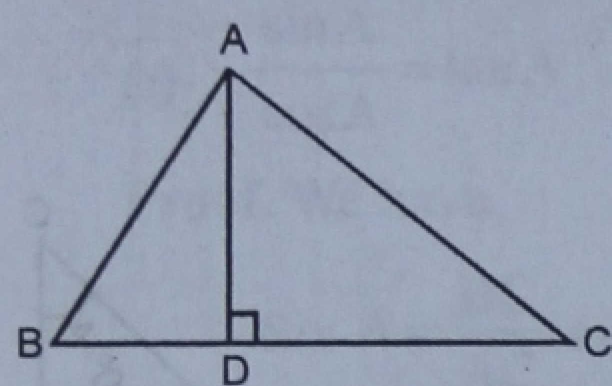
Relation Between Degree and Minutes :

$$1^\circ = 60' \Leftrightarrow 1' = \left(\frac{1}{60}\right)^\circ \Leftrightarrow 6' = \left(\frac{6}{60}\right)^\circ = \left(\frac{1}{10}\right)^\circ = (0\cdot1)^\circ.$$

EXERCISE 25(A)

Q. 1. In the given figure, AD is perpendicular to BC, $\tan B = \frac{3}{4}$ and $\tan C = \frac{5}{12}$. BC = 56 cm.

Calculate the length of AD. **(1995)**

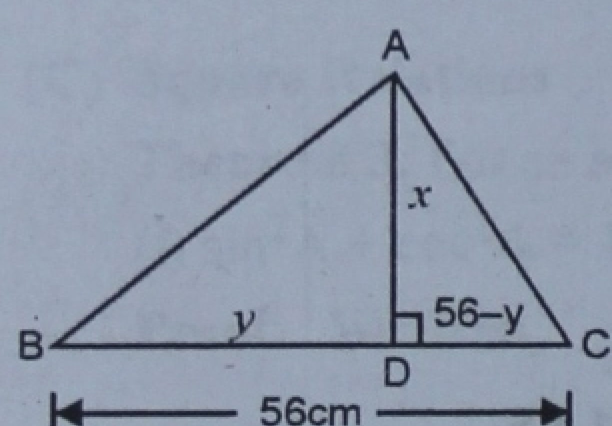


Sol. In the given figure AD ⊥ BC, $\tan B = \frac{3}{4}$

$\tan C = 5 : 12 = \frac{5}{12}$, BC = 56 cm.

Let AD = x and BD = y

Then, DC = $(56 - y)$ m



In right ΔABD,

$$\tan B = \frac{AD}{BD}$$

$$\Rightarrow \frac{3}{4} = \frac{x}{y} \Rightarrow x = \frac{3}{4}y \quad ...(i)$$

Again, in right ΔACD,

$$\tan C = \frac{AD}{DC} \Rightarrow \frac{5}{12} = \frac{x}{56 - y}$$

$$\Rightarrow x = \frac{5(56 - y)}{12} \quad ...(ii)$$

$$\frac{3}{4}y = \frac{5(56 - y)}{12}$$

$$\Rightarrow 36y = 20(56 - y)$$

$$\Rightarrow 36y = 1120 - 20y$$

$$\Rightarrow 36y + 20y = 1120$$

$$\Rightarrow 56y = 1120$$

$$\Rightarrow y = \frac{1120}{56} = 20 \text{ cm}$$

Substituting the value y in (i)

$$x = \frac{3}{4} \times 20 \text{ cm}$$

$$x = 15 \text{ cm}$$

Hence, AD = 15 cm. **Ans.**

Q. 2. If $2\cos\theta = \frac{2}{5}$, find $\sin\theta$ without using tables.

Sol. $2\cos\theta = \frac{2}{5} \Rightarrow \cos\theta = \frac{1}{5}$

$$\therefore \sin\theta = \sqrt{1 - \cos^2\theta}$$

$$= \sqrt{1 - \left(\frac{1}{5}\right)^2} = \sqrt{1 - \frac{1}{25}}$$

$$= \sqrt{\frac{25 - 1}{25}} = \sqrt{\frac{24}{25}} = \frac{\sqrt{24}}{\sqrt{25}}$$

$$= \frac{\sqrt{4 \times 6}}{\sqrt{25}} = \frac{2\sqrt{6}}{5} \quad \textbf{Ans.}$$

Q. 3. If $\sin x = \frac{12}{13}$, find the value of :

(i) $\tan x$ (ii) $\sec x$

(iii) $(\operatorname{cosec}^2 x - \cot^2 x)$

Sol. $\sin x = \frac{12}{13}$

$\therefore \cos x = \sqrt{1-\sin^2 x} = \sqrt{1-\left(\frac{12}{13}\right)^2}$

$= \sqrt{1-\frac{144}{169}} = \sqrt{\frac{169-144}{169}}$

$= \sqrt{\frac{25}{169}} = \frac{5}{13}$

(*i*) $\tan x = \frac{\sin x}{\cos x} = \frac{\frac{12}{13}}{\frac{5}{13}} = \frac{12}{13} \times \frac{13}{5} = \frac{12}{5}$

$\cot x = \frac{1}{\tan x} = \frac{5}{12}$

$\sec x = \frac{1}{\cos x} = \frac{13}{5}$

and $\operatorname{cosec} x = \frac{1}{\sin x} = \frac{13}{12}$

Now (*i*) $\tan x = \frac{12}{5}$

(*ii*) $\sec x = \frac{13}{5}$

(*iii*) $\operatorname{cosec}^2 x - \cot^2 x = \left(\frac{13}{12}\right)^2 - \left(\frac{5}{12}\right)^2$

$= \frac{169}{144} - \frac{25}{144}$

$= \frac{169-25}{144} = \frac{144}{144} = 1$ **Ans.**

Q. 4. Given A is an acute angle and 13 sin A = 5, evaluate without using tables :

$\frac{(5 \sin A - 2 \cos A)}{\tan A}$ **(1994)**

Sol. 13 sin A = 5

$\Rightarrow \sin A = \frac{5}{13}$

$\therefore \cos A = \sqrt{1-\sin^2 A} = \sqrt{1-\left(\frac{5}{13}\right)^2}$

$= \sqrt{1-\frac{25}{169}} = \sqrt{\frac{169-25}{169}} = \sqrt{\frac{144}{169}}$

$= \frac{12}{13}$

$\tan A = \frac{\sin A}{\cos A} = \frac{\frac{5}{13}}{\frac{12}{13}} = \frac{5}{13} \times \frac{13}{12} = \frac{5}{12}$

Now, $\frac{5 \sin A - 2 \cos A}{\tan A}$

$= \frac{5 \times \frac{5}{13} - 2 \times \frac{12}{13}}{\frac{5}{12}}$

$= \frac{\frac{25}{13} - \frac{24}{13}}{\frac{5}{12}} = \frac{\frac{1}{13}}{\frac{5}{12}} = \frac{1}{13} \times \frac{12}{5}$

$= \frac{12}{65}$ **Ans.**

Q. 5. If θ is an acute angle such as $\sin \theta = \frac{\sqrt{3}}{2}$ then find the value of (cosec θ + cot θ).

Sol. $\sin \theta = \frac{\sqrt{3}}{2}$

$\therefore \cos \theta = \sqrt{1-\sin^2 \theta} = \sqrt{1-\left(\frac{\sqrt{3}}{2}\right)^2}$

$= \sqrt{1-\frac{3}{4}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

$\operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{2}{\sqrt{3}}$

and $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

$$= \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{2} \times \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

Now, $\operatorname{cosec}\theta + \cot\theta = \dfrac{2}{\sqrt{3}} + \dfrac{1}{\sqrt{3}} = \dfrac{3}{\sqrt{3}}$

$$= \frac{3 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{3\sqrt{3}}{3} = \sqrt{3} \text{ Ans.}$$

Q. 6. If $5\cos\theta = 3$, evaluate :

$$\frac{3\sin\theta + 2\cos\theta}{3\sin\theta - 2\cos\theta}.$$

Sol. $5\cos\theta = 3 \Rightarrow \cos\theta = \dfrac{3}{5}$

$\therefore \sin\theta = \sqrt{1 - \cos^2\theta}$

$$= \sqrt{1 - \left(\frac{3}{5}\right)^2} = \sqrt{1 - \frac{9}{25}}$$

$$= \sqrt{\frac{25 - 9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

Now, $\dfrac{3\sin\theta + 2\cos\theta}{3\sin\theta - 2\cos\theta} = \dfrac{3 \times \frac{4}{5} + 2 \times \frac{3}{5}}{3 \times \frac{4}{5} - 2 \times \frac{3}{5}}$

$$= \frac{\frac{12}{5} + \frac{6}{5}}{\frac{12}{5} - \frac{6}{5}} = \frac{\frac{18}{5}}{\frac{6}{5}} = \frac{18}{5} \times \frac{5}{6}$$

$= 3$ **Ans.**

Q. 7. (*i*) $\dfrac{\sec A - 1}{\sec A + 1} = \dfrac{1 - \cos A}{1 + \cos A}$

(*ii*) $\dfrac{\operatorname{cosec}\theta + 1}{\operatorname{cose}\theta - 1} = \dfrac{1 + \sin\theta}{1 - \sin\theta}$

(*iii*) Prove that : $\dfrac{\sin\theta\ \tan\theta}{1 - \cos\theta} = 1 + \sec\theta$

Sol. (*i*) L.H.S. $= \dfrac{\sec A - 1}{\sec A + 1} = \dfrac{\frac{1}{\cos A} - 1}{\frac{1}{\cos A} + 1}$ $\left(\because \sec A = \dfrac{1}{\cos A}\right)$

$$= \frac{\frac{1 - \cos A}{\cos A}}{\frac{1 + \cos A}{\cos A}} = \frac{1 - \cos A}{\cos A} \times \frac{\cos A}{1 + \cos A}$$

$$= \frac{1 - \cos A}{1 + \cos A} = \text{R.H.S.}$$

(*ii*) L.H.S. $= \dfrac{\operatorname{cosec}\theta + 1}{\operatorname{cose}\theta - 1}$

$$\frac{\frac{1}{\sin\theta} + 1}{\frac{1}{\sin\theta} - 1} = \frac{\frac{1 + \sin\theta}{\sin\theta}}{\frac{1 - \sin\theta}{\sin\theta}}$$ $\left(\because \operatorname{cosec}\theta = \dfrac{1}{\sin\theta}\right)$

$$= \frac{1 + \sin\theta}{\sin\theta} \times \frac{\sin\theta}{1 - \sin\theta} = \frac{1 + \sin\theta}{1 - \sin\theta} = \text{R.H.S.}$$

(*iii*) L.H.S. $= \dfrac{\sin\theta\ \tan\theta}{1 - \cos\theta}$

$$= \frac{\sin\theta \frac{\sin\theta}{\cos\theta}}{1 - \cos\theta} = \frac{\sin^2\theta}{\cos\theta\ (1 - \cos\theta)}$$

$$= \frac{(1 - \cos^2\theta)}{\cos\theta\ (1 - \cos\theta)} = \frac{(1 + \cos\theta)(1 - \cos\theta)}{\cos\theta\ (1 - \cos\theta)}$$

$$= \frac{1}{\cos\theta} + \frac{\cos\theta}{\cos\theta} = \frac{1}{\cos\theta} + 1$$

$= 1 + \sec\theta$ = R.H.S. **Proved.**

Q. 8. (*i*) $(1 - \cos^2 A) \operatorname{cosec}^2 A = 1$

(*ii*) $(1 - \tan^2 A) \cos^2 A = 1$

Sol. (*i*) L.H.S. $= (1 - \cos^2 A) \operatorname{cosec}^2 A$

$= \sin^2 A \operatorname{cosec}^2 A$

$\{\because 1 - \cos^2 A = \sin^2 A\}$

$= \sin^2 A \times \dfrac{1}{\sin^2 A} = 1 \Rightarrow$ R.H.S.

(*ii*) L.H.S. $= (1 - \tan^2 A) \cos^2 A$

$= \sec^2 A \cos^2 A$

$\{\because 1 - \tan^2 A = \sec^2 A\}$

$= \dfrac{1}{\cos^2 A} \times \cos^2 A = 1 =$ R.H.S.

Q.9. (*i*) $(\sec^2 A - 1) \cot^2 A = 1$

(*ii*) $\left(\cot^2 A - \dfrac{1}{\sin^2 A}\right) = -1$

Sol. (*i*) L.H.S. $= (\sec^2 A - 1) \cot^2 A$

$= \tan^2 A \cot^2 A (\because \sec^2 A - 1 = \tan^2 A)$

$= \tan^2 A \times \dfrac{1}{\tan^2 A} = 1 =$ R.H.S.

$\left(\because \cot^2 A = \dfrac{1}{\tan^2 A}\right)$

(*ii*) L.H.S. $= \cot^2 A - \dfrac{1}{\sin^2 A}$

$= \dfrac{\cos^2 A}{\sin^2 A} - \dfrac{1}{\sin^2 A}$

$= \dfrac{\cos^2 A - 1}{\sin^2 A} = -\dfrac{1 - \cos^2 A}{\sin^2 A}$

$= -\dfrac{\sin^2 A}{\sin^2 A} = -1 =$ R.H.S.

$(\because 1 - \cos^2 A = \sin^2 A)$

Q. 10. (*i*) $(1 + \cos A)(1 - \cos A)(1 + \cot^2 A) = 1$

(*ii*) $\operatorname{cosec} A (1 + \cos A)(\operatorname{cosec} A - \cot A) = 1$

Sol. (*i*) L.H.S. $= (1 + \cos A)(1 - \cos A)(1 + \cot^2 A)$

$= (1 - \cos^2 A)(1 + \cot^2 A)$

$\{\because (a + b)(a - b) = a^2 - b^2\}$

$= (\sin^2 A)\left(1 + \dfrac{\cos^2 A}{\sin^2 A}\right)$

$= \sin^2 A \left(\dfrac{\sin^2 A + \cos^2 A}{\sin^2 A}\right)$

$= \sin^2 A + \cos^2 A = 1$

$(\because \sin^2 A + \cos^2 A = 1)$

= R.H.S.

(*ii*) L.H.S. $= \operatorname{cosec} A (1 + \cos A)(\operatorname{cosec} A - \cot A)$

$= \dfrac{1}{\sin A}(1 + \cos A)\left(\dfrac{1}{\sin A} - \dfrac{\cos A}{\sin A}\right)$

$= \dfrac{1 + \cos A}{\sin A} \times \dfrac{1 - \cos A}{\sin A}$

$= \dfrac{(1 + \cos A)(1 - \cos A)}{\sin^2 A}$

$= \dfrac{1 - \cos^2 A}{\sin^2 A} = \dfrac{\sin^2 A}{\sin^2 A} = 1 =$ R.H.S.

$\{\because 1 - \cos^2 A = \sin^2 A\}$

Q. 11. $(\sec A + \cos A)(\sec A - \cos A) = \tan^2 A + \sin^2 A$

Sol. L.H.S. $= (\sec A + \cos A)(\sec A - \cos A)$

$= (\sec^2 A - \cos^2 A)$

$(\because (a+b)(a-b) = a^2 - b^2)$

$= (1 + \tan^2 A) - (1 - \sin^2 A)$

$\{\because \sec^2 A = 1 + \tan^2 A \text{ and } \cos^2 A = 1 - \sin^2 A\}$

$= 1 + \tan^2 A - 1 + \sin^2 A$

$= \tan^2 A + \sin^2 A = \text{R.H.S.}$

Q. 12. (*i*) $(\sin A - \cos A)^2 = 1 - 2 \sin A \cos A$

(*ii*) $(1 + \cot A)^2 + (1 - \cot A)^2 = 2 \operatorname{cosec}^2 A$

Sol. (*i*) L.H.S. $= (\sin A - \cos A)^2$

$= \sin^2 A + \cos^2 A - 2 \sin A \cos A$ $\{(a-b)^2 = a^2 + b^2 - 2ab\}$

$= 1 - 2 \sin A \cos A = \text{R.H.S.}$

$(\because \sin^2 A + \cos^2 A = 1)$

(*ii*) L.H.S. $= (1 + \cot A)^2 + (1 - \cot A)^2$

$= 1 + \cot^2 A + 2 \cot A + 1 + \cot^2 A - 2 \cot A$

$= 2 + 2 \cot^2 A = 2(1 + \cot^2 A)$

$= 2 \operatorname{cosec}^2 A$

$(\because 1 + \cot^2 A = \operatorname{cosec}^2 A)$

= R.H.S.

Q.13. $\dfrac{1}{(1+\tan^2 A)} + \dfrac{1}{(1+\cot^2 A)} = 1$

Sol. $= \dfrac{1}{1+\tan^2 A} + \dfrac{1}{1+\cot^2 A}$

$= \dfrac{1}{\sec^2 A} + \dfrac{1}{\operatorname{cosec}^2 A}$

$\{\because 1 + \tan^2 A = \sec^2 A,\ 1 + \cot^2 A = \operatorname{cosec}^2 A\}$

$= \cos^2 A + \sin^2 A = 1$

$(\because \sin^2 A + \cos^2 A = 1)$

= R.H.S.

Q. 14. $\dfrac{1}{1+\sin A} + \dfrac{1}{1-\sin A} = 2 \sec^2 A$

Sol. L.H.S. $= \dfrac{1}{1+\sin A} + \dfrac{1}{1-\sin A}$

$= \dfrac{1 - \sin A + 1 + \sin A}{(1+\sin A)(1-\sin A)}$

$= \dfrac{2}{1-\sin^2 A} = \dfrac{2}{\cos^2 A}$

$(\because (a+b)(a-b) = a^2 - b^2)$

$= 2 \sec^2 A = \text{R.H.S.}$

Q. 15. $\dfrac{(1+\sin A)}{\cos A} + \dfrac{\cos A}{(1+\sin A)} = 2 \sec A$

Sol. L.H.S. $= \dfrac{(1+\sin A)}{\cos A} + \dfrac{\cos A}{(1+\sin A)}$

$= \dfrac{(1+\sin A)^2 + \cos^2 A}{\cos A\,(1+\sin A)}$

$= \dfrac{1 + \sin^2 A + 2 \sin A + \cos^2 A}{\cos A\,(1+\sin A)}$

$= \dfrac{1 + \sin^2 A + \cos^2 A + 2 \sin A}{\cos A\,(1+\sin A)}$

$= \dfrac{1 + 1 + 2 \sin A}{\cos A\,(1+\sin A)}$

$(\because \sin^2 A + \cos^2 A = 1)$

$= \dfrac{2 + 2 \sin A}{\cos A\,(1+\sin A)} = \dfrac{2(1+\sin A)}{\cos A\,(1+\sin A)}$

$= \dfrac{2}{\cos A} = 2 \sec A = \text{R.H.S.}$

Q.16. $\dfrac{\text{cosec A}}{(\text{cosec A} - 1)} + \dfrac{\text{cosec A}}{\text{cosec A} + 1} = 2\sec^2 A$

Sol. L.H.S. $= \dfrac{\text{cosec A}}{\text{cosec A} - 1} + \dfrac{\text{cosec A}}{\text{cosec A} + 1}$

$= \dfrac{\text{cosec}^2 A + \text{cosec A} + \text{cosec}^2 A - \text{cosec A}}{(\text{cosec A} - 1)(\text{cosec A} + 1)}$

$= \dfrac{2\,\text{cosec}^2 A}{\text{cosec}^2 A - 1}$

$= \dfrac{2\,\text{cosec}^2 A}{\cot^2 A}$

$= \dfrac{2}{\sin^2 A} \times \dfrac{\sin^2 A}{\cos^2 A}$

$= \dfrac{2}{\cos^2 A} = 2\sec^2 A$

= R.H.S.

Q. 17. (*i*) $\sec^2 A + \text{cosec}^2 A = \sec^2 A\,\text{cosec}^2 A$

(*ii*) $\tan^2 A - \sin^2 A = \tan^2 A \sin^2 A$

Sol. (*i*) L.H.S. $= \sec^2 A + \text{cosec}^2 A$

$= \dfrac{1}{\cos^2 A} + \dfrac{1}{\sin^2 A}$

$= \dfrac{\sin^2 A \times \cos^2 A}{\sin^2 A \cos^2 A}$

$= \dfrac{1}{\sin^2 A \cos^2 A}$

$(\because \sin^2 A + \cos^2 A = 1)$

$= \sec^2 A\,\text{cosec}^2 A$ = R.H.S.

(*ii*) L.H.S. $= \tan^2 A - \sin^2 A$

$= \dfrac{\sin^2 A}{\cos^2 A} - \sin^2 A$

$= \dfrac{\sin^2 A - \sin^2 A \cos^2 A}{\cos^2 A}$

$= \dfrac{\sin^2 A(1 - \cos^2 A)}{\cos^2 A}$

$= \dfrac{\sin^2 A \times \sin^2 A}{\cos^2 A}$

$= \tan^2 A \,.\, \sin^2 A$ = R.H.S.

Q. 18. Prove that : $(1 + \tan A)^2 + (1 - \tan A)^2 = 2\sec^2 A$. **(2005)**

Sol. L.H.S. $= (1 + \tan A)^2 + (1 - \tan A)^2$

$= 1 + 2\tan A + \tan^2 A + 1 - 2\tan A + \tan^2 A$.

$= 2(1 + \tan^2 A)$

$= 2\sec^2 A$ = R.H.S. **Hence proved**

Q.19. $(\sin A + \text{cosec A})^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

Sol. L.H.S. $= (\sin A + \text{cosec A})^2 + (\cos A + \sec A)^2$

$= \sin^2 A + \text{cosec}^2 A + 2 + \cos^2 A + \sec^2 A + 2$

$= \sin^2 A + \cos^2 A + \sec^2 A + \text{cosec}^2 A + 4$

$= 1 + \sec^2 A + \text{cosec}^2 A + 4 = 5 + \sec^2 A + \text{cosec}^2 A$

$= 5 + (1 + \tan^2 A) + (1 + \cot^2 A)$

$= 5 + 1 + \tan^2 A + 1 + \cot^2 A$ $\{\sec^2 A = 1 + \tan^2 A$ and $\text{cosec}^2 A = 1 + \cot^2 A\}$

$= 7 + \tan^2 A + \cot^2 A$

= R.H.S.

Q. 20. (cosec A – sin A) (sec A – cos A) (tan A + cot A) = 1

Sol. L.H.S. = (cosec A – sin A) (sec A – cos A) (tan A + cot A)

$$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)\left(\tan A + \frac{1}{\tan A}\right)$$

$$= \left(\frac{1-\sin^2 A}{\sin A}\right) \times \left(\frac{1-\cos^2 A}{\cos A}\right) \times \left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right)$$

$$= \frac{\cos^2 A \times \sin^2 A}{\sin A \cos A} \times \frac{\sin^2 A + \cos^2 A}{\sin A \cos A}$$

$\because 1 - \sin^2 A = \cos^2 A$

$1 - \cos^2 A = \sin^2 A$

$$= \frac{(\sin^2 A \cos^2 A) \times 1}{\sin^2 A \cos^2 A} = \frac{\sin^2 A \cos^2 A}{\sin^2 A \cos^2 A}$$

$= 1 = $ R.H.S.

Q. 21. $(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2$

Sol. L.H.S. $= (1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A)$

$$= \left(1 + \frac{\cos A}{\sin A} - \frac{1}{\sin A}\right)\left(1 + \frac{\sin A}{\cos A} + \frac{1}{\cos A}\right)$$

$$= \frac{\sin A + \cos A - 1}{\sin A} \times \frac{\cos A + \sin A + 1}{\cos A}$$

$$= \frac{(\sin A + \cos A)^2 - (1)^2}{\sin A \cos A}$$

$$= \frac{\sin^2 A + \cos^2 A + 2\sin A \cos A - 1}{\sin A \cos A}$$

$$= \frac{1 + 2\sin A \cos A - 1}{\sin A \cos A}$$

$$= \frac{2\sin A \cos A}{\sin A \cos A}$$

$= 2 = $ R.H.S.

Q. 22. $\sqrt{\sec^2 A + \operatorname{cosec}^2 A} = (\tan A + \cot A)$

Sol. L.H.S. $= \sqrt{\sec^2 A + \operatorname{cosec}^2 A}$

$= \sqrt{1 + \tan^2 A + 1 + \cot^2 A}$

$= \sqrt{\tan^2 A + \cot^2 A + 2}$

$= \sqrt{(\tan A + \cot A)^2}$

$= \tan A + \cot A = $ R.H.S.

Q. 23. $\tan^2 A + \cot^2 A + 2 = \sec^2 A \operatorname{cosec}^2 A$

Sol. L.H.S. $= \tan^2 A + \cot^2 A + 2$

$$= (\tan A + \cot A)^2 = \left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right)^2$$

$$= \left[\frac{\sin^2 A + \cos^2 A}{\sin A \cos A}\right]^2 = \frac{1}{\sin^2 A \cos^2 A}$$

$= \operatorname{cosec}^2 A \sec^2 A$

$= \sec^2 A \operatorname{cosec}^2 A = $ R.H.S.

Q. 24. $\sin A (1 + \tan A) + \cos A (1 + \cot A) = \sec A + \operatorname{cosec} A.$

Sol. L.H.S. $= \sin A (1 + \tan A) + \cos A (1 + \cot A)$

$$= \sin A\left(1 + \frac{\sin A}{\cos A}\right) + \cos A\left(1 + \frac{\cos A}{\sin A}\right)$$

$$= \frac{\sin A(\cos A + \sin A)}{\cos A} + \frac{\cos A(\sin A + \cos A)}{\sin A}$$

$$= (\sin A + \cos A)\left[\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right]$$

$$= (\sin A + \cos A)\left[\frac{\sin^2 A + \cos^2 A}{\sin A \cos A}\right]$$

$$= (\sin A + \cos A) \times \frac{1}{\sin A \cos A}$$

$\{\because \sin^2 A + \cos^2 A = 1\}$

$$= \frac{\sin A + \cos A}{\sin A \cos A}$$

$$= \frac{\sin A}{\sin A \cos A} + \frac{\cos A}{\sin A \cos A}$$

$= \frac{1}{\cos A} + \frac{1}{\sin A} = \sec A + \operatorname{cosec} A$

= R.H.S.

Q. 25. (*i*) $\frac{1}{(\sec A - \tan A)} = (\sec A + \tan A)$

(*ii*) $\frac{1}{(\operatorname{cosec} A + \cot A)} = (\operatorname{cosec} A - \cot A)$

Sol. (*i*) L.H.S. $= \frac{1}{\sec A - \tan A}$

$= \frac{1}{\frac{1}{\cos A} - \frac{\sin A}{\cos A}}$

$= \frac{1}{\frac{1-\sin A}{\cos A}} = \frac{\cos A}{1-\sin A}$

R.H.S. = sec A + tan A

$= \frac{1}{\cos A} + \frac{\sin A}{\cos A}$

$= \frac{1+\sin A}{\cos A} \times \frac{1-\sin A}{1-\sin A}$

{Multiplying and dividing by (1 – sin A)}

$= \frac{1-\sin^2 A}{\cos A(1-\sin A)} = \frac{\cos^2 A}{\cos A(1-\sin A)}$

$\{\because 1 - \sin^2 A = \cos^2 A\}$

$= \frac{\cos A}{1-\sin A}$

Hence, L.H.S. = R.H.S.

(*ii*) L.H.S. $= \frac{1}{\operatorname{cosec} A + \cot A}$

$= \frac{1}{\frac{1}{\sin A} + \frac{\cos A}{\sin A}} = \frac{1}{\frac{1+\cos A}{\sin A}}$

$= \frac{\sin A}{1+\cos A}$

R.H.S. = cosec A – cot A

$= \frac{1}{\sin A} - \frac{\cos A}{\sin A} = \frac{1-\cos A}{\sin A}$

$= \frac{(1-\cos A)(1+\cos A)}{\sin A(1+\cos A)}$

{Multiplying and dividing by (1 + cos A)}

$= \frac{1-\cos^2 A}{\sin A(1+\cos A)} = \frac{\sin^2 A}{\sin A(1+\cos A)}$

$= \frac{\sin A}{1+\cos A}$

∴ L.H.S. = R.H.S.

Q. 26. $\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A}$

$= \sec A \operatorname{cosec} A + 1$

Sol. L.H.S. $= \frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A}$

$= \frac{\frac{\sin A}{\cos A}}{1-\frac{\cos A}{\sin A}} + \frac{\frac{\cos A}{\sin A}}{1-\frac{\sin A}{\cos A}}$

$= \frac{\frac{\sin A}{\cos A}}{\frac{\sin A - \cos A}{\sin A}} + \frac{\frac{\cos A}{\sin A}}{\frac{\cos A - \sin A}{\cos A}}$

$= \frac{\sin A}{\cos A} \times \frac{\sin A}{\sin A - \cos A}$

$+ \frac{\cos A}{\sin A} \times \frac{\cos A}{\cos A - \sin A}$

$= \frac{\sin^2 A}{\cos A(\sin A - \cos A)} - \frac{\cos^2 A}{\sin A(\sin A - \cos A)}$

$= \frac{1}{(\sin A - \cos A)}\left[\frac{\sin^3 A - \cos^3 A}{\sin A \cos A}\right]$

$= \frac{(\sin A - \cos A)(\sin^2 A + \cos^2 A + \sin A \cos A)}{(\sin A - \cos A)(\sin A \cos A)}$

$= \frac{1 + \sin A \cos A}{\sin A \cos A} = \frac{1}{\sin A \cos A} + 1$

= sec A cosec A + 1 = R.H.S.

Q. 27. $\dfrac{\cos^3 A + \sin^3 A}{\cos A + \sin A} + \dfrac{\cos^3 A - \sin^3 A}{\cos A - \sin A} = 2$

Sol. L.H.S. $= \dfrac{\cos^3 A + \sin^3 A}{\cos A + \sin A} + \dfrac{\cos^3 A - \sin^3 A}{\cos A - \sin A}$

$= \dfrac{(\cos A + \sin A)(\cos^2 A - \sin A \cos A + \cos^2 A)}{(\cos A + \sin A)} + \dfrac{(\cos A - \sin A)(\cos^2 A + \sin A \cos A + \cos^2 A)}{(\cos A - \sin A)}$

$= (1 - \sin A \cos A) + (1 + \sin A \cos A)$

$= 1 - \sin A \cos A + 1 + \sin A \cos A = 2 =$ R.H.S.

Q. 28. (*i*) $\dfrac{1 - \tan^2 A}{\cot^2 A - 1} = \tan^2 A$

(*ii*) $\cot A - \tan A = \dfrac{2\cos^2 A - 1}{\sin A \cos A}$

Sol. (*i*) L.H.S. $= \dfrac{1 - \tan^2 A}{\cot^2 A - 1} = \dfrac{1 - \dfrac{\sin^2 A}{\cos^2 A}}{\dfrac{\cos^2 A}{\sin^2 A} - 1}$

$= \dfrac{\dfrac{\cos^2 A - \sin^2 A}{\cos^2 A}}{\dfrac{\cos^2 A - \sin^2 A}{\sin^2 A}}$

$= \dfrac{\cos^2 A - \sin^2 A}{\cos^2 A} \times \dfrac{\sin^2 A}{\cos^2 A - \sin^2 A}$

$= \dfrac{\sin^2 A}{\cos^2 A} = \tan^2 A =$ R.H.S.

(*ii*) L.H.S. $= \cot A - \tan A$

$= \dfrac{\cos A}{\sin A} - \dfrac{\sin A}{\cos A} = \dfrac{\cos^2 A - \sin^2 A}{\sin A \cos A}$

$= \dfrac{\cos^2 A - (1 - \cos^2 A)}{\sin A \cos A}$

$\{\because \sin^2 A = 1 - \cos^2 A\}$

$= \dfrac{\cos^2 - 1 + \cos^2 A}{\sin A \cos A} = \dfrac{2\cos^2 A - 1}{\sin A \cos A}$

= R.H.S.

Q. 29. $\left(1 - \dfrac{\cos^2\theta}{1 + \sin\theta}\right) = \sin\theta$ **(2001)**

Sol. L.H.S. $= 1 - \dfrac{\cos^2\theta}{1 + \sin\theta} = 1 - \dfrac{1 - \sin^2\theta}{1 + \sin\theta}$

$\{\because \cos^2\theta = 1 - \sin^2\theta\}$

$= 1 - \dfrac{(1 + \sin\theta)(1 - \sin\theta)}{(1 + \sin\theta)}$

$\{\because a^2 - b^2 = (a + b)(a - b)\}$

$= 1 - (1 - \sin\theta) = 1 - 1 + \sin\theta = \sin\theta$

= R.H.S.

Q. 30. $\dfrac{\cot\theta + \operatorname{cosec}\theta - 1}{\cot\theta - \operatorname{cosec}\theta + 1} = \dfrac{1 + \cos\theta}{\sin\theta}$

Sol. L.H.S. $= \dfrac{\cot\theta + \operatorname{cosec}\theta - 1}{\cot\theta - \operatorname{cosec}\theta + 1}$

$= \dfrac{\cot\theta + \operatorname{cosec}\theta - (\operatorname{cosec}^2\theta - \cot^2\theta)}{\cot\theta - \operatorname{cosec}\theta + 1}$

$= \dfrac{(\cot\theta + \operatorname{cosec}\theta) + (\cot^2\theta - \operatorname{cosec}^2\theta)}{1 + \cot\theta - \operatorname{cosec}\theta}$

$= \dfrac{(\cot\theta + \operatorname{cosec}\theta) + (\cot\theta + \operatorname{cosec}\theta)(\cot\theta - \operatorname{cosec}\theta)}{1 + \cot\theta - \operatorname{cosec}\theta}$

$= \dfrac{(\cot\theta + \operatorname{cosec}\theta)[1 + \cot\theta - \operatorname{cosec}\theta]}{(1 + \cot\theta - \operatorname{cosec}\theta)}$

$= \cot\theta + \operatorname{cosec}\theta = \dfrac{\cos\theta}{\sin\theta} + \dfrac{1}{\sin\theta}$

$= \dfrac{\cos\theta + 1}{\sin\theta} = \dfrac{1 + \cos\theta}{\sin\theta} =$ R.H.S.

Q. 31. $\dfrac{\tan\theta + \sin\theta}{\tan\theta - \sin\theta} = \dfrac{\sec\theta + 1}{\sec\theta - 1}$

Sol. L.H.S. $= \dfrac{\tan\theta + \sin\theta}{\tan\theta - \sin\theta}$

$= \dfrac{\dfrac{\sin\theta}{\cos\theta} + \sin A}{\dfrac{\sin\theta}{\cos\theta} - \sin A}$

$$= \frac{\sin\theta\left(\frac{1}{\cos\theta}+1\right)}{\sin\theta\left(\frac{1}{\cos\theta}-1\right)}$$

$$= \frac{\frac{1}{\cos\theta}+1}{\frac{1}{\cos\theta}-1} = \frac{\sec\theta+1}{\sec\theta-1} = \text{R.H.S.}$$

Q. 32. $\frac{1}{(\sec A+\tan A)} - \frac{1}{\cos A}$

$$= \frac{1}{\cos A} - \frac{1}{(\sec A-\tan A)}$$

Sol. L.H.S. $= \frac{1}{\sec A+\tan A} - \frac{1}{\cos A}$

$$= \frac{\sec^2 A-\tan^2 A}{\sec A+\tan A} - \frac{1}{\cos A}$$

$\{\because \sec^2 A - \tan^2 A = 1$ and $\frac{1}{\cos A} = \sec A\}$

$$= \frac{(\sec A+\tan A)(\sec A-\tan A)}{(\sec A+\tan A)} - \sec A$$

$= \sec A - \tan A - \sec A = -\tan A$

R.H.S. $= \frac{1}{\cos A} - \frac{1}{\sec A-\tan A}$

$$= \sec A - \frac{\sec^2 A-\tan^2 A}{\sec A-\tan A}$$

$(\because \sec^2 A - \tan^2 A = 1)$

$$= \sec A - \frac{(\sec A+\tan A)(\sec A-\tan A)}{(\sec A-\tan A)}$$

$= \sec A - (\sec A + \tan A)$

$= \sec A - \sec A - \tan A = -\tan A$

Hence, L.H.S. = R.H.S.

Q. 33. $\frac{1-\cos A}{1+\cos A} = (\cot A - \operatorname{cosec} A)^2$

Sol. R.H.S. $= (\cot A - \operatorname{cosec} A)^2$

$$= \left(\frac{\cos A}{\sin A} - \frac{1}{\sin A}\right)^2 = \left(\frac{\cos A-1}{\sin A}\right)^2$$

$$= \frac{(\cos A-1)^2}{\sin^2 A} = \frac{(\cos A-1)^2}{1-\cos^2 A}$$

$$= \frac{(1-\cos A)^2}{(1+\cos A)(1-\cos A)}$$

$$= \frac{1-\cos A}{1+\cos A} = \text{L.H.S.}$$

Q. 34. $\frac{\cot A-1}{2-\sec^2 A} = \frac{\cot A}{1+\tan A}$

Sol. L.H.S. $= \frac{\cot A-1}{2-\sec^2 A} = \frac{\frac{\cos A}{\sin A}-1}{2-\frac{1}{\cos^2 A}}$

$$= \frac{\frac{\cos A-\sin A}{\sin A}}{\frac{2\cos^2 A-1}{\cos^2 A}}$$

$$= \frac{\cos A-\sin A}{\sin A} \times \frac{\cos^2 A}{2\cos^2 A-1}$$

$$= \frac{\cos A-\sin A}{\sin A} \times \frac{\cos^2 A}{\cos^2 A+\cos^2 A-1}$$

$$= \frac{\cos A-\sin A}{\sin A} \times \frac{\cos^2 A}{\cos^2 A-(1-\cos^2 A)}$$

$$= \frac{\cos A-\sin A}{\sin A} \times \frac{\cos^2 A}{(\cos^2 A-\sin^2 A)}$$

$$= \frac{\cos A-\sin A}{\sin A} \times \frac{\cos^2 A}{(\cos A+\sin A)(\cos A-\sin A)}$$

$$= \frac{\cos^2 A}{\sin A(\cos A+\sin A)}$$

R.H.S. $= \frac{\cot A}{1+\tan A} = \frac{\frac{\cos A}{\sin A}}{1+\frac{\sin A}{\cos A}}$

$$= \frac{\frac{\cos A}{\sin A}}{\frac{\cos A + \sin A}{\cos A}}$$

$$= \frac{\cos A}{\sin A} \times \frac{\cos A}{\cos A + \sin A}$$

$$= \frac{\cos^2 A}{\sin A\,(\cos A + \sin A)}$$

Hence, L.H.S. = R.H.S.

Q. 35. $\sqrt{\frac{1+\cos A}{1-\cos A}} = (\operatorname{cosec} A + \cot A)$

Sol. $\sqrt{\frac{1+\cos A}{1-\cos A}}$

$$= \sqrt{\frac{(1+\cos A)\,(1+\cos A)}{(1-\cos A)(1+\cos A)}}$$

[(Multiplying and dividing by (1 + cos A)]

$$= \sqrt{\frac{(1+\cos A)^2}{1-\cos^2 A}}$$

$$= \sqrt{\frac{(1+\cos A)^2}{\sin^2 A}} = \frac{1+\cos A}{\sin A}$$

$$= \frac{1}{\sin A} + \frac{\cos A}{\sin A} = (\operatorname{cosec} A + \cot A)$$

= R.H.S.

Q. 36. $\frac{\sin A}{(\cot A + \operatorname{cosec} A)}$

$$= 2 + \frac{\sin A}{(\cot A - \operatorname{cosec} A)}$$

Sol. L.H.S. $= \frac{\sin A}{\cot A + \operatorname{cosec} A}$

$$= \frac{\sin A}{\frac{\cos A}{\sin A} + \frac{1}{\sin A}} = \frac{\sin A}{\frac{\cos A + 1}{\sin A}}$$

$$= \frac{\sin A \times \sin A}{\cos A + 1} = \frac{\sin^2 A}{\cos A + 1} = \frac{1-\cos^2 A}{1+\cos A}$$

$$= \frac{(1+\cos A)(1-\cos A)}{1+\cos A} = 1 - \cos A$$

R.H.S. $= 2 + \frac{\sin A}{\cot A - \operatorname{cosec} A}$

$$= 2 + \frac{\sin A}{\frac{\cos A}{\sin A} - \frac{1}{\sin A}}$$

$$= 2 + \frac{\sin A}{\frac{\cos A - 1}{\sin A}} = 2 + \frac{\sin^2 A}{\cos A - 1}$$

$$= 2 + \frac{1-\cos^2 A}{\cos A - 1}$$

$$= 2 + \frac{(1+\cos A)\,(1-\cos A)}{-(1-\cos A)}$$

$$= 2 - (1 + \cos A)$$

$$= 2 - 1 - \cos A = 1 - \cos A$$

Hence, L.H.S. = R.H.S.

Eliminate θ between the given equations : (Q. 37 to Q. 42)

Q. 37. $x = a \operatorname{cosec} \theta, y = b \cot \theta$.

Sol. $x = a \operatorname{cosec} \theta, y = b \cot \theta$

$$\Rightarrow \frac{x}{a} = \operatorname{cosec} \theta, \frac{y}{b} = \cot \theta$$

Squaring and subtracting,

$$\frac{x^2}{a^2} = \operatorname{cosec}^2 \theta, \frac{y^2}{b^2} = \cot^2 \theta$$

$$\Rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = \operatorname{cosec}^2 \theta - \cot^2 \theta = 1$$

Hence, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ **Ans.**

Q. 38. $x = a \sec^3 \theta, y = b \tan^3 \theta$.

Sol. $\frac{x}{a} = \sec^3 \theta, \frac{y}{b} = \tan^3 \theta$

$$\Rightarrow \left(\frac{x}{a}\right)^{\frac{1}{3}} = \sec \theta, \left(\frac{y}{b}\right)^{\frac{1}{3}} = \tan \theta$$

Now, squaring and subtracting,

$$\left(\frac{x}{a}\right)^{\frac{2}{3}} = \sec^2 \theta, \left(\frac{y}{b}\right)^{\frac{2}{3}} = \tan^2 \theta$$

$$\left(\frac{x}{a}\right)^{\frac{2}{3}} - \left(\frac{y}{b}\right)^{\frac{2}{3}} = \sec^2\theta - \tan^2\theta = 1$$

Hence, $\left(\frac{x}{a}\right)^{\frac{2}{3}} - \left(\frac{y}{b}\right)^{\frac{2}{3}} = 1$

$$\Rightarrow \frac{x^{\frac{2}{3}}}{a^{\frac{2}{3}}} - \frac{y^{\frac{2}{3}}}{b^{\frac{2}{3}}} = 1 \text{ Ans.}$$

Q. 39. $x = h + a\cos\theta,\ y = k + b\sin\theta$.

Sol. $a\cos\theta = x - k \Rightarrow \cos\theta = \frac{x-k}{a}$

$b\sin\theta = y - k \Rightarrow \sin\theta = \frac{y-k}{b}$

Squaring and adding,

$$\cos^2\theta = \left(\frac{x-k}{a}\right)^2$$

and $\sin^2\theta = \left(\frac{y-k}{b}\right)^2$

$$\Rightarrow \left(\frac{x-k}{a}\right)^2 + \left(\frac{y-k}{b}\right)^2 = \cos^2\theta + \sin^2\theta$$

$$\Rightarrow \frac{(x-k)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \text{ Ans.}$$

Q. 40. $a\cot\theta + b\,\text{cosec}\,\theta = x$,
$a\,\text{cosec}\,\theta + b\cot\theta = y$.

Sol. Squaring, we get

$(a\cot\theta + b\,\text{cosec}\,\theta)^2 = x^2$,

$(a\,\text{cosec}\,\theta + b\cot\theta)^2 = y^2$

$\Rightarrow a^2\cot^2\theta + b^2\,\text{cosec}^2\theta + 2ab\cot\theta\,\text{cosec}\,\theta = x^2$

$a^2\,\text{cosec}^2\theta + b^2\cot^2\theta + 2ab\cot\theta\,\text{cosec}\,\theta = y^2$

Subtracting

$a^2(\cot^2\theta - \text{cosec}^2\theta) + b^2(\text{cosec}^2\theta - \cot^2\theta) = x^2 - y^2$

$\Rightarrow a^2 \times (-1) + b^2(1) = x^2 - y^2$

$\Rightarrow -a^2 + b^2 = x^2 - y^2$

$\Rightarrow b^2 - a^2 = x^2 - y^2$ **Ans.**

Q. 41. $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$,

$\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta = -1$.

Sol. $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ ...(*i*)

$\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta = -1$ ...(*ii*)

Squaring both sides,

$$\frac{x^2}{a^2}\cos^2\theta + \frac{y^2}{b^2}\sin^2\theta + 2\frac{xy}{ab}\sin\theta\cos\theta = 1$$

and $\frac{x^2}{a^2}\sin^2\theta + \frac{y^2}{b^2}\cos^2\theta - 2\frac{xy}{ab}\sin\theta\cos\theta = 1$

Adding, we get

$$\frac{x^2}{a^2}(\cos^2\theta + \sin^2\theta) + \frac{y^2}{b^2}(\sin^2\theta + \cos^2\theta) = 2$$

$$\frac{x^2}{a^2} \times 1 + \frac{y^2}{b^2} \times 1 = 2$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 2 \text{ Ans.}$$

Q. 42. If $(\sin\theta + \cos\theta) = m$ and $(\sec\theta + \text{cosec}\,\theta) = n$, prove that $n(m^2 - 1) = 2m$.

Sol. $\sin\theta + \cos\theta = m$

Squaring and adding

$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = m^2$

$\Rightarrow 1 + 2\sin\theta\cos\theta = m^2$

$\Rightarrow 2\sin\theta\cos\theta = m^2 - 1$ ...(*i*)

$\sec\theta + \text{cosec}\,\theta = n$

$$\frac{1}{\cos\theta} + \frac{1}{\sin\theta} = n \Rightarrow \frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta} = n$$

$$\Rightarrow \frac{m}{\sin\theta\cos\theta} = n \quad ....(ii)$$

Multiplying (*i*) and (*ii*),

$$2\sin\theta\cos\theta \times \frac{m}{\sin\theta\cos\theta} = n(m^2-1)$$

$$\Rightarrow 2m = n(m^2-1) \Rightarrow n(m^2-1) = 2m$$

Hence proved.

Prove the following :

Q. 43. $\sin^2 A\cos^2 B - \cos^2 A\sin^2 B = \sin^2 A - \sin^2 B$

Sol. L.H.S. $= \sin^2 A\cos^2 B - \cos^2 A\sin^2 B$

$= \sin^2 A(1-\sin^2 B) - (1-\sin^2 A)\sin^2 B$

$= (\sin^2 A - \sin^2 A\sin^2 B) - (\sin^2 B - \sin^2 A\sin^2 B)$

$= \sin^2 A - \sin^2 A\sin^2 B - \sin^2 B + \sin^2 A\sin^2 B$

$= \sin^2 A - \sin^2 B =$ R.H.S.

Q. 44. $\dfrac{\tan A + \tan B}{\cot A + \cot B} = \tan A\tan B$

Sol. L.H.S. $= \dfrac{\tan A + \tan B}{\cot A + \cot B}$

$$= \frac{\tan A + \tan B}{\frac{1}{\tan A} + \frac{1}{\tan B}} = \frac{\tan A + \tan B}{\frac{\tan B + \tan A}{\tan A\tan B}}$$

$$= \frac{(\tan A + \tan B) \times \tan A\tan B}{(\tan A + \tan B)}$$

$= \tan A\tan B =$ R.H.S.

Q. 45. $\tan^2 A\sec^2 B - \sec^2 A\tan^2 B = \tan^2 A - \tan^2 B.$

Sol. L.H.S. $= \tan^2 A\sec^2 B - \sec^2 A\tan^2 B$

$= \tan^2 A(\tan^2 B + 1) - (\tan^2 A + 1)\tan^2 B$

$= (\tan^2 A\tan^2 B + \tan^2 A) - (\tan^2 A\tan^2 B + \tan^2 B)$

$= \tan^2 A\tan^2 B + \tan^2 A - \tan^2 A\tan^2 B - \tan^2 B$

$= \tan^2 A - \tan^2 B =$ R.H.S.

Q. 46. Solve the following equations, when $0° < \theta < 90°$.

(*i*) $2\sin^2\theta = \frac{1}{2}$ (*ii*) $2\cos^2\theta = \frac{1}{2}$

(*iii*) $\sin\theta = \cos\theta$ (*iv*) $2\cos 3\theta = 1$

(*v*) $2\cos^2\theta - 1 = 0.$

(*vi*) $2\cos^2\theta + \sin\theta - 2 = 0$

Sol. (*i*) $2\sin^2\theta = \frac{1}{2} \Rightarrow \sin^2\theta = \frac{1}{2\times 2} = \frac{1}{4}$

$$\Rightarrow \sin^2\theta = \left(\frac{1}{2}\right)^2$$

$$\Rightarrow \sin\theta = \frac{1}{2} = \sin 30°$$

$\therefore\quad \theta = 30°$ **Ans.**

(*ii*) $2\cos^2\theta = \frac{1}{2} \Rightarrow \cos^2\theta = \frac{1}{2\times 2} = \frac{1}{4}$

$$\Rightarrow \cos^2\theta = \left(\frac{1}{2}\right)^2$$

$$\Rightarrow \cos\theta = \frac{1}{2} = \cos 60°$$

$\therefore\quad \theta = 60°$ **Ans.**

(*iii*) $\sin\theta = \cos\theta \Rightarrow \dfrac{\sin\theta}{\cos\theta} = 1$

$$\Rightarrow \tan\theta = 1 = \tan 45°$$

$\therefore \quad \theta = 45°$ **Ans.**

(*iv*) $2 \cos 3\theta = 1$

$\Rightarrow \cos 3\theta = \frac{1}{2} = \cos 60°$

$\Rightarrow 3\theta = 60° \Rightarrow \theta = \frac{60°}{3} = 20°$ **Ans.**

(*v*) $2 \cos^2 \theta - 1 = 0 \quad \Rightarrow 2 \cos^2 \theta = 1$

$\cos^2 \theta = \frac{1}{2} \quad \Rightarrow \cos^2 \theta = \left(\frac{1}{\sqrt{2}}\right)^2$

$\cos\theta = \frac{1}{\sqrt{2}} = \cos 45°$

$\therefore \quad \theta = 45°$ **Ans.**

(*vi*) $2 \cos^2 \theta + \sin \theta - 2 = 0$

$2 (1 - \sin^2 \theta) + \sin \theta - 2 = 0$

$\Rightarrow 2 - 2 \sin^2 \theta + \sin \theta - 2 = 0$

$\Rightarrow \quad -2 \sin^2 \theta + \sin \theta = 0$

$\Rightarrow \quad 2 \sin^2 \theta - \sin \theta = 0$

$\Rightarrow \quad \sin \theta (2 \sin \theta - 1) = 0$

Either, $\sin \theta = 0$, then $\theta = 0°$

or $2 \sin \theta - 1 = 0$, then $2 \sin \theta = 1$

$\Rightarrow \sin \theta = \frac{1}{2}$

$\therefore \quad \theta = 30° \quad \left[\because \sin 30° = \frac{1}{2}\right]$

Hence, $\theta = 30°$ **Ans.**

EXERCISE 25 (B)

Q. 1. (*i*) $\sin^2 45° + \cos^2 30° - \tan^2 45°$

(*ii*) $\cos 90° + \cos^2 45° \sin 30° \tan 45°$

(*iii*) $\frac{4}{3}\tan^2 30° + \sin^2 60° - 3 \cos^2 60° + \frac{3}{4} \tan^2 60° - 2 \tan^2 45°$

Sol. We know that,

$\sin 45° = \frac{1}{\sqrt{2}}, \cos 30° = \frac{\sqrt{3}}{2}, \tan 45° = 1$

$\cos 90° = 0, \cos 45° = \frac{\sqrt{3}}{2}, \sin 30° = \frac{1}{2},$

$\tan 30° = \frac{1}{\sqrt{3}}, \sin 60° = \frac{\sqrt{3}}{2},$

$\cos 60° = \frac{1}{2}, \tan 60° = \sqrt{3}$

(*i*) Now, $\sin^2 45° + \cos^2 30° - \tan^2 45°$

$= \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - (1)^2$

$= \frac{1}{2} + \frac{3}{4} - 1 = \frac{2+3-4}{4} = \frac{5-4}{4}$

$= \frac{1}{4}$ **Ans.**

(*ii*) $\cos 90° + \cos^2 45° \sin 30° \tan 45°$

$= (0) + \left(\frac{1}{\sqrt{2}}\right)^2 \times \frac{1}{2} \times 1$

$= 0 + \frac{1}{2} \times \frac{1}{2} \times 1 = 0 + \frac{1}{4} = \frac{1}{4}$ **Ans.**

(*iii*) $\frac{4}{3}\tan^2 30° + \sin^2 60°$

$-3\cos^2 60° + \frac{3}{4} \tan^2 60° - 2\tan^2 45°$

$= \frac{4}{3}\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2$

$$-3\left(\frac{1}{2}\right)^2+\frac{3}{4}(\sqrt{3})^2-2(1)^2$$

$$=\frac{4}{3}\times\frac{1}{3}+\frac{3}{4}-\frac{3}{4}+\frac{3}{4}\times 3-2\times 1$$

$$=\frac{4}{9}+\frac{3}{4}-\frac{3}{4}+\frac{9}{4}-2=\frac{4}{9}+\frac{9}{4}-2$$

$$=\frac{16+81-72}{36}=\frac{25}{36} \textbf{ Ans.}$$

Q. 2. (*i*) $\frac{\sin 27^\circ}{\cos 63^\circ}$

(*ii*) $\frac{\sec 38^\circ}{\operatorname{cosec} 52^\circ}$

(*iii*) $\frac{\tan 19^\circ}{\cot 71^\circ}$

Sol. (*i*) $\frac{\sin 27^\circ}{\cos 63^\circ}=\frac{\sin 27^\circ}{\cos(90^\circ-27^\circ)}$

$$=\frac{\sin 27^\circ}{\sin 27^\circ}=1 \textbf{ Ans.}$$

(*ii*) $\frac{\sec 38^\circ}{\operatorname{cosec} 52^\circ}=\frac{\sec 38^\circ}{\operatorname{cosec}(90^\circ-38^\circ)}$

$$=\frac{\sec 38^\circ}{\sec 38^\circ}=1 \textbf{ Ans.}$$

(*iii*) $\frac{\tan 19^\circ}{\cot 71^\circ}=\frac{\tan 19^\circ}{\cot(90^\circ-19^\circ)}$

$$=\frac{\tan 19^\circ}{\tan 19^\circ}=1 \textbf{ Ans.}$$

Q.3. (*i*) $\frac{\sin 72^\circ}{\cos 18^\circ}-\frac{\sec 32^\circ}{\operatorname{cosec} 58^\circ}$

(*ii*) $\frac{2\tan 53^\circ}{\cot 37^\circ}-\frac{\cot 80^\circ}{\tan 10^\circ}$ **(2006)**

Sol. (*i*) $\frac{\sin 72^\circ}{\cos 18^\circ}-\frac{\sec 32^\circ}{\operatorname{cosec} 58^\circ}$

$$=\frac{\sin 72^\circ}{\cos(90^\circ-72^\circ)}-\frac{\sec 32^\circ}{\operatorname{cosec}(90^\circ-32^\circ)}$$

$$=\frac{\sin 72^\circ}{\sin 72^\circ}-\frac{\sec 32^\circ}{\sec 32^\circ}=1-1=0 \textbf{ Ans.}$$

(*ii*) $\frac{2\tan 53^\circ}{\cot 37^\circ}-\frac{\cot 80^\circ}{\tan 10^\circ}$

$$=\frac{2\tan(90^\circ-37^\circ)}{\cot 37^\circ}-\frac{\cot(90^\circ-10^\circ)}{\tan 10^\circ}$$

$$=\frac{2\cot 37^\circ}{\cot 37^\circ}-\frac{\tan 10^\circ}{\tan 10^\circ}$$

$[\because \tan(90^\circ-\theta)=\cot\theta]$

$$=2-1=1 \textbf{ Ans.}$$

Q. 4. (*i*) $\frac{\cos 70^\circ}{\sin 20^\circ}+\frac{\cos 59^\circ}{\sin 31^\circ}-8\sin^2 30^\circ$

(*ii*) $\frac{\cos 35^\circ}{\sin 55^\circ}+\frac{\sin 11^\circ}{\cos 79^\circ}-\cos 28^\circ \operatorname{cosec} 62^\circ$

Sol. (*i*) $\frac{\cos 70^\circ}{\sin(90^\circ-70^\circ)}+\frac{\cos 59^\circ}{\sin(90^\circ-59^\circ)}-8\sin^2 30^\circ$

$$=\frac{\cos 70^\circ}{\cos 70^\circ}+\frac{\cos 59^\circ}{\cos 59}-8\left(\frac{1}{2}\right)^2$$

$\left(\because \sin 30^\circ=\frac{1}{2}\right)$

$$=1+1-8\times\frac{1}{4}=2-2=0 \textbf{ Ans.}$$

(*ii*) $\frac{\cos 35^\circ}{\sin 55^\circ}+\frac{\sin 11^\circ}{\cos 79^\circ}-\cos 28^\circ \operatorname{cosec} 62^\circ$

$$=\frac{\cos 35^\circ}{\sin 55^\circ}+\frac{\sin 11^\circ}{\cos 79^\circ}-\frac{\cos 28^\circ}{\sin 62^\circ}$$

$$=\frac{\cos 35^\circ}{\sin(90^\circ-35^\circ)}+\frac{\sin 11^\circ}{\cos(90^\circ-11^\circ)}$$

$$-\frac{\cos 28^\circ}{\sin(90^\circ-28^\circ)}$$

$$=\frac{\cos 35^\circ}{\cos 35^\circ}+\frac{\sin 11^\circ}{\sin 11^\circ}-\frac{\cos 28^\circ}{\cos 28^\circ}$$

$= 1 + 1 - 1 = 1$ **Ans.**

Q. 5. (*i*) $\frac{\sin 29^\circ 31'}{\cos 60^\circ 29'}$ (*ii*) $\frac{\tan 20^\circ 43'}{\cot 69^\circ 17'}$

(*iii*) $\frac{\sin 28^\circ 54'}{\sec 61^\circ 6'}+\frac{\cos 61^\circ 6'}{\text{cosec } 28^\circ 54'}$

Sol. (*i*) $\frac{\sin 29^\circ 31'}{\cos 60^\circ 29'}=\frac{\sin[90^\circ-(60^\circ 29')]}{\cos 60^\circ 29'}$

$$=\frac{\cos 60^\circ 29'}{\cos 60^\circ 29'}=1 \textbf{ Ans.}$$

(*ii*) $\frac{\tan 20^\circ 43'}{\cot 69^\circ 17'}=\frac{\tan[90^\circ-(69^\circ 17')]}{\cot 69^\circ 17'}$

$$=\frac{\cot 69^\circ 17'}{\cot 69^\circ 17'}=1 \textbf{ Ans.}$$

(*iii*) $\frac{\sin 28^\circ 54'}{\sec 61^\circ 6'}+\frac{\cos 61^\circ 6'}{\text{cosec } 28^\circ 54'}$

$= \sin 28^\circ 54' \times \cos 61^\circ 6' + \cos 61^\circ 6' \sin 28^\circ 54'$

$= \sin 28^\circ 54' \cdot \cos(90^\circ - 28^\circ 54') + \cos(90^\circ - 28^\circ 54') \sin 28^\circ 54'$

$= \sin 28^\circ 54' \sin 28^\circ 54' + \sin 28^\circ 54' \sin 28^\circ 54'$

$= \sin^2 28^\circ 54' + \sin^2 28^\circ 54'$

$= 2 \sin^2 28^\circ 54'$

Q. 6. 3 cos 80° cosec 10° + 2 cos 59° cosec 31°.

Sol. 3 cos 80° cosec 10° + 2 cos 59° cosec 31°.

$$= 3\cos 80^\circ\left(\frac{1}{\sin 10^\circ}\right)+2\cos 59^\circ\frac{1}{\sin 31^\circ}$$

$$= 3\frac{\cos 80^\circ}{\sin(90^\circ-80^\circ)}+2\frac{\cos 59^\circ}{\sin(90^\circ-59^\circ)}$$

$$= 3\frac{\cos 80^\circ}{\cos 80^\circ}+2\frac{\cos 59^\circ}{\cos 59^\circ}$$

$= 3 \times 1 + 2 \times 1 = 3 + 2 = 5$ **Ans.**

Q. 7. (*i*) $\frac{\sin 80^\circ}{\cos 10^\circ}+\sin 59^\circ \sec 31^\circ$

(*ii*) $\left(\frac{\sin 39^\circ}{\cos 51^\circ}\right)^2+\left(\frac{\cos 51^\circ}{\sin 39^\circ}\right)^2$

(*iii*) $\frac{\sec 17^\circ}{\text{cosec } 73^\circ}+\frac{\tan 68^\circ}{\cot 22^\circ}+\cos^2 44^\circ+\cos^2 46^\circ$

Sol. (*i*) $\frac{\text{Sin}80^\circ}{\text{Cos}10^\circ}+\text{Sin } 59^\circ \text{ Sec } 31^\circ$

$$=\frac{\text{Sin } 80^\circ}{\text{Cos}(90^\circ-80^\circ)}+\text{Sin } 59^\circ \text{ Sec }(90^\circ-59^\circ)$$

$$=\frac{\text{Sin } 80^\circ}{\text{Sin } 80^\circ}+\text{Sin } 59^\circ \text{ Cosec } 59^\circ$$

$= 1 + 1 = 2$

[$\because$ Cos $(90^\circ-\theta)=$ Sinθ, Sec $(90^\circ-\theta)=$ Cosec θ and Sin 59° Cosec 59° = 1]

(*ii*) $\left(\frac{\sin 39^\circ}{\cos 51^\circ}\right)^2+\left(\frac{\cos 51^\circ}{\sin 39^\circ}\right)^2$

$$=\left[\frac{\sin 39^\circ}{\cos(90^\circ-39^\circ)}\right]^2+\left[\frac{\cos 51^\circ}{\sin(90^\circ-51^\circ)}\right]^2$$

$$=\left[\frac{\sin 39^\circ}{\sin 39^\circ}\right]^2+\left[\frac{\cos 51^\circ}{\cos 51^\circ}\right]^2$$

$= (1)^2 + (1)^2 = 1 + 1 = 2$ **Ans.**

(*iii*) $\frac{\sec 17^\circ}{\text{cosec } 73^\circ}+\frac{\tan 68^\circ}{\cot 22^\circ}+\cos^2 44^\circ+\cos^2 46^\circ$

$$=\frac{\sec 17^\circ}{\text{cosec}(90^\circ-17^\circ)}+\frac{\tan 68^\circ}{\cot(90^\circ-68^\circ)}+\cos^2(90^\circ-46^\circ)+\cos^2 46^\circ$$

$$=\frac{\sec 17^\circ}{\sec 17^\circ}+\frac{\tan 68^\circ}{\tan 68^\circ}+\sin^2 46^\circ+\cos^2 46^\circ$$

$= 1 + 1 + 1 = 3$ $\quad(\because \sin^2\theta+\cos^2\theta=1)$

Q. 8. (*i*) $\left(\dfrac{\sin 47^\circ}{\cos 43^\circ}\right)^2 + \left(\dfrac{\cos 43^\circ}{\sin 47^\circ}\right)^2 - 4\cos^2 45^\circ$

(*ii*) $2\left(\dfrac{\cos 67^\circ}{\sin 23^\circ}\right)^2 - \left(\dfrac{\tan 40^\circ}{\cot 50^\circ}\right) - \sin 90^\circ$.

Sol. (*i*) $\left(\dfrac{\sin 47^\circ}{\cos 43^\circ}\right)^2 + \left(\dfrac{\cos 43^\circ}{\sin 47^\circ}\right)^2 - 4\cos^2 45^\circ$

$$= \left[\frac{\sin 47^\circ}{\cos(90^\circ - 47^\circ)}\right]^2 + \left[\frac{\cos 43^\circ}{\sin(90^\circ - 43^\circ)}\right]^2 - 4\cos^2 45^\circ$$

$$= \left(\frac{\sin 47^\circ}{\sin 47^\circ}\right)^2 + \left(\frac{\cos 43^\circ}{\cos 43^\circ}\right)^2 - 4\cos^2 45^\circ$$

$$= (1)^2 + (1)^2 - 4 \times \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= 1 + 1 - 4 \times \frac{1}{2} = 2 - 2 = 0 \textbf{ Ans.}$$

(*ii*) $2\left[\dfrac{\cos 67^\circ}{\sin(90^\circ - 67^\circ)}\right] - \dfrac{\tan 40^\circ}{\cot(90^\circ - 40^\circ)} - \sin 90^\circ$

$$= 2\left(\frac{\cos 67^\circ}{\cos 67^\circ}\right)^2 - \left(\frac{\tan 40^\circ}{\tan 40^\circ}\right) - 1$$

$$= 2(1)^2 - (1) - 1 \quad \left\{\begin{array}{l}\sin(90^\circ - \theta) = \cos\theta \\ \cot(90^\circ - \theta) = \tan\theta \\ \sin 90^\circ = 1\end{array}\right\}$$

$$= 2 \times 1 - 1 - 1$$
$$= 2 - 1 - 1 = 0$$

Q. 9. (*i*) $\sin 37^\circ - \cos 53^\circ$

(*ii*) $\cos^2 25^\circ - \sin^2 65^\circ$

(*iii*) $\sin^2 23^\circ + \sin^2 67^\circ$

Sol. (*i*) $\sin 37^\circ - \cos 53^\circ$

$$= \sin 37^\circ - \cos(90^\circ - 37^\circ)$$
$$= \sin 37^\circ - \sin 37^\circ = 0 \quad \textbf{Ans.}$$

(*ii*) $\cos^2 25^\circ - \sin^2 65^\circ$

$$= \cos^2 25^\circ - \sin^2(90^\circ - 25^\circ)$$
$$= \cos^2 25^\circ - \cos^2 25^\circ = 0 \quad \textbf{Ans.}$$

(*iii*) $\sin^2 23^\circ + \sin^2 67^\circ$

$$= \sin^2 23^\circ + \sin^2(90^\circ - 23^\circ)$$
$$= \sin^2 23^\circ + \cos^2 23^\circ = 1 \quad \textbf{Ans.}$$
$$(\because \sin^2\theta + \cos^2\theta = 1)$$

Q. 10. (*i*) $\sec^2 36^\circ - \cot^2 54^\circ$

(*ii*) $\text{cosec}^2 48^\circ - \tan^2 52^\circ$

(*iii*) $\cos^2 24^\circ + \cos^2 66^\circ - \tan^2 45^\circ$.

Sol. (*i*) $\sec^2 36^\circ - \cot^2 54^\circ$

$$= \sec^2 36^\circ - \cot^2(90^\circ - 36^\circ)$$
$$= \sec^2 36^\circ - \tan^2 36^\circ = 1 \quad \textbf{Ans.}$$
$$\{\because \sec^2\theta - \tan^2\theta = 1\}$$

(*ii*) $\text{cosec}^2 48^\circ - \tan^2 52^\circ$

$$= \text{cosec}^2 48^\circ - \tan^2(90^\circ - 48^\circ)$$
$$= \text{cosec}^2 48^\circ - \cot^2 48^\circ \quad \{\tan(90^\circ - \theta) = \cot 0\}$$
$$= 1 \quad \{\text{cosec}^2\theta - \cot^2\theta = 1)$$

(*iii*) $\cos^2 24^\circ + \cos^2 66^\circ - \tan^2 45^\circ$

$$= \cos^2 24^\circ + \cos^2(90^\circ - 24^\circ) - \tan^2 45^\circ$$
$$= \cos^2 24^\circ + \sin^2 24^\circ - \tan^2 45^\circ$$
$$(\because \sin^2\theta + \cos^2\theta = 1)$$
$$= 1 - (1)^2 = 1 - 1 = 0 \textbf{ Ans.}$$

Q. 11. (*i*) $\dfrac{\cos^2 34^\circ + \cos^2 56^\circ}{\sin^2 59^\circ + \sin^2 31^\circ}$

(*ii*) $\dfrac{\text{cosec}^2 67^\circ - \tan^2 23^\circ}{\sec^2 20^\circ - \cot^2 70^\circ}$

Sol. (*i*) $\frac{\cos^2 34° + \cos^2 56°}{\sin^2 59° + \sin^2 31°}$

$= \frac{\cos^2 34° + \cos^2(90° - 34°)}{\sin^2 59° + \sin^2(90° - 59°)}$

$= \frac{\cos^2 34° + \sin^2 34°}{\sin^2 59° + \cos^2 59°}$

$= \frac{1}{1} = 1$ **Ans.** $\{\because \sin^2\theta + \cos^2\theta = 1\}$

(*ii*) $\frac{\text{cosec}^2 67° - \tan^2 23°}{\sec^2 20° - \cot^2 70°}$

$= \frac{\text{cosec}^2 67° - \tan^2(90° - 67°)}{\sec^2 20° - \cot^2(90° - 20°)}$

$= \frac{\text{cosec}^2 67° - \cot^2 67°}{\sec^2 20° - \tan^2 20°} = \frac{1}{1}$

$\{\because \text{cosec}^2\theta - \cot^2\theta = 1$ and $\sec^2\theta - \tan^2\theta = 1\}$

$= 1$ **Ans.**

Q. 12. Without using trigonometric tables, prove that :

(*i*) $\sin 73° \cos 17° + \cos 73° \sin 17° = 1$

(*ii*) $\sin 40° \sec 50° + \cos 40° \text{ cosec } 50° = 2$

(*iii*) $\sec^2 75° - \cot^2 15° = 1$

(*iv*) $\cos^2 23° + \cos^2 67° = 1$

(*v*) $\text{cosec}^2 56° - \tan^2 34° = 1$

(*vi*) $\tan 10° \tan 15° \tan 75° \tan 80° = 1$

(*vii*) $\tan 10° \tan 20° \tan 30° \tan 70° \tan 80° = \frac{1}{\sqrt{3}}$.

Sol. (*i*) L.H.S. $= \sin 73° \cos 17° + \cos 73° \sin 17°$

$= \sin 73° \cos(90° - 73°) + \cos 73° \sin(90° - 73°)$

$= \sin 73° \sin 73° + \cos 73° . \cos 73°$

$= \sin^2 73° + \cos^2 73° = 1 =$ R.H.S.

$(\because \sin^2\theta + \cos^2\theta = 1)$

(*ii*) L.H.S. $= \sin 40° \sec 50° + \cos 40° \text{ cosec } 50°$

$= \sin 40° \sec(90° - 40°) + \cos 40° \text{ cosec } (90° - 40°)$

$= \sin 40° \text{ cosec } 40° + \cos 40° \sec 40°$

$= \sin 40° \times \frac{1}{\sin 40°} + \cos 40° \times \frac{1}{\cos 40°}$

$= 1 + 1 = 2 =$ R.H.S.

(*iii*) L.H.S. $= \sec^2 75° - \cot^2 15°$

$= \sec^2(90° - 15°) - \cot^2 15°$

$= \text{cosec}^2 15° - \cot^2 15° = 1$

$(\because \text{cosec}^2\theta - \cot^2\theta = 1)$

= R.H.S.

(*iv*) L.H.S. $= \cos^2 23° + \cos^2 67°$

$= \cos^2(90° - 67°) + \cos^2 67°$

$= \sin^2 67° + \cos^2 67°$

$= 1 =$ R.H.S. $(\sin^2\theta + \cos^2\theta = 1)$

(*v*) L.H.S. $= \text{cosec}^2 56° - \tan^2 34°$

$= \text{cosec}^2(90° - 34°) - \tan^2 34°$

$= \sec^2 34° - \tan^2 34° = 1$

$(\because \sec^2\theta - \tan^2\theta = 1)$

= R.H.S.

(*vi*) L.H.S. $= \tan 10° \tan 15° \tan 75° \tan 80°$

$= \tan 10° \tan 80° \tan 15° \tan 75°$

$= \tan 10° \tan(90° - 10°) \tan 15° \tan(90° - 15°)$

$= (\tan 10° \cot 10°)(\tan 15° \cot 15°)$

$= 1 \times 1 = 1 =$ R.H.S.

(*vii*) L.H.S. $= \tan 10° \tan 20° \tan 30° \tan 70° \tan 80°$

$= \tan 10° \tan 80° \tan 20° \tan 70° \tan 30°$

$= \tan 10° \tan(90° - 10°) \tan 20° \tan(90° - 20°) \tan 30°$

$= (\tan 10° \cot 10°)(\tan 20° \cot 20°) \tan 30°$

$= 1 \times 1 \times \tan 30° = \frac{1}{\sqrt{3}} =$ R.H.S.

($\tan\theta \cot\theta = 1$)

Q.13. Express each of the following in terms of trigonometric ratios of angles between 0° and 45°.

(*i*) cos 81° + cot 81°

(*ii*) cos 76° + sec 76°

(*iii*) sec 63° + cosec 49°

(*iv*) sin 59° + cos 56°

Sol. (*i*) cos 81° + cot 81°

= cos (90° – 9°) + cot (90° – 9°)

= sin 9° + tan 9°

(*ii*) cos 76° + sec 76°

= cos (90° – 14°) + sec (90° – 14°)

= sin 14° + cosec 14°

(*iii*) sec 63° + cosec 49°

= sec (90° – 27°) + cosec (90° – 41°)

= cosec 27° + sec 41°

(*iv*) sin 59° + cos 56°

= sin (90° – 31°) + cos (90° – 34°)

= cos 31° + sin 34° **Ans.**

Q.14. If 0° < θ° < 25°, prove that

$\cos(65° + \theta) - \sin(25° - \theta) = 0$.

Sol. $\cos(65° + \theta) - \sin(25° - \theta)$

$= \cos(65° + \theta) - \sin[90° - (65° + \theta)]$

$= \cos(65° + \theta) - \cos(65° + \theta) = 0$

= R.H.S.

Q. 15. Prove that : $\sin(50° + \theta) - \cos(40° - \theta) = 0$.

Sol. L.H.S. $= \sin(50° + \theta) - \cos(40° - \theta)$

$= \cos[90° - (50° + \theta) - \cos(40° - \theta)$

[$\because \sin\theta = \cos(90° - \theta)$]

$= \cos(40° - \theta) - \cos(40° - \theta)$

= 0 = R.H.S.

Q.16. Prove that :

(*i*) $\frac{\cos A}{\sin(90° - A)} + \frac{\sin A}{\cos(90° - A)} = 2$.

(*ii*) $\frac{\sin A}{\sin(90° - A)} + \frac{\cos A}{\cos(90° - A)}$ = sec A cosec A.

(*iii*) $\frac{\sin A}{\sin(90° - A)} + \frac{\cos A}{\cos(90° - A)}$ = sec (90° – A) cosec (90° – A).

(*iv*) $\sin(90° - A)\cos(90° - A) = \frac{\tan A}{1 + \tan^2 A}$.

(*v*) tan (45° – A) tan (45° + A) = 1.

Sol. (*i*) L.H.S. $= \frac{\cos A}{\sin(90° - A)} + \frac{\sin A}{\cos(90° - A)}$

$= \frac{\cos A}{\cos A} + \frac{\sin A}{\sin A} = 1 + 1 = 2 =$ R.H.S.

(*ii*) L.H.S. $= \frac{\sin A}{\sin(90° - A)} + \frac{\cos A}{\cos(90° - A)}$

$= \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$

$= \frac{\sin^2 A + \cos^2 A}{\cos A \sin A} = \frac{1}{\cos A \sin A}$

= sec A cosec A

= R.H.S. ($\because \sin^2\theta + \cos^2\theta = 1$)

(*iii*) L.H.S. $= \frac{\sin A}{\sin(90° - A)} + \frac{\cos A}{\cos(90° - A)}$

$= \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A} = \frac{\sin^2 A + \cos^2 A}{\cos A \sin A}$

$= \frac{1}{\cos A \sin A}$ = sec A cosec A

R.H.S. = sec (90° – A) cosec (90° – A)

= cosec A sec A

Hence, L.H.S. = R.H.S.

(*iv*) L.H.S. = sin (90° – A) cos (90° – A)

= cos A sin A

R.H.S. $= \frac{\tan A}{1 + \tan^2 A} = \frac{\frac{\sin A}{\cos A}}{1 + \frac{\sin^2 A}{\cos^2 A}}$

$$= \frac{\frac{\sin A}{\cos A}}{\frac{\cos^2 A + \sin^2 A}{\cos^2 A}}$$

$$= \frac{\sin A}{\cos A} \times \frac{\cos^2 A}{\sin^2 A + \cos^2 A}$$

$$= \frac{\sin A \cos A}{1} (\because \sin^2 A + \cos^2 A = 1)$$

$= \sin A \cos A$

Hence, L.H.S. = R.H.S.

(*v*) L.H.S. = tan (45° – A) tan (45° + A)

= tan (90° – 45° – A) tan (45° + A) = tan {90° – (45° + A)} tan (45° + A)

= cot (45° + A) tan (45° + A)

$$\frac{1}{\tan(45^\circ + A)} \times \tan(45^\circ + A) = 1 = \text{R.H.S.}$$

Q.17. In ΔABC, prove that :

(*i*) $\sin\left(\frac{A+B}{2}\right) = \cos\frac{C}{2}$

(*ii*) $\tan\left(\frac{B+C}{2}\right) = \cot\frac{A}{2}$

Sol. (*i*) ∵ In a triangle,

$\angle A + \angle B + \angle C = 180^\circ$

$\therefore \angle A + \angle B = 180^\circ - \angle C$

$\Rightarrow \frac{A+B}{2} = 90^\circ - \frac{C}{2}$

Now, L.H.S. $= \sin\frac{A+B}{2}$

$= \sin\left(90^\circ - \frac{C}{2}\right) = \cos\frac{C}{2}$ = R.H.S.

(*ii*) $\because A + B + C = 180^\circ$

$\therefore B + C = 180^\circ - A$

$\Rightarrow \frac{B+C}{2} = 90^\circ - \frac{A}{2}$

Now, $\tan\frac{B+C}{2}$

$= \tan\left(90^\circ - \frac{A}{2}\right) = \cot\frac{A}{2}$

Hence proved.

EXERCISE 25 (C)

Q.1. Using tables find the value of :

(*i*) sin 83° 12′ (*ii*) sin 61° 14′

(*iii*) sin 9° 55′ (*iv*) sin 32° 40′

Sol. (*i*) sin 83° 12′ = **0·9930**

(From the table of sin x°)

(*ii*) sin 61° 12′ = 0·8763

Mean difference 2′ = 3

(Adding mean difference)

sin 61°14′ = **0·8766**

(*iii*) sin 9°54′ = ·01719

(From the table)

Mean difference 1′ = 3

(Adding mean difference)

sin 9° 55 = **0·1722**

(*iv*) sin 32°36′ = 0·5388 (From the table)

Mean of 4′ = 10 (Adding mean difference)

∴ sin 32° 40′ = **0·5398 Ans.**

Q.2. Using the table, find the value of :

(*i*) cos 48°36′ (*ii*) cos 23°6′

(*iii*) cos 70°17′ (*iv*) cos 85°8′

Sol. (*i*) cos 48° 36′ = 0·6613 (From the table)

(*ii*) cos 23° 6′ = 0·6613 (From the table)

(*iii*) cos 70°12′ = 0·3387

(From the table)

mean difference 5′ = – 14

∴ cos 70°17′ = **0·3373**

(subtracting the mean difference).

(*iv*) cos 85° 6′ = 0·0854 (From the table)

Mean difference of 2′ = – 6

(Subtracting the mean difference)

∴ cos 85° 8′ = **0·0848 Ans.**

Q. 3. Using tables find the values of :

(*i*) tan 24°24′ (*ii*) tan 9° 38′

(*iii*) tan 31°27′ (*iv*) tan 65°50′

Sol. (*i*) tan 24°24′ = 0·4536 using table

(*ii*) tan 9°36′ = 0·1691 using table

Mean difference of 2′ = + 6

(Adding the mean difference)

∴ tan 9° 38′ = **0·1697**

(*iii*) tan 31° 24′ = 0·6104 using table

ean difference of 3′ = + 12 (Adding the mean difference)

n 31° 27′ = **0·6116**

tan 65°48′ = 0·2251 (using the table)

Mean difference of 2′ = + 34 (Adding the mean difference)

∴ tan 65°50′ = **0·2285 Ans.**

Q. 4. Using tables, find the acute angle θ, when

(*i*) sin θ = 0·36 (*ii*) sin θ = 0·4274

(*iii*) sin θ = 0·5955

(*iv*) sin θ = 0·8229.

Sol. (*i*) sin θ = 0·36

From the table, we find that

sin 21° 6′ = 0·3600

Hence, θ = 21°6′

(*ii*) sin θ = 0·4274

From the table, we find that

sin 25°18′ = 0·4274

Hence, θ = 25°18′

(*iii*) sin θ = 0·5955

From the table, we find that

sin 36° 30′ = 0·5948

Mean difference of 3′ is 7

∴ sin 36°30′ = 0·5948

+ 3 + 7

sin 36° 33′ = 0·5955

Hence, θ = 36°33′

(*iv*) sin θ = 0·8229

From the table, we find that

sin 55°18′ = 0·8221

Mean difference of 5′ = + 8

∴ sin 55°23′ = 0·8229

Hence, θ = 55°23′ **Ans.**

Q. 5. If sin θ = 0·42, find :

(*i*) θ (*ii*) cos θ (*iii*) tan θ.

Sol. sin θ = 0·42

From the table, we find that

sin 24°48′ = 0·4195

Mean difference of 2′ = + 5

∴ sin 24°50′ = **0·4200 = 0·42**

(*i*) ∴ θ = 24°50′

(*ii*) cos 24°48′ = 0·9078 [From the table]

Mean difference of 2′ = –2 [subtracting the mean difference]

∴ cos 24°50′ = **0·9076**

(*iii*) tan 24°48′ = 0·4621 [From the Table]

Mean difference of 2′ = + 7 [Adding mean difference]

∴ tan 24°50′ = **0·4628 Ans.**

Q. 6. Using tables, find the acute angle θ, when

(*i*) cos θ = 0·94 (*ii*) cos θ = 0·8092

(*iii*) cos θ = 0·1679

Sol. (*i*) cos θ = 0·94

From the table, we find that

cos 19°54′ = 0·9403

Mean difference of 3′ = – 3

∴ cos 19°57′ = **0·9400** = 0·94

Hence, θ = 19°57′

(*ii*) cos θ = 0·8092

From the table, we find that

cos 35°54′ = 0·8100

Common difference of 5′ = – 8

∴ cos 35°59′ = **0·8092**

Hence, θ = 35°59′

(*iii*) cos θ = 0·1679

From the table, we see that

cos 80°18′ = 0·1685

Mean difference of 2′ = – 6

∴ cos 80°20′ = **0·1679**

Hence, $\theta = 80°20'$ **Ans.**

Q.7. If $\cos\theta = 0{\cdot}51$, find

(*i*) θ (*ii*) $\sin\theta$

(*iii*) $\tan\theta$.

Sol. (*i*) $\cos\theta = 0{\cdot}51$

From the table, we find that

$\cos 59°18' = 0{\cdot}5105$

Mean difference of $2' = -5$

[Subtracting the mean difference]

$\therefore \cos 59°20' = \mathbf{0{\cdot}5100 = 0{\cdot}51}$

Hence, $\theta = \mathbf{59°20'}$

(*ii*) $\sin 59°18' = 0{\cdot}8599$ [From the table]

Mean difference of $2' = +3$ [Adding mean difference]

$\therefore \sin 59°20' = \mathbf{0{\cdot}8602}$

(*iii*) $\tan 59°18' = 0{\cdot}6842$ [From the table]

Mean difference of $2' = +23$ [Adding mean difference]

$\therefore \tan 59°20' = \mathbf{0{\cdot}6865}$ **Ans.**

Q.8. Using tables to find the acute angle θ, when :

(*i*) $\tan\theta = 1{\cdot}476$

(*ii*) $\tan\theta = 2{\cdot}91$

(*iii*) $\tan\theta = 0{\cdot}3$

Sol. (*i*) $\tan\theta = 1{\cdot}476$

From the table, we find that

$\tan 55°\,48' = 1{\cdot}4715$

Mean difference of $5' = +45$

$\tan 55°53' = 1{\cdot}4760 = \mathbf{1{\cdot}4760}$

$\therefore \theta = \mathbf{55°53'}$ **Ans.**

(*ii*) $\tan\theta = 2{\cdot}91$

From the table, we find that

$\tan 71°{\cdot}0' = 2{\cdot}9042$

Mean difference of $2' = +58$

$\tan 71°\,2' = \mathbf{2{\cdot}9100}$

$\therefore \theta = \mathbf{71°2'}$ **Ans.**

(*iii*) $\tan\theta = 0{\cdot}3$

From the table, we find that

$\tan 16°42' = 0{\cdot}3000 = 0{\cdot}3$

$\therefore \theta = \mathbf{16°42'}$ **Ans.**