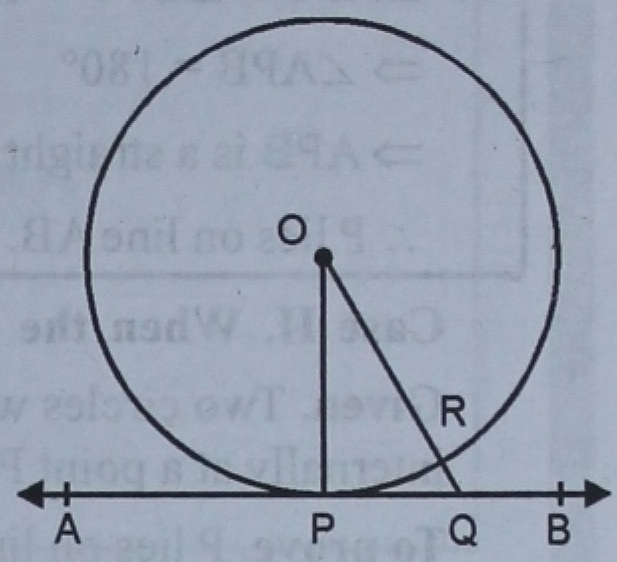
Chapter 21

Tangent Properties of Circles

POINTS TO REMEMBER

1. Some Results (Theorems)



Theorem 1. *The tangent at any point of a circle and the radius through the point are perpendicular to each other.*

Given. A circle with centre O, AB is a tangent to the circle at a point P and OP is the radius through P.

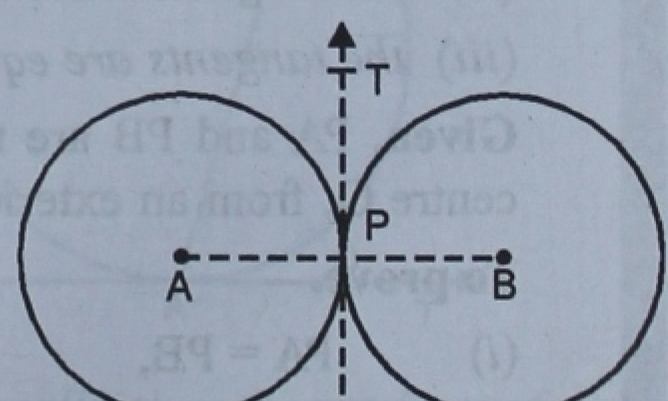
To prove. $OP \perp AB$.

Construction. Take a point Q, other than P, on tangent AB. Join OQ.

Proof.

Statement	Reason
1. Since Q is a point on tangent AB, other than the point P, so Q will be outside the circle. $\therefore$ OQ will intersect the circle at some point R.	Tangent at P intersects the circle at point P only.
2. $\therefore OR < OQ$ $\Rightarrow OP < OQ$	A part is less than its whole. $OR = OP =$ radius.
3. Thus, OP is shorter than any other line segment joining O to any point of AB.	
4. $OP \perp AB$	Of all line segments drawn from O to line AB, the perpendicular is the shortest.
Hence, the radius OP is perpendicular to tangent at P.	

Theorem 2. *If two circles touch each other, the point of contact lies on the straight line through their centres.*



Case 1. When the given two circles touch each other externally.

Given. Two circles with centres A and B, touching each other externally at a point P.

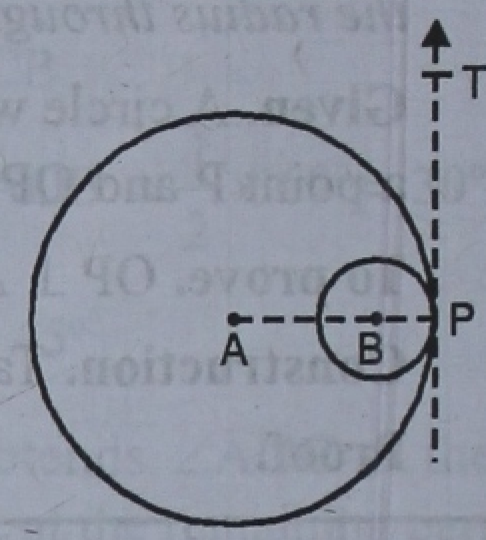
To prove. P lies on line AB.

Construction. At the point P, draw a common tangent PT to the two circles. Join AP and BP.

Proof.

Statement	Reason
1. ∠APT = 90°	Radius through the point of contact is perpendicular to the tangent.
2. ∠BPT = 90°	Same as above.
3. ∠APT + ∠BPT = 180° ⇒ ∠APB = 180° ⇒ APB is a straight line. ∴ P lies on line AB.	Adding 1 and 2.

Case II. When the given two circles touch each other internally.



Given. Two circles with centres A and B, touching each other internally at a point P.

To prove. P lies on line AB.

Construction. At the point P, draw a common tangent PT. Join AP and BP.

Proof.

Statement	Reason
1. AP ⊥ PT	Radius through the point of contact is perpendicular to the tangent.
2. BP ⊥ PT	Same as above.
3. AP and BP are both perpendicular to the same line PT.	From 1 and 2.
4. AP and BP lie in the same line ⇒ ABP is a straight line. ∴ P lies on line AB.	

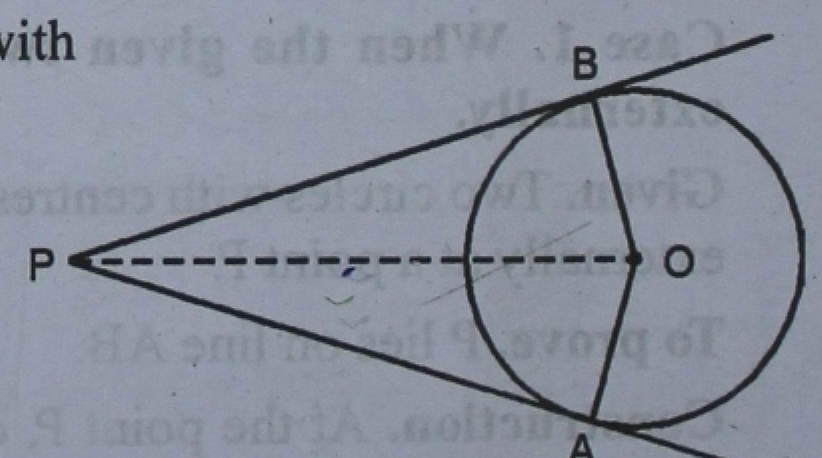
Theorem 3. *If two tangents are drawn to a circle from an exterior point, then*

(*i*) *the tangents are equal in length;*

(*ii*) *the tangents subtend equal angles at the centre;*

(*iii*) *the tangents are equally inclined to the line joining the point and the centre of the circle.*

Given. PA and PB are two tangents drawn to a circle with centre O, from an exterior point P.



To prove.

(*i*) PA = PB,

(*ii*) ∠AOP = ∠BOP,

(*iii*) ∠APO = ∠BPO.

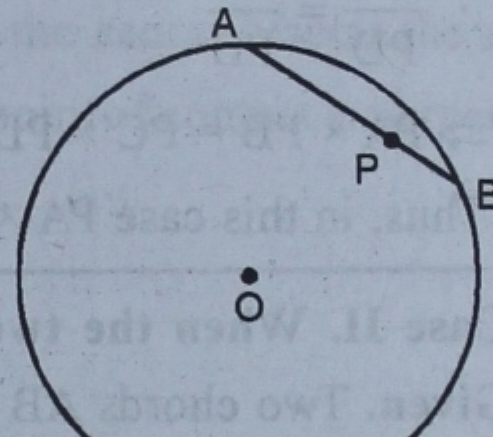
Proof.

Statement	Reason
1. In ΔAOP and ΔBOP :	
OA = OB	Radii of the same circle.
∠OAP = ∠OBP = 90°	Radius through point of contact is perpendicular to the tangent.
OP = OP	common.
∴ ΔAOP ≅ ΔBOP	S.S.A. (axiom of congruency)
2. Hence, we have	
(*i*) PA = PB	c.p.c.t.
(*ii*) ∠AOP = ∠BOP	c.p.c.t.
(*iii*) ∠APO = ∠BPO	c.p.c.t.

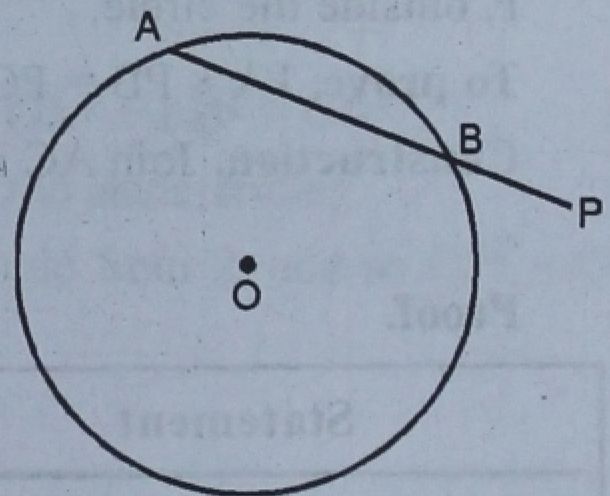
Intersecting chords and Tangents :

(*a*) Segments of a chord :

(*i*) If P is a point on a chord AB of a circle, then we say that P divides AB internally into two segments PA and PB.



(*ii*) If AB is a chord of a circle and P is a point on AB produced, we say that P divides AB externally into two segments PA and PB.



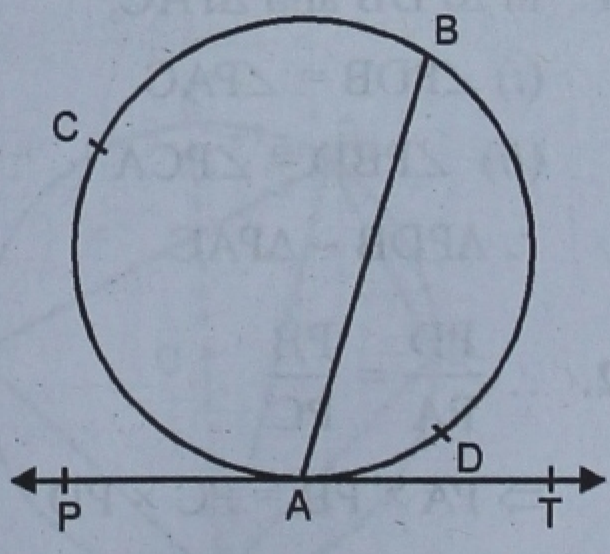
. **Alternate Segments :**

In the given figure, PAT is a tangent to the circle at a point A and AB is a chord.

The chord AB divides the circle into two segments, namely ADB and BCA, called the **alternate segments.**

For ∠BAT, the alternate segment is BCA.

For ∠BAP, the alternate segment is ADB.



. **Some more Results (Theorems)**

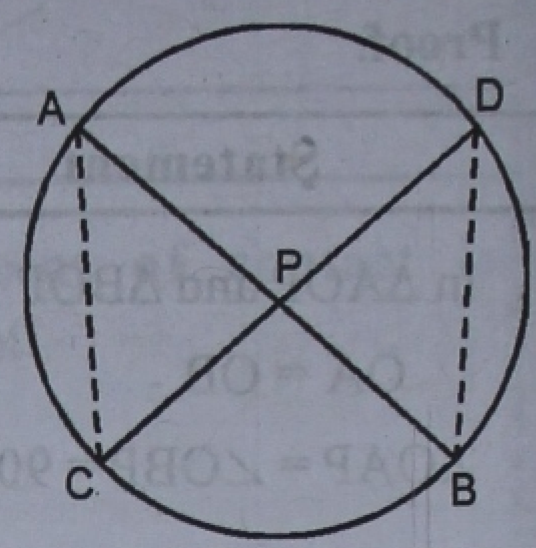
Theorem 1. *If two chords of a circle intersect internally or externally, then the products of the lengths of their segments are equal.*

Case 1. When the two chords intersect internally.

Given. Two chords AB and CD of a circle intersect each other at a point P inside the circle.

To prove. PA × PB = PC × PD.

Construction. Join AC and BD.



Proof.

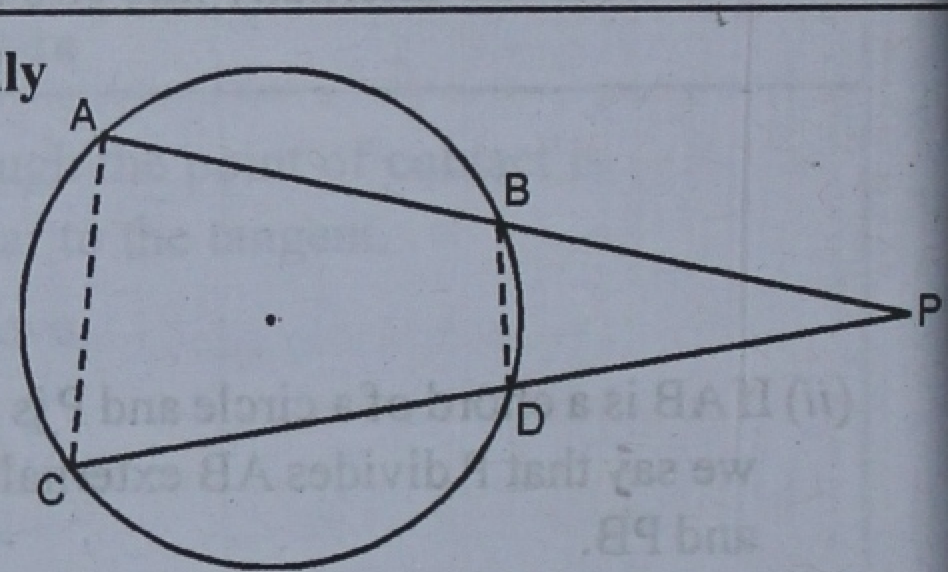
Statement	Reason
1. In ΔAPC and ΔDPB,	
(*i*) ∠APC = ∠DPB	Vert. opp. angles
(*ii*) ∠PAC = ∠PDB	angles in the same segment.
∴ ΔAPC ~ ΔDPB	By AA-similarity axiom.
2. ∴ $\frac{PA}{PD} = \frac{PC}{PB}$	Corresponding sides of similar Δ *s* are proportional
⇒ PA × PB = PC × PD.	
Thus, in this case PA × PB = PC × PD.	

Case II. When the two chords intersect externally

Given. Two chords AB and CD of a circle, when produced, intersect each other at a point P, outside the circle.

To prove. PA × PB = PC × PD.

Construction. Join AC and BD.

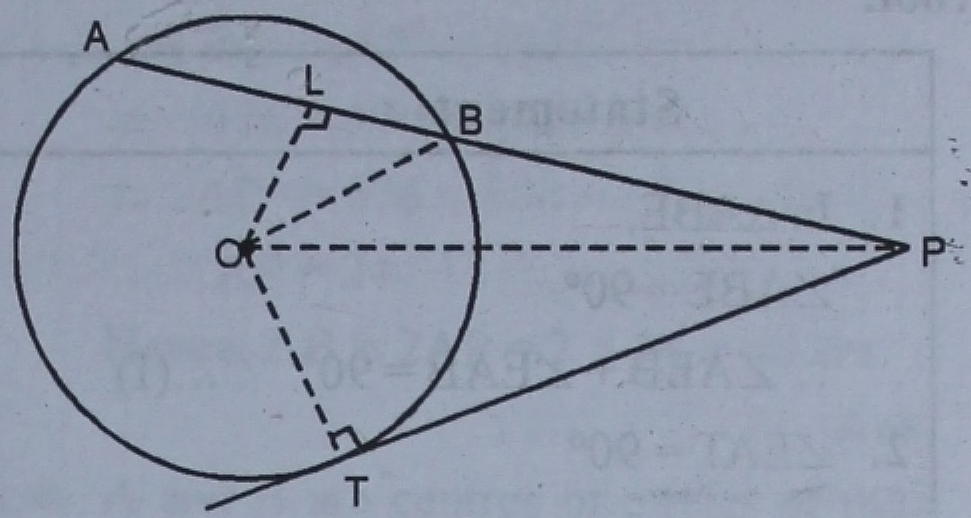


Proof.

Statement	Reason
1. In ΔPDB and ΔPAC,	
(*i*) ∠PDB = ∠PAC	Exterior angle of a cyclic quad. = Int. opp. angle.
(*ii*) ∠PBD = ∠PCA	Exterior angle of a cyclic quad. = Int. opp. angle.
∴ ΔPDB ~ ΔPAC	By AA–similarity axiom.
2. ∴ $\frac{PD}{PA} = \frac{PB}{PC}$	Corresponding sides of similar Δ *s* are proportional.
⇒ PA × PB = PC × PD.	
∴ In this case also, PA × PB = PC × PD.	

Hence, in both the cases, we have PA × PB = PC × PD.

Theorem 2. *If a chord and a tangent intersect externally, then the product of the lengths of segments of the chord is equal to the square of the length of the tangent from the point of contact to the point of intersection.*



Given. A circle with centre O and tangent to the circle at a point T intersects the chord AB produced at a point P outside the circle.

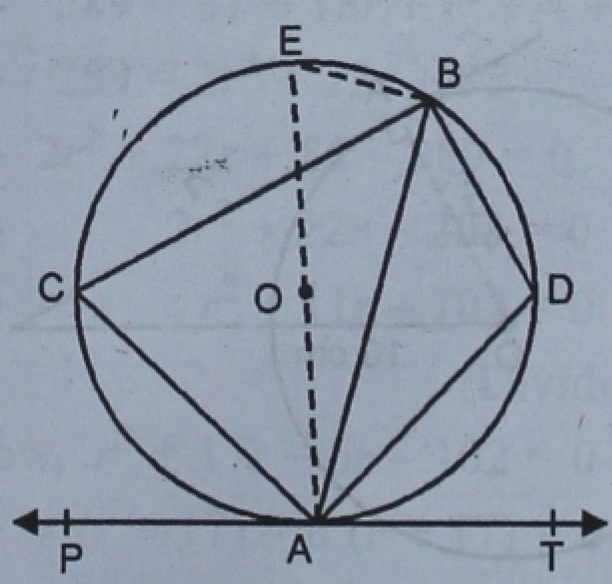
To prove. $PA \times PB = PT^2$.

Construction. Draw $OL \perp AB$. Join OB, OP and OT.

Proof.

Statement	Reason
1. AL = LB	Perpendicular from the centre bisects the chord.
2. $OT \perp TP$	Radius through the point of contact is perpendicular to the tangent.
3. $PA \times PB = (PL + AL) \cdot (PL - LB)$	
$= (PL + LB) \cdot (PL - LB)$	AL = LB, from 1.
$= PL^2 - LB^2$	
$= (OP^2 - OL^2) - LB^2$	From right Δ OLP, $OP^2 = OL^2 + PL^2$.
$= OP^2 - (OL^2 + LB^2)$	
$= OP^2 - OB^2$	From right Δ OLB, $OL^2 + LB^2 = OB^2$.
$= OP^2 - OT^2$	OB = OT (Radii of the same circle)
$= PT^2$	Δ OTP is right triangle from 2, and so $OP^2 = OT^2 + PT^2$.
$\therefore PA \times PB = PT^2$.	

Theorem 3. *The angle between a tangent and a chord through the point of contact is equal to an angle in the alternate segment.*



Given. A circle with centre O and PAT is the tangent at A. Through A, chord AB is drawn. Points C and D are taken in alternate segments BA and AB respectively.

To prove. (*i*) $\angle BAT = \angle ACB$ and

(*ii*) $\angle BAP = \angle ADB$.

Construction. Draw the diameter AOE and join EB.

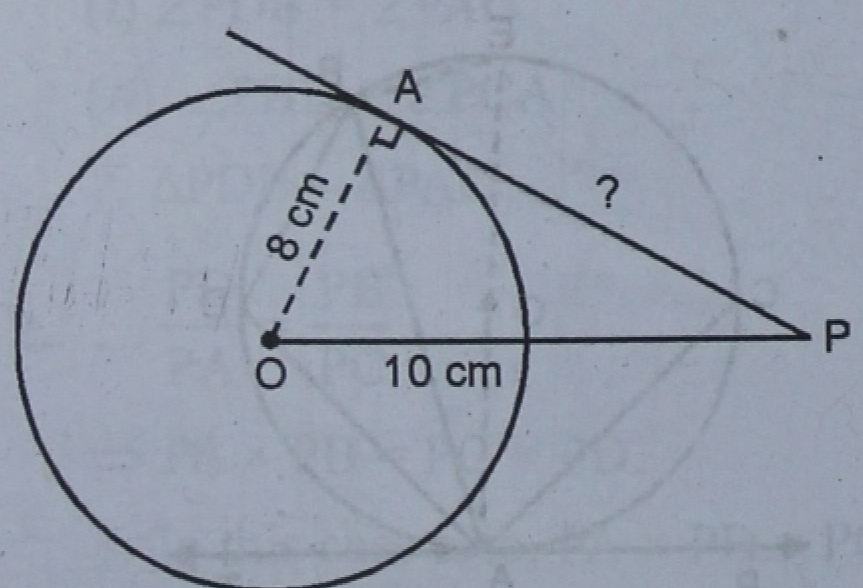
Proof.

Statement	Reason
1. In ΔABE,	
$\angle ABE = 90^\circ$	Angle in a semi-circle.
$\therefore \angle AEB + \angle EAB = 90^\circ$ ...(I)	Sum of the $\angle s$ of a Δ is 180°.
2. $\angle EAT = 90^\circ$	Diameter through the point of contact is perpendicular to the tangent.
$\Rightarrow \angle EAB + \angle BAT = 90^\circ$ ...(II)	$\angle EAT = \angle EAB + \angle BAT$
3. $\therefore \angle AEB + \angle EAB = \angle EAB + \angle BAT$	From (I) and (II).
$\Rightarrow \angle AEB = \angle BAT$	
4. But, $\angle AEB = \angle ACB$	Angles in the same segment.
5. $\therefore \angle BAT = \angle ACB$	From 3 and 4.
This proves one part of the theorem.	
6. Now, $\angle BAP + \angle BAT = 180^\circ$	
$\Rightarrow \angle BAP + \angle ACB = 180^\circ$	$\angle BAT = \angle ACB$
7. Also, $\angle ADB + \angle ACB = 180^\circ$	Opposite $\angle s$ of a cyclic quadrilateral.
8. $\therefore \angle BAP + \angle ACB = \angle ADB + \angle ACB$	From 6 and 7.
$\Rightarrow \angle BAP = \angle ADB$ ...(III)	
This proves the second result.	
Hence, $\angle BAT = \angle ACB$ and $\angle BAP = \angle ADB$.	

EXERCISE 21 (A)

Q. 1. Find the length of the tangent drawn to a circle of radius 8 cm., from a point which is at a distance of 10 cm. from the centre of the circle.

Sol. In the circle, OA is radius and AP is the tangent to the circle



$\therefore$ OA = 8 cm, OP = 10 cm.

$\because OA \perp AP$ or $\angle OAP = 90^\circ$

$\therefore$ In right ΔOAP,

$$OP^2 = OA^2 + AP^2$$

(Pythagoras Theorem)

$\Rightarrow (10)^2 = (8)^2 + AP^2$

$\Rightarrow 100 = 64 + AP^2$

$\Rightarrow AP^2 = 100 - 64 = 36 = (6)^2$

$\therefore$ AP = 6 cm. **Ans.**

Q. 2. A point P is 17 cm. away from the centre of the circle and the length of the tangent drawn from P to the circle is 15 cm. Find the radius of the circle.

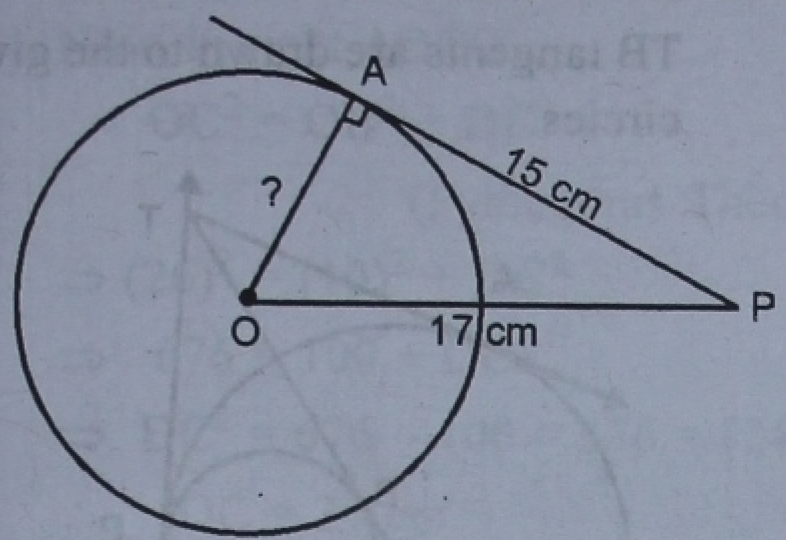
Sol. In the circle, OA is the radius, AP is the tangent drawn from P

$\therefore \angle OAP = 90^\circ$ or $OA \perp AP$

Now, in right ΔOAP,

$$OP^2 = OA^2 + AP^2$$

(Pythagoras Theorem)



$\Rightarrow (17)^2 = OA^2 + (15)^2$

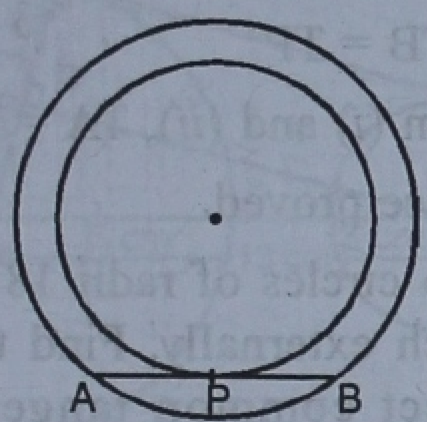
$\Rightarrow 289 = OA^2 + 225$

$\Rightarrow OA^2 = 289 - 225 = 64 = (8)^2$

$\therefore \quad OA = 8$

Hence, radius of the circle = 8 cm. **Ans.**

Q.3. There are two concentric circles, each with centre O and of raddi 10 cm and 26 cm respectively. Find the length of the chord AB of the outer circle which touches the inner circle at P.

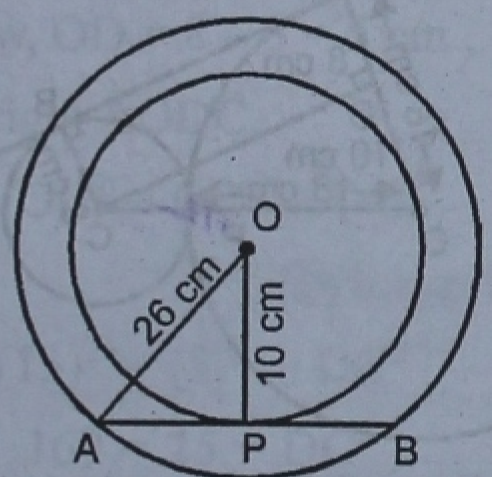


Sol. Radius (r) of the inner circle = 10 cm.

Radius (R) of the outer circle = 26 cm.

AB is the chord of the outer circle and tangent to the inner circle at P.

Join OA and OP.



$\because$ AB is tangent and OP the radius of the inner circle.

$\therefore$ OP ⊥ AB and P bisects the chord AB of the outer circle.

Now, in right ΔOAP,

$^2 = AP^2 + OP^2$ (Pythagoras Theorem)

$\Rightarrow (26)^2 = AP^2 + (10)^2$

$\Rightarrow 676 = AP^2 + 100$

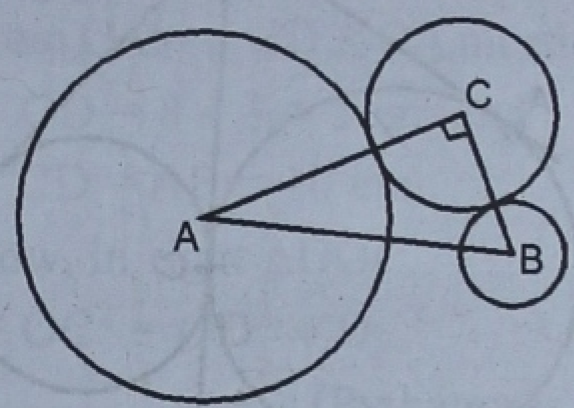
$\Rightarrow AP^2 = 676 - 100 = 576 = (24)^2$

$\therefore \quad AP = 24$

Hence, AB = 2AP = 2 × 24 = 48 cm.

Ans.

Q. 4. A and B are centres of circles of radii 9 cm and 2 cm such that AB = 17 cm and C is the centre of the circle of radius r cm which touches the above circles externally. If ∠ACB = 90°, write an equation in r and solve it.



Sol. A, B and C are the centres of the three circles, such that circle with centre C touches the other two circles externally.

Radius of circle with centre A = 9 cm.

Radius of circle with centre B = 2 cm.

AB = 17 cm. and ∠ACB = 90°

Let, radius of the third circle = r

$\therefore$ AC = $(9 + r)$ = cm.

arc BC = $(2 + r)$ = cm.

Now, in right ΔACB,

$AB^2 = AC^2 + BC^2$

$\Rightarrow (17)^2 = (9 + r)^2 + (2 + r)^2$

$\Rightarrow 289 = 81 + 18r + r^2 + 4 + 4r + r^2$

$\Rightarrow 289 = 2r^2 + 22r + 85$

$\Rightarrow 2r^2 + 22r + 85 - 289 = 0$

$\Rightarrow 2r^2 + 22r - 204 = 0$

$\Rightarrow r^2 + 11r - 102 = 0$

(Dividing by 2)

Now, $r^2 + 17r - 6r - 102 = 0$

$\Rightarrow r(r + 17) - 6(r + 17) = 0$

$\Rightarrow (r + 17)(r - 6) = 0$

(Zero Product Rule)

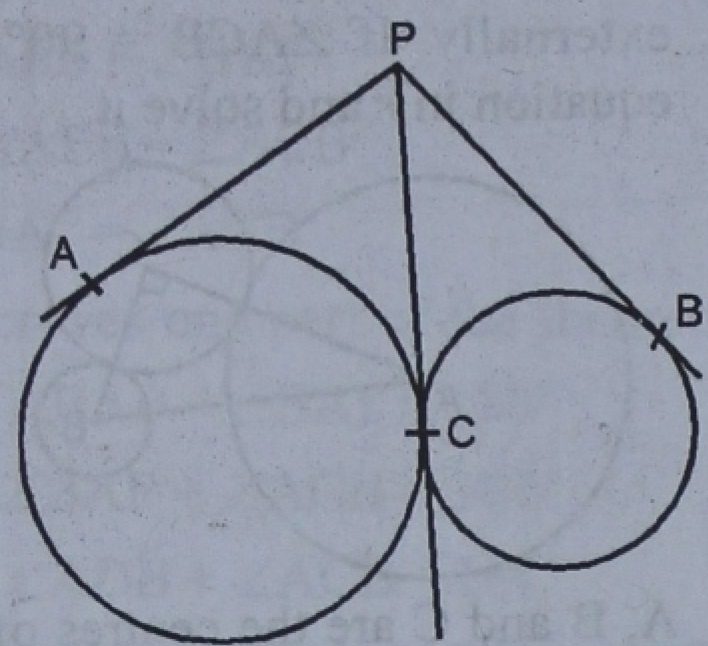
Either $r + 17 = 0$, then $r = -17$, but it is not possible.

Or $r - 6 = 0$, then $r = 6$

Hence, radius of the third circle (r)

$= 6$ cm. **Ans.**

Q. 5. Two circles touch each other externally at a point C and P is a point on the common tangent at C. If PA and PB are tangents to the two circles, prove that PA = PB.



Sol. Given. Two circles touch each other externally at C. Through C, a common tangent is drawn. From a point P on it, tangents PA and PB are drawn to their respective circles.

To prove. PA = PB

Proof. From P, PA and PC are the tangents drawn to the first circle

$\therefore$ PA = PC ...(*i*)

Similarly, from P, PB and PC are the tangents drawn to the second circle

$\therefore$ PB = PC ...(*ii*)

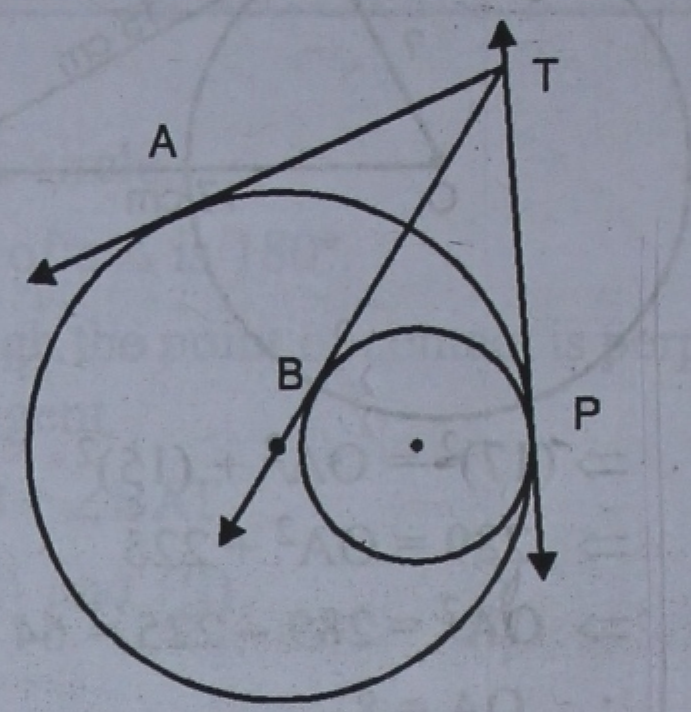
From (*i*) and (*ii*),

PA = PB

Hence proved.

Q. 6. Two circles touch each other internally. Prove that the tangents drawn to the two circles from any point on the common tangent are equal in length.

Sol. Given. Two circles touch each other at P internally. A common tangent is drawn from P. From a point T on it, TA and TB tangents are drawn to the given tw circles.



To prove. TA = TB.

Proof. $\because$ From T , TA and TP are th tangents to the first circle.

$\therefore$ TA = TP ...(i

Similarly, from T, TB and TP are th tangents to the second circle.

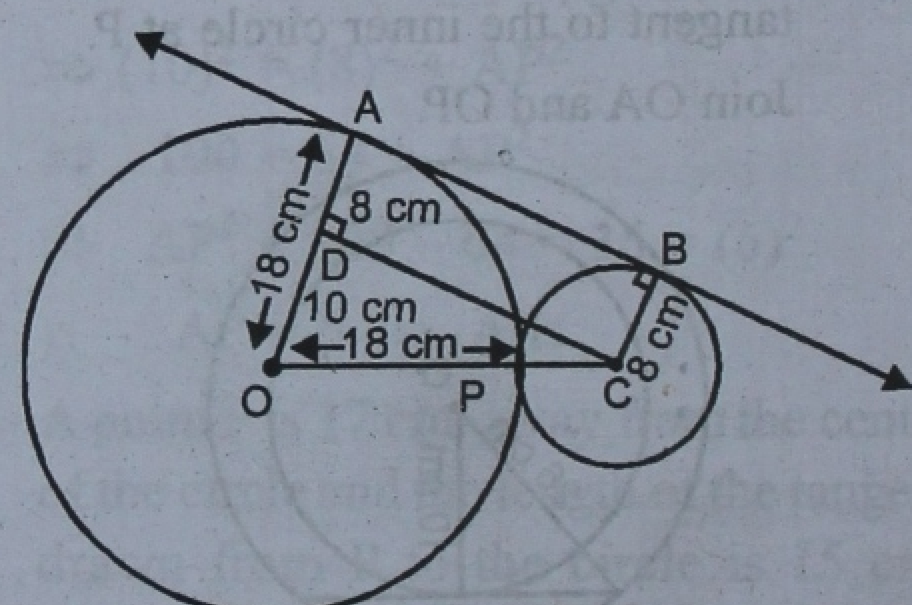
$\therefore$ TB = TP ...(ii

From (*i*) and (*ii*), TA = TB

Hence proved.

Q. 7. Two circles of radii 18 cm. and 8 cr touch externally. Find the length of direct common tangent to the tw circles.

Sol. Two circles with centres O and C touc each other externally at P.



Radius of the first circle is 18 cm an second circle is 8 cm.

AB is the direct common tangent. Fro C, draw CD $\perp$ AO meeting OA at D.

$\therefore$ OD = OA – AD = 18 – 8 = 10 cm.

OC = OP + PC = 18 + 8 = 26 cm.

Now, in right ΔODC,

$OC^2 = OD^2 + DC^2$ (Pythagoras Theorem)

$\Rightarrow (26)^2 = (10)^2 + DC^2$

$\Rightarrow 676 = 100 + DC^2$

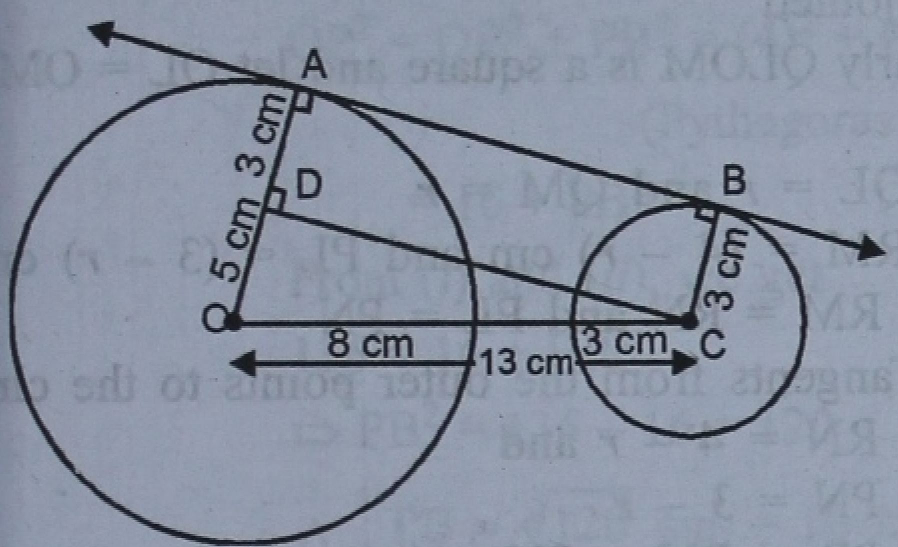
$\Rightarrow DC^2 = 676 - 100 = 576 = (24)^2$

$\therefore$ DC = 24 cm.

$\therefore$ AB = DC = 24 cm. **Ans.**

Q. 8. Two circles of radii 8 cm and 3 cm have their centres 13 cm apart. Find the length of a direct common tangent to the two, circles.

Sol. Two circles with centres O and C are drawn of the radii 8 cm and 3 cm. Their, centres are 13 cm apart.



AB is their common direct tangent. Join OA and CB

Through C, draw a perpendicular CD to OA meeting it at D.

Now, OD = 8 – 3 = 5 cm., OC = 13 cm.

In right ΔODC,

$OC^2 = OD^2 + DC^2$ (Pythagoras Theorem)

$\Rightarrow (13)^2 = (5)^2 + DC^2$

$\Rightarrow 169 = 25 + DC^2$

$\Rightarrow DC^2 = 169 - 25 = 144.$

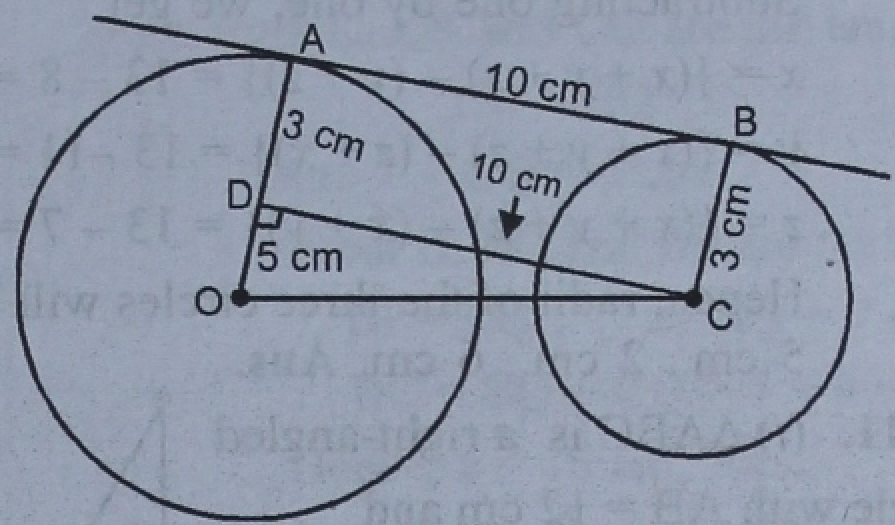
$\therefore$ DC = $\sqrt{144}$ = 12 cm. **Ans.**

Q.9. Two circles of radii 8 cm and 3 cm have a direct common tangent of length 10 cm. Find the distance between their centres, up to two places of decimal.

Sol. Two circles of radii 8 cm and 3cm have O and C as their centres respectively.

AB is their common direct tangent

OA = 8 cm, CB = 3 cm, AB = 10 cm.



From C, draw CD $\perp$ OA meeting OA at D

$\therefore$ OD = 8 – 3 = 5 cm and

CD = AB = 10 cm.

Now, in right ΔDOC,

$OC^2 = OD^2 + DC^2$ (Pythagoras Theorem)

$= (5)^2 + (10)^2 = 25 + 100 = 125$

$= 25 \times 5$

$\therefore OC = \sqrt{25 \times 5} = 5\sqrt{5}$

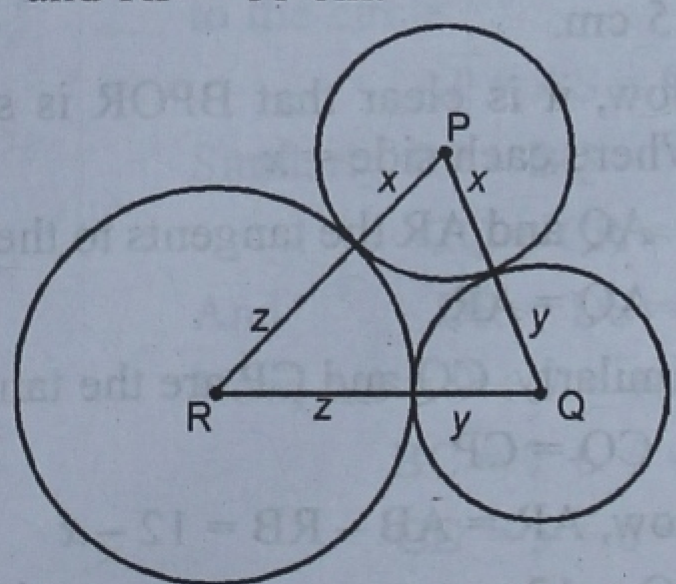
$= 5 \times (2{\cdot}236) = 11{\cdot}18$ cm.

$\therefore$ Distance between their centres

= 11·18 cm. **Ans.**

Q. 10. With the vertices of ΔPQR as centres, three circles are described, each touching the other two externally. If the sides of the triangle are 7 cm., 8 cm. and 11 cm. find the radii of the three circles.

Sol. Let PQ = 7 cm., QR = 8 cm and RP = 11 cm.



Let x, y, z be the radii of the three circles, then

$x + y = 7, y + z = 8, z + x = 11$

Adding, we get

$2(x + y + z) = 26 \Rightarrow x + y + z = \frac{26}{2} = 13$

Subtracting one by one, we get

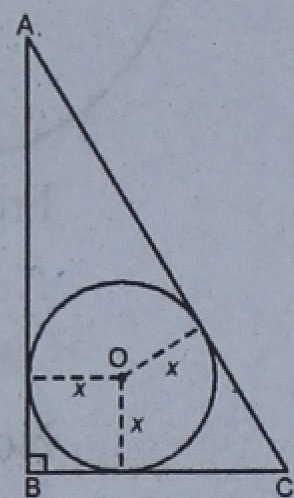
$x = \{(x + y + z) - (y + z)\} = 13 - 8 = 5$

$y = \{(x + y + z) - (z + x)\} = 13 - 11 = 2$

$z = \{(x + y + z) - (x + y)\} = 13 - 7 = 6$

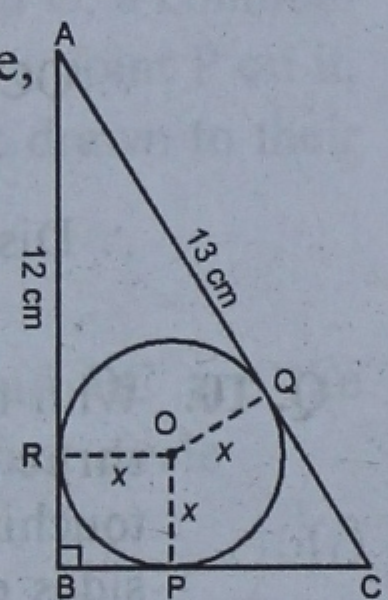
Hence, radii of the three circles will be 5 cm., 2 cm., 6 cm. **Ans.**

Q. 11. (*i*) ΔABC is a right-angled triangle with AB = 12 cm and AC = 13 cm. A circle with centre O has been inscribed inside the triangle. Calculate the value of x, the radius of the inscribed circle.



(*ii*) PQR is a right angled triangle with PQ = 3 cm and QR = 4 cm A circle which touches all the sides of the triangle is inscribed in the triangle. Calculate the radius of the circle.

Sol. ΔABC is a right-angled triangle, right angle at B, AB = 12 cm AC = 13 cm. A circle with centre O is drawn in the triangle touching its sides at P, Q, R respectively.



Now, in right ΔABC,

$AC^2 = AB^2 + BC^2$ (Pythagoras Theorem)

$\Rightarrow (13)^2 = (12)^2 + BC^2 \Rightarrow 169 = 144 + BC^2$

$\Rightarrow BC^2 = 169 - 144 = 25 = (5)^2$

$\therefore$ BC = 5 cm.

Now, it is clear that BPOR is square. Where each side = x

$\because$ AQ and AR the tangents to the circle

$\therefore$ AQ = AR

Similarly, CQ and CP are the tangents

$\therefore$ CQ = CP

Now, AR = AB – RB = 12 – x

$\Rightarrow$ AQ = 12 – x ...(*i*)

CP = BC – BP = 5 – x

$\Rightarrow$ CQ = 5 – x ...(ii)

Adding (*i*) and (*ii*), we get

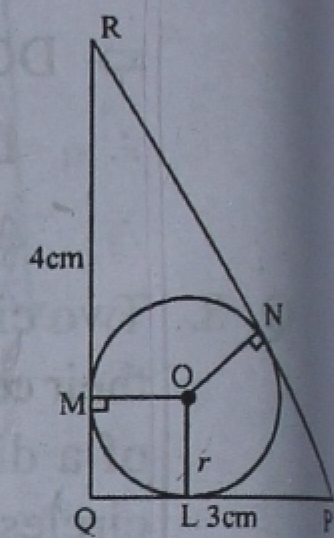
AQ + CQ = 12 – x + 5 – x

AC = 17 – 2x $\Rightarrow$ 13 = 17 – 2x $\Rightarrow$ 2x = 17 – 13 = 4

$\therefore x = \frac{4}{2} = 2$

Hence, value of x = 2 cm. **Ans.**

(*ii*) In Δ PQR, $\angle$Q = 90° and PQ = 3 cm, QR = 4cm



$\therefore$ PR $= \sqrt{PQ^2 + QR^2}$

$= \sqrt{(3)^2 + (4)^2}$

$= \sqrt{9 + 16} = \sqrt{25}$ = 5 cm

A circle with centre O, is drawn which touches the sides of the Δ PQR at L, M and N respectively. Let O be the centre of the circle OL, OM, ON are joined

clearly QLOM is a square and let OL = OM = r

$\therefore$ QL = r and QM = r

$\therefore$ RM = (4 – r) cm and PL = (3 – r) cm

But RM = RN and PL = PN

(Tangents from the outer points to the circle)

$\therefore$ RN = 4 – r and

PN = 3 – r

$\therefore$ PR = RN + PN = 4 – r + 3 – r

= 7 – 2r

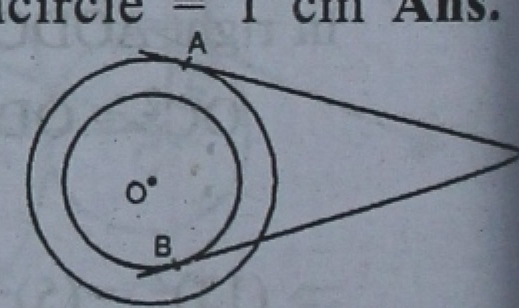
But PR = 5cm

$\therefore$ 5 = 7 – 2r

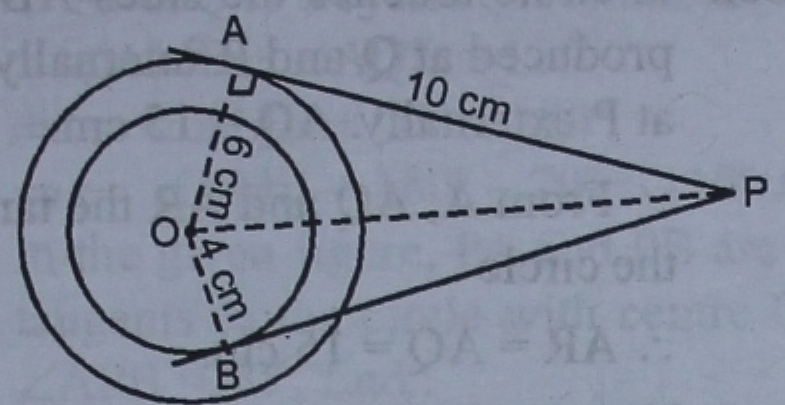
$\Rightarrow 2r = 7 - 5 = 2 \Rightarrow r = \frac{2}{2} = 1$

Hence radius of the incircle = 1 cm **Ans.**

Q. 12. In the given figure, O is the centre of each of two concentric circles of radii 4 cm and 6 cm respectively. PA and PB are tangents to outer and inner circle respectively. If PA = 10 cm., find the length of PB, upto two places of decimal.



Sol. Two concentric circles with centre O and radius OA and OB respectively. AP and BP are the tangents drawn from P in the circles. Join OA, OB and OP. AP = 10 cm., OA = 6 cm, OB = 4 cm



$\because$ AP is tangent and OA is radius

$\therefore$ OA ⊥ AP

Similarly, OB ⊥ BP

Now, in right ΔOAP,

$OP^2 = OA^2 + AP^2 = (6)^2 + (10)^2$ (Pythagoras Theorem)

$= 36 + 100 = 136$ ...(*i*)

Similarly, in right ΔOBP

$OP^2 = OB^2 + PB^2 = (4)^2 + PB^2$ (Pythagoras Theorem)

$= 16 + PB^2$ ...(*ii*)

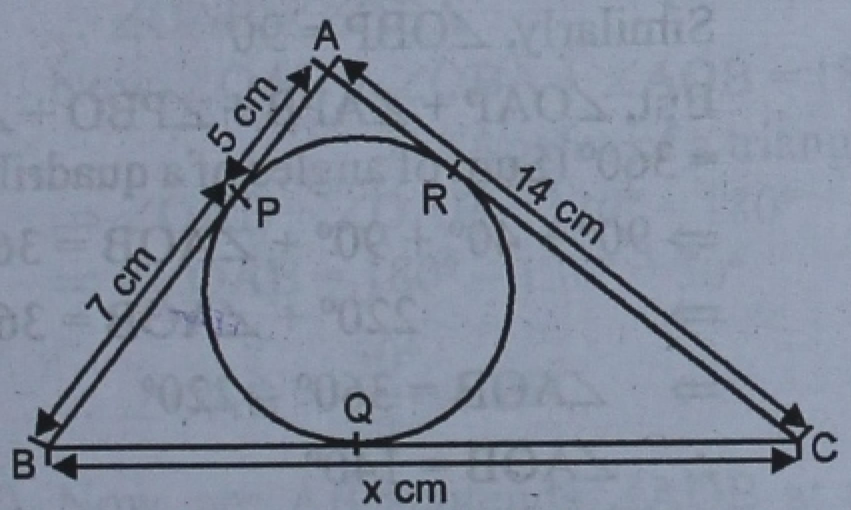
From (*i*) and (*ii*), we get

$136 = 16 + BP^2$

$\Rightarrow PB^2 = 136 - 16 = 120$

$\therefore$ $PB = \sqrt{120}$ cm. = 10·95 cm. **Ans.**

Q. 13. In the given figure, ΔABC is circumscribed. The circle touches the sides AB, BC and CA at P, Q, R respectively.



If AP = 5 cm, BP = 7 cm, AC = 14 cm and BC = *x* cm, find the value of *x*.

Sol. ΔABC is circumscribed and circle touches its sides AB, BC, CA at P, Q and R respectively.

AP = 5 cm., BP = 7 cm.,

AC = 14 cm.

and BC = *x*

From A, AP and AR arc the tangents to the circle

$\therefore$ AP = AR $\Rightarrow$ AR = 5 cm.

$\therefore$ CR = 14 – 5 = 9 cm.

Now, from C, CR and CQ are the tangents

$\therefore$ CR = CQ

$\Rightarrow$ CQ = 9 cm

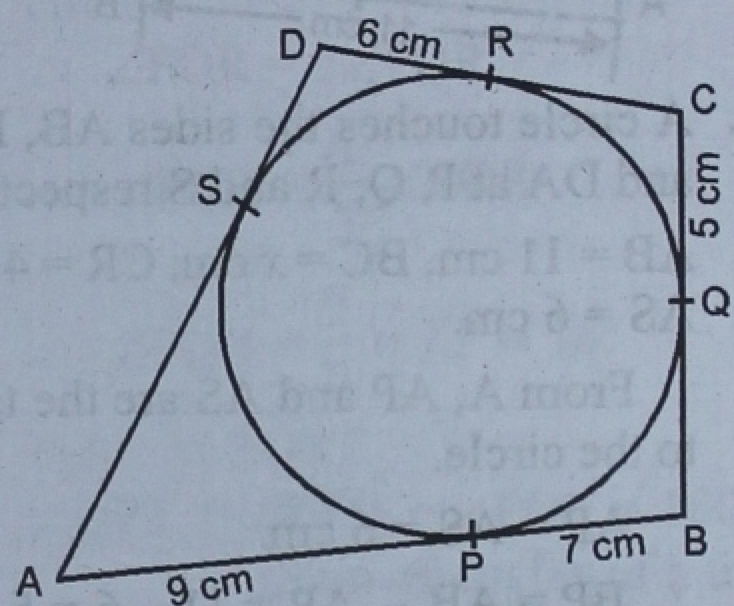
From B, BQ and BP are the tangents.

$\therefore$ BP = BQ $\Rightarrow$ BQ = 7 cm.

$\therefore$ BC = BQ + CQ = 7 + 9 = 16 cm.

Hence, *x* = 16 cm. **Ans.**

Q. 14. In the given figure, quadrilateral ABCD is circumscribed. The circle touches the sides AB, BC, CD and DA at P, Q, R, S, respectively. If AP = 9 cm, BP = 7 cm, CQ = 5 cm and DR = 6 cm, find the perimeter of quad. ABCD.



Sol. Quadrilateral ABCD is circumscribed. A circle touches its sides AB, BC, CD and DA at P, Q, R and S respectively.

AP = 9 cm, BP = 7 cm, CQ = 5 cm and DR = 6 cm.

$\because$ From A, AP and AS are the tangents to the circle.

$\therefore$ AP = AS = 9 cm.

Similarly, BP = BQ = 7 cm.

CQ = CR = 5 cm.

And DR = DS = 6 cm.

Now, AB = 9 + 7 = 16 cm.

BC = 7 + 5 = 12 cm.

CD = 5 + 6 = 11 cm.

and DA = 6 + 9 = 15 cm.

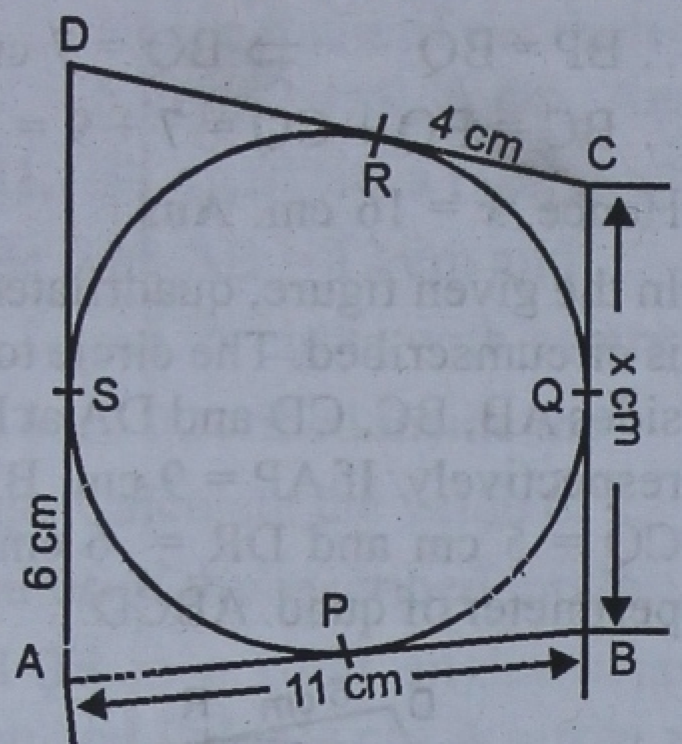
∴ Perimeter of quadrilateral ABCD

= AB + BC + CD + DA

= 16 + 12 + 11 + 15 = 54 cm. **Ans.**

Q. 15. In the given figure, the circle touches the sides AB, BC, CD and DA of a quadrilateral ABCD at the points P, Q, R, S respectively.

If AB = 11 cm, BC = x cm, CR = 4 cm and AS = 6 cm, find the value of x.



Sol. A circle touches the sides AB, BC, CA and DA at P, Q, R and S respectively.

AB = 11 cm, BC = x cm, CR = 4 cm and AS = 6 cm.

∵ From A, AP and AS are the tangents to the circle.

AP = AS = 6 cm.

∴ BP = AB – AP = 11 – 6 = 5 cm.

Similarly, BP = PQ = 5 cm.

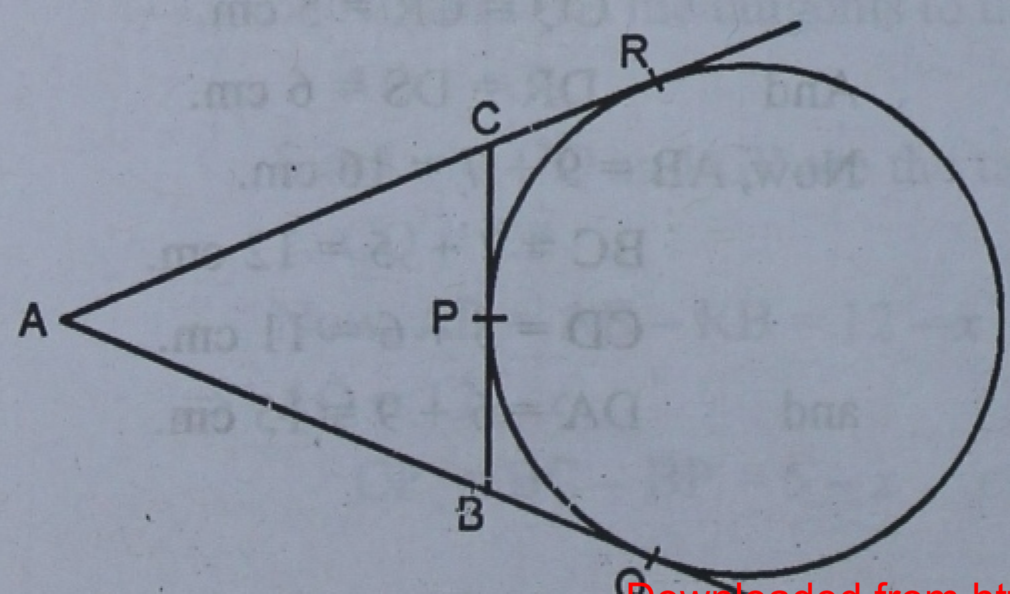
and CQ = CR = 4 cm.

Now, BC = BQ + CQ

= BP + CR = 5 + 4 = 9 cm.

Hence, x = 9 cm. **Ans.**

Q. 16. In the given figure, a circle touches the side BC of ΔABC at P and AB and AC produced at Q and R respectively. If AQ = 15 cm. find the perimeter of ΔABC.



Sol. A circle touches the sides AB and AC produced at Q and R internally and BC at P externally. AQ = 15 cm.

∵ From A, AQ and AR the tangents to the circle

∴ AR = AQ = 15 cm.

Now, perimeter of ΔABC

= AB + AC + BC

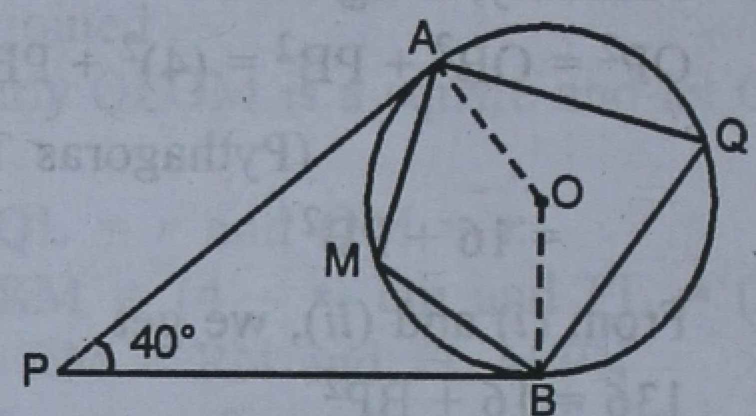
= AB + AC + BP + CP

= AB + AC + BQ + CR

= AB + BQ + AC + CR = AQ + AR

= 15 + 15 = 30 cm. **Ans.**

Q. 17. In the given figure, PA and PB are two tangents to the circle with centre O. If ∠APB = 40°, find ∠AQB and ∠AMB.



Sol. In the figure,

PA and PB are two tangents to the circle with centre O. ∠APB = 40°

Join OA and OB.

Now, ∠OAP = 90°

(∵ OA is radius and PA tangent)

Similarly, ∠OBP = 90°

But, ∠OAP + ∠APB + ∠PBO + ∠AOB = 360° (Sum of angles of a quadrilateral)

⇒ 90° + 40° + 90° + ∠AOB = 360°

⇒ 220° + ∠AOB = 360°

⇒ ∠AOB = 360° – 220°

∴ ∠AOB = 140°

(*i*) Now, arc AMP, subtends ∠AOB at the centre and ∠AQB at the remaining of the circle

∴ ∠AOB = 2∠AQB

$$\Rightarrow \angle AQB = \frac{1}{2}\angle AOB$$

$$\Rightarrow \angle AQB = \frac{1}{2} \times 140^\circ = 70^\circ$$

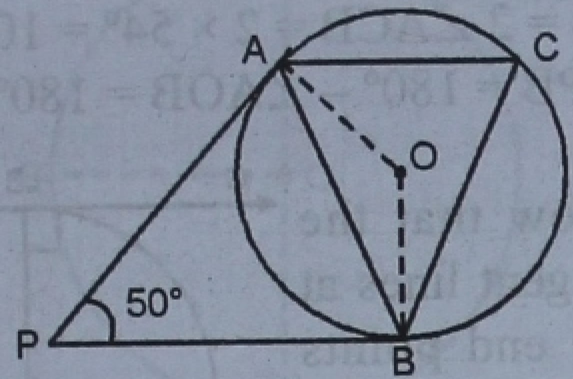
$\because$ AMBQ is a cyclic quadrilateral

$\therefore \angle AMB + \angle AQB = 180^\circ$

$\Rightarrow \angle AMB + 70^\circ = 180^\circ$

$\Rightarrow \angle AMB = 180^\circ - 70^\circ = 110^\circ$ **Ans.**

Q. 18. In the given figure, PA and PB are two tangents to the circle with centre O. If $\angle APB = 50^\circ$, find :
(*i*) $\angle AOB$ (*ii*) $\angle OAB$ (*iii*) $\angle ACB$



Sol. PA and PB are the tangents to the circle with centre O. $\angle APB = 50^\circ$
Join OA and OB.

(*i*) $\because$ OA is radius and AP is the tangent to the circle

$\therefore OA \perp AP$

Similarly, $OB \perp BP$

Now, $\angle APB + \angle OAP + \angle OBP + \angle AOB = 360^\circ$
(Sum of angles of a quadrilateral)

$\Rightarrow 50^\circ + 90^\circ + 90^\circ + \angle AOB = 360^\circ$

$\Rightarrow 230^\circ + \angle AOB = 360^\circ$

$\Rightarrow \angle AOB = 360^\circ - 230^\circ$

$\therefore \angle AOB = 130^\circ$

(*ii*) In ΔOAB,
OA = OB (radii of the same circle)

$\therefore \angle OAB = \angle OBA$

Now, $\angle OAB + \angle OBA + \angle AOB = 180^\circ$
(Angles of a triangle)

$\Rightarrow \angle OAB + \angle OAB + 130^\circ = 180^\circ$

$\Rightarrow 2\angle OAB = 180^\circ - 130^\circ = 50^\circ$

$\therefore \angle OAB = \frac{50^\circ}{2} = 25^\circ$

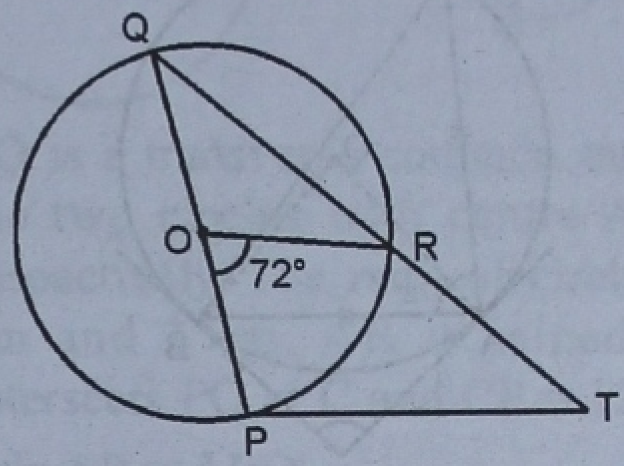
(*iii*) Now, arc AB subtends $\angle AOB$ at the centre and $\angle ACB$ at the remaining part of the circle.

$\therefore \angle AOB = 2\angle ACB$

$\Rightarrow \angle ACB = \frac{1}{2}\angle AOB$

$= \frac{1}{2} \times 130^\circ = 65^\circ$ **Ans.**

Q. 19. In the given figure, PQ is a diameter of a circle with centre O and PT is a tangent at P. QT meets the circle at R. If $\angle POR = 72^\circ$, find $\angle PTR$.



Sol. PQ is the diameter of the circle with centre O. PT is the tangent at P. QT meets the circle at R.

$\angle POR = 72^\circ$

Arc PR subtends $\angle POR$ at the centre and $\angle PQR$ at the remaining part of the circle.

$\therefore \angle POR = 2\angle PQR$

$\Rightarrow \angle PQR = \frac{1}{2}\angle POR$

$\therefore \angle PQR = \frac{1}{2} \times 72^\circ = 36^\circ$

Now, in ΔQPT,

$\angle QPT + \angle PTQ + \angle PQT = 180^\circ$
(Sum of angles of a triangle)

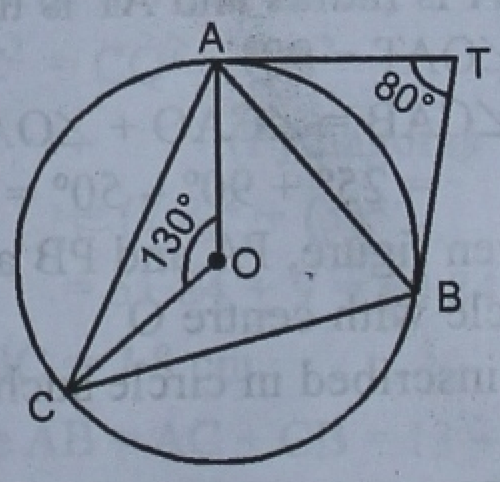
$\Rightarrow 90^\circ + 36^\circ + \angle PTQ = 180^\circ$

$\Rightarrow 126^\circ + \angle PTQ = 180^\circ$

$\Rightarrow \angle PTQ = 180^\circ - 126^\circ$

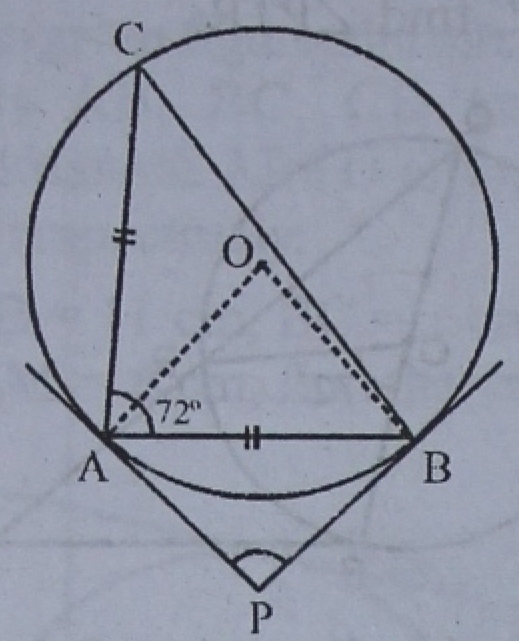
$\therefore \angle PTQ = 54^\circ$ or $\angle PTR = 54^\circ$ **Ans.**

Q. 20. (*i*) In the given figure, O is the centre of the circumcircle of ΔABC. Tangents at A and B intersect at T. If $\angle ATB = 80^\circ$ and $\angle AOC = 130^\circ$, calculate $\angle CAB$.



(*ii*) In the given figure, PA and PB are tangents to a circle with centre O and ΔABC has been inscribed in the circle such that AB = AC. If $\angle BAC = 72^\circ$, calculate

(a) $\angle AOB$ (b) $\angle APB$



Sol. (*i*) O is the centre of the circumcircle of ΔABC. At A and B, tangents AT and BT are drawn to meet at T.

$\angle ATB = 80^\circ$ and $\angle AOC = 130^\circ$

$\because$ TA = TB (Tangents from T)

$\therefore \angle TAB = \angle TBA$

But in ΔTAB,

$\angle TAB + \angle TBA + \angle ATB = 180^\circ$ (Sum of angles of a triangle)

$\Rightarrow \angle TAB + \angle TAB + 80^\circ = 180^\circ$

$\Rightarrow 2\angle TAB = 180^\circ - 80^\circ = 100^\circ$

$\therefore \angle TAB = \frac{100^\circ}{2} = 50^\circ$

$\because$ OA = OC (Radii of the same circle)

$\therefore \angle OAC = \angle OCA$

Now in ΔOAC,

$\therefore \angle OAC + \angle OCA + \angle AOC = 180^\circ$ (Angles of triangle)

$\Rightarrow \angle OAC + \angle OAC + 130^\circ = 180^\circ$

$\Rightarrow 2\angle OAC = 180^\circ - 130^\circ = 50^\circ$

$= \angle OAC = \frac{50^\circ}{2} = 25^\circ$

$\because$ OA is radius and AT is the tangent.

$\therefore \angle OAT = 90^\circ$

Now, $\angle CAB = \angle CAO + \angle OAT - \angle TAB$
$= 25^\circ + 90^\circ - 50^\circ = 65^\circ$ **Ans.**

(*ii*) In the given figure, PA and PB are tangents to the circle with centre O

ΔABC is inscribed in circle such that AB = AC

$\angle BAC = 72^\circ$

Now in ΔABC,

$\therefore \angle ABC + \angle ACB = 180^\circ - 72^\circ = 108^\circ$

But $\angle ABC = \angle ACB$ (Angles opposite to equal sides)

$\therefore \angle ACB + \angle ACB = 108^\circ$

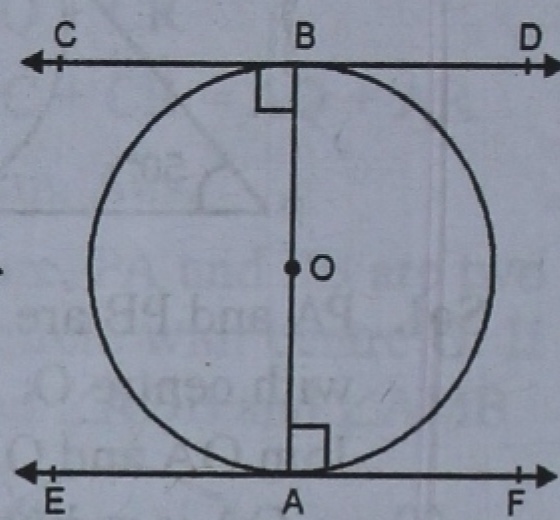
$\Rightarrow 2\angle ACB = 108^\circ \Rightarrow \angle ACB = \frac{108^\circ}{2} = 54^\circ$

(a) Arc AB, subtends $\angle AOB$ at the centre and $\angle ACB$ on the remaining part of the circle

$\therefore \angle AOB = 2\angle ACB = 2 \times 54^\circ = 108^\circ$

(b) $\therefore \angle APB = 180^\circ - \angle AOB = 180^\circ - 108^\circ = 72^\circ$

Q. 21. Show that the tangent lines at the end points of a diameter of a circle are parallel.



Sol. Given. AB is the diameter of the circle with centre O. At A and B, tangents EAF and CBD are drawn.

To prove. CD ∥ EF

Proof. $\because$ OA is radius and EAF is the tangent.

$\therefore OA \perp EF$ or $\angle OAE = 90^\circ$ ...(*i*)

Again, OB is radius and CBD is the tangent $= \angle OBD = 90^\circ$ ...(*ii*)

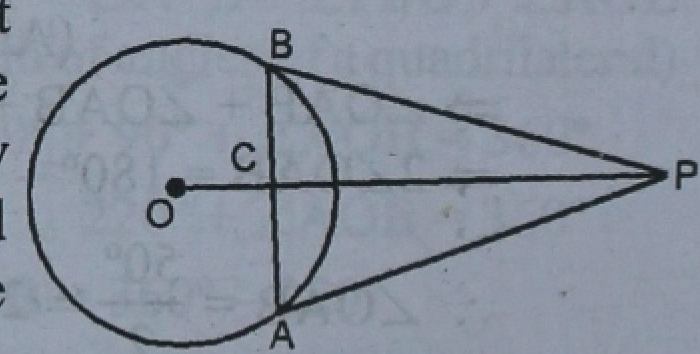
From (*i*) and (*ii*),

$\angle OAE = \angle OBD$

But, these are alternate angles

$\therefore$ CD ∥ EF Hence proved.

Q. 22. Prove that the tangents at the extremities of any chord make equal angles with the chord.



Sol. AB is the chord of the circle with centre O. BP and AP are the tangents drawn meeting each other at P. OP is joined intersecting AB at C.

To prove. $\angle PAC = \angle BPC$

Proof. In ΔPAC and ΔPBC,

PA = PB (Tangents from P)

PC = PC (Common)

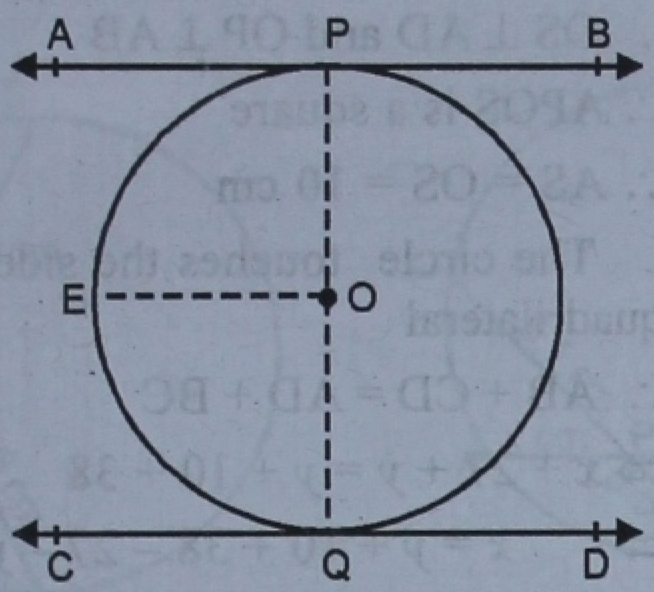
$\angle APC = \angle BPC$ (Angles with OP)

$\therefore \quad \Delta PAC \cong \Delta PBC$ (S.A.S. axiom)

Hence, $\angle PAC = \angle BPC$ (c.p.c.t.)

Hence proved.

Q. 23. Show that the line segment joining the points of contact of two parallel tangents passes through the centre.



Sol. Given. AB and CD are two tangents such that AB ∥ CD. PO and QO are joined.

To prove. POQ is a straight line.

Construction. Draw OE ∥ AB ∥ CD.

Proof. $\because$ OP is the radius and AB is the tangent.

$\therefore \quad \angle OPA = 90°$

Similarly, $\angle OQC = 90°$

$\because$ OE ∥ AB

$\therefore \angle OPA + \angle POE = 180°$

(Angles on the same side of the transversal)

$\Rightarrow 90° + \angle POE = 180°$

$\Rightarrow \quad \angle POE = 180° - 90° = 90°$

Similarly, OE ∥ CD

$\because \angle QOE + \angle OQC = 180°$

$\Rightarrow \quad \angle QOE + 90° = 180°$

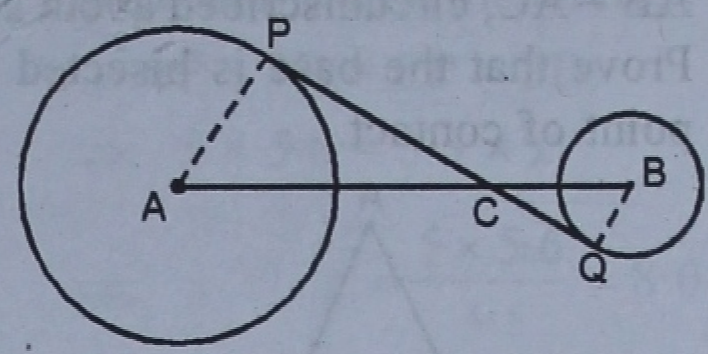
$\Rightarrow \quad \angle QOE = 180° - 90° = 90°$

$\because \quad \angle POE + QOE = 90° + 90° = 180°$

$\therefore$ POQ is a straight line

Hence proved.

Q. 24. In the given figure, PQ is a transverse common tangent to two circles with centres A and B and of radii 5 cm and 3 cm respectively. If PQ intersects AB at C such that CP = 12 cm, calculate AB.



Sol. PQ is a transverse common tangent to the two circles with centre A and B respectively. The radii of circles are 5 cm and 3 cm. AB is joined which intersects PQ at C and CP = 12 cm.

Join AP and BQ.

$\because$ AP is radius and PQ is tangent

$\therefore \quad \angle APQ = 90°$

Similarly, $\angle BQC = 90°$

Now, in ΔPAC and ΔQBC,

$\angle APC = \angle BQC$ (each 90°)

$\angle PCA = \angle QCB$

(Vertically opposite anlges)

$\therefore \Delta PAC \sim \Delta QBC$ (AA axiom)

$$\therefore \frac{AC}{CB} = \frac{PC}{CQ} = \frac{AP}{BQ}$$

$$\Rightarrow \frac{PC}{CQ} = \frac{AP}{BQ} \Rightarrow \frac{12}{CQ} = \frac{5}{3}$$

$$\Rightarrow CQ = \frac{12 \times 3}{5} = \frac{36}{5} \text{ cm} = 7{\cdot}2 \text{ cm.}$$

Now, In right ΔAPC

$AC^2 = PC^2 + AP^2$

(Pythagoras Theorem)

$AC^2 = 12^2 + 5^2 \quad = 144 + 25 = 169 = (13)^2$

$\therefore \quad AC = 13$ cm.

Similarly, In right ΔBCQ

$BC^2 = CQ^2 + QB^2$

(Pythagoras Theorem)

$= (7{\cdot}2)^2 + (3)^2$

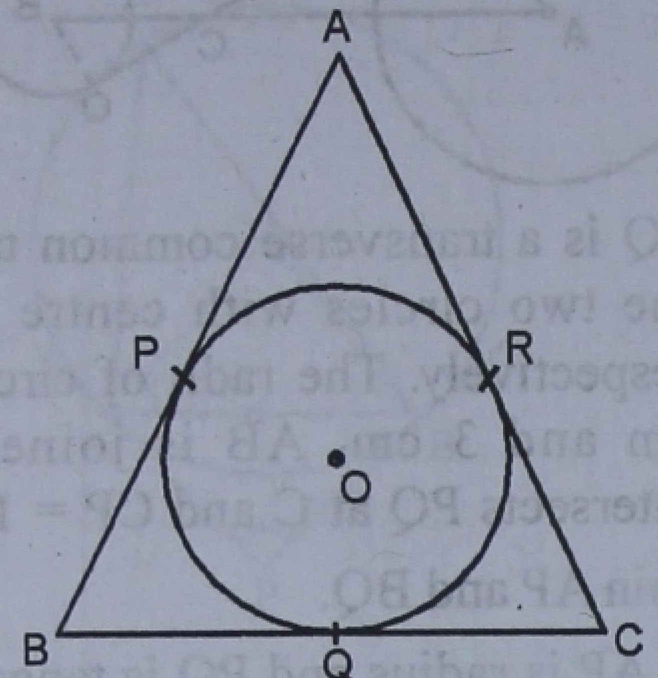
$= 51{\cdot}84 + 9 = 60{\cdot}84 = (7{\cdot}8)^2$

$\therefore \quad BC = 7{\cdot}8$ cm.

Hence AB = AC + CB = 13 + 7·8

= 20·8 cm. **Ans.**

Q. 25. ΔABC is an isosceles triangle in which AB = AC, circumscribed about a circle. Prove that the base is bisected by the point of contact.



Sol. ΔABC is circumscribed about a circle with centre O. AB = AC and the circle touches the sides AB, BC and CA at P, Q and R respectively.

To prove. Q bisects BC.

Proof. AP and AR the tangents to the circle

$\therefore$ AP = AR

Similarly, BP = BQ and CQ = CR

$\because$ AB = AC and AP = AR

$\therefore$ AB – AP = AC – AR

$\Rightarrow$ BP = CR

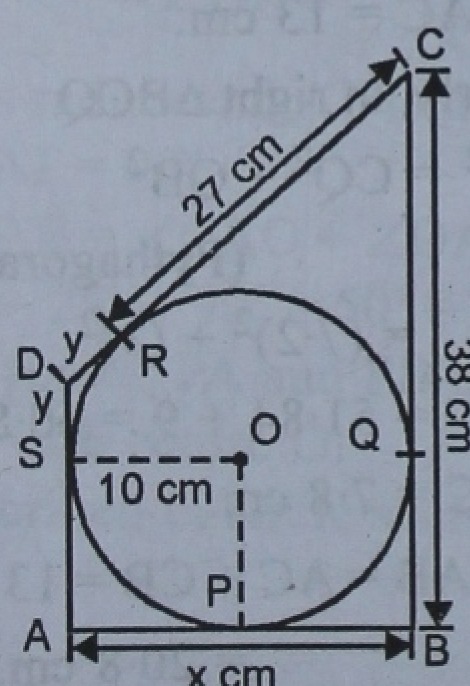
But BQ = BP and CQ = CR

$\therefore$ BQ = CQ

Hence, Q is the mid-point of BC

Hence proved.

Q. 26. In the given figure, quadrilateral ABCD is circumscribed and AD $\perp$ AB. If the radius of incircle is 10 cm, find the value of x.



Sol. Quadrilateral ABCD is circumscribed about a circle with centre O. AD $\perp$ AB Radius of circle = 10 cm. AB = x cm. BC = 38 cm., CR = 27 cm.

$\because$ DR and DS are the tangents to the circle from D

$\therefore$ DR = DS = y (Say)

$\because$ OS $\perp$ AD and OP $\perp$ AB

$\therefore$ APOS is a square

$\therefore$ AS = OS = 10 cm

$\because$ The circle touches the sides of the quadrilateral

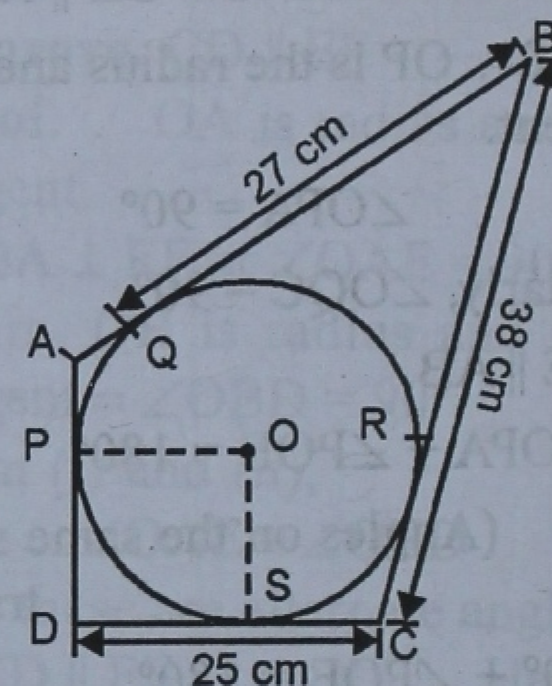
$\therefore$ AB + CD = AD + BC

$\Rightarrow x + 27 + y = y + 10 + 38$

$\Rightarrow \quad x = y + 10 + 38 - 27 - y = 21$

Hence, x = 21 cm. **Ans.**

Q. 27. In the given figure, a circle is inscribed in quad. ABCD. If BC = 38 cm, BQ = 27 cm, DC = 25 cm and AD $\perp$ DC, find the radius of the circle.



Sol. In the figure, a circle with centre O is inscribed in a quadrilateral ABCD DC = 25 cm, CB = 38 cm. BQ = 27 cm. AD $\perp$ DC.

$\because$ BQ and BR are the tangents to the circle from B

$\therefore$ BR = BQ = 27 cm

$\therefore$ CR = BC – BR = 38 – 27 = 11 cm.

Similarly, CS = CR = 11 cm.

$\therefore$ DS = DC – CS = 25 – 11 = 14 cm.

$\because$ OP $\perp$ AD and OS $\perp$ DC

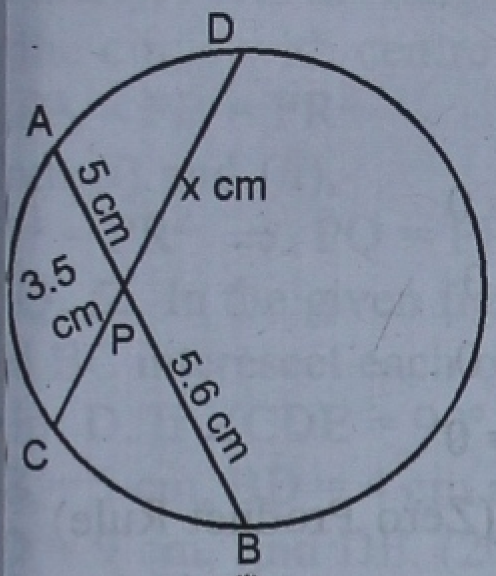
$\therefore$ DSOP is a square

$\therefore$ DS = PO = radius of the circle

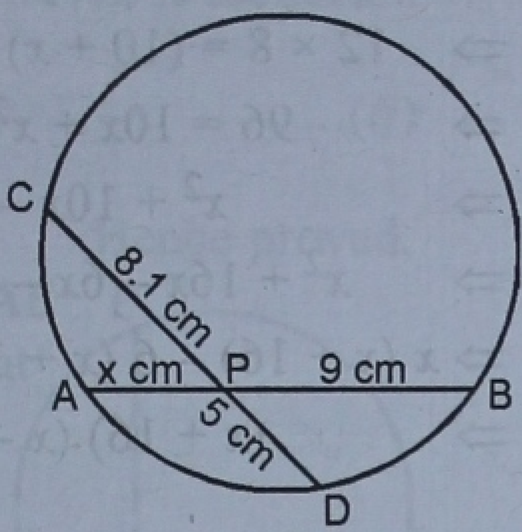
$\therefore$ Radius of the circle = 14 cm. **Ans.**

EXERCISE 21 (B)

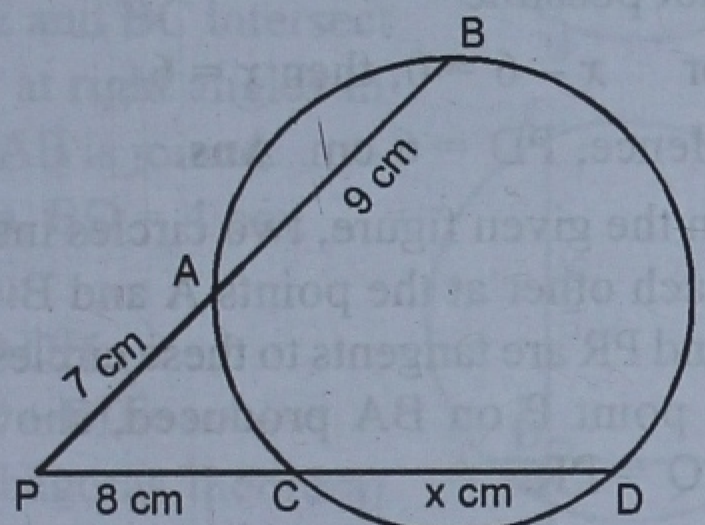
Q. 1. *Find the unknown length x in each of the following figures :*



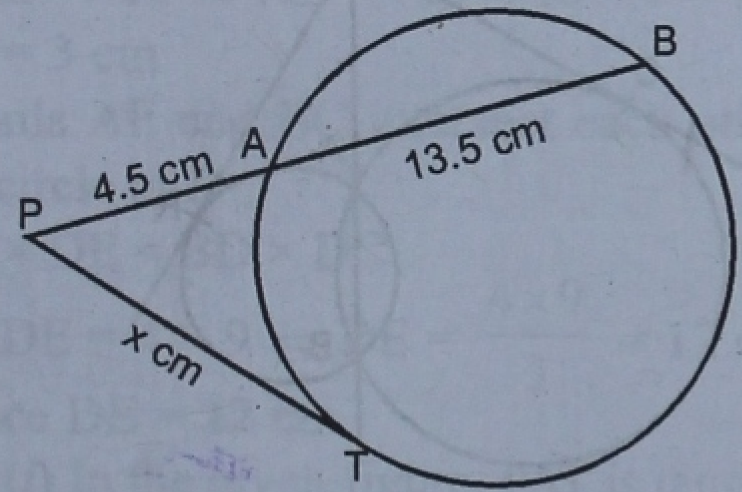
(i)



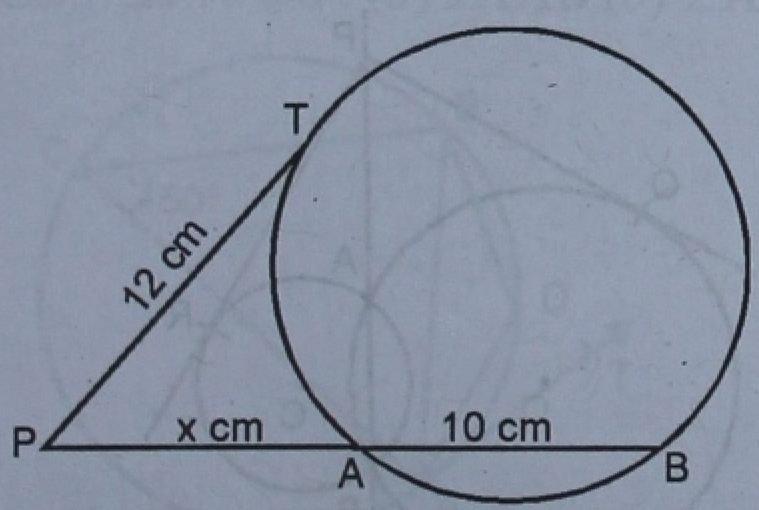
(ii)



(iii)



(iv)



(v)

Sol. (*i*) $\because$ Chords AB and CD intersect each other at P inside the circle.

$\therefore$ AP × PB = CP × PD

$\Rightarrow$ $5 \times 5{\cdot}6 = 3{\cdot}5 \times x$

$\Rightarrow$ $x = \dfrac{5 \times 5{\cdot}6}{3{\cdot}5} = 8{\cdot}0$

$\therefore$ $x = 8{\cdot}0$ cm. **Ans.**

(*ii*) $\because$ Chords AB and CD intersect each other at P inside the circle

$\therefore$ AP × PB = CP × PD

$\Rightarrow$ $x \times 9 = 8{\cdot}1 \times 5$

$\Rightarrow$ $x = \dfrac{8{\cdot}1 \times 5}{9}$

$\therefore$ $x = 4{\cdot}5$ cm. **Ans.**

(*iii*) $\because$ Chords AB and CD intersect each other at P outside the circle

$\therefore$ AP × PB = CP × PD

$7 \times (7 + 9) = 8\,(8 + x)$

$\Rightarrow$ $7 \times 16 = 8\,(8 + x)$

$\Rightarrow$ $8\,(8 + x) = 112.$

$\Rightarrow$ $8 + x = \dfrac{112}{8} = 14$

$\therefore$ $x = 14 - 8 = 6$ cm. **Ans.**

(*iv*) $\because$ PAB is the secant and PT is the tangent to the circle

$\therefore$ $PT^2 = PA \times PB$

$\Rightarrow$ $x^2 = 4{\cdot}5\,(4{\cdot}5 + 13{\cdot}5)$

$= 4{\cdot}5 \times 18 = 81{\cdot}0 = 81$

$\therefore$ $x = \sqrt{81} = 9$ cm. **Ans.**

(*v*) $\because$ PAB is the secant and PT is the tangent to the circle

$\therefore$ $PT^2 = PA \times PB$

$\Rightarrow$ $(12)^2 = x \times (x + 10)$

$\Rightarrow$ $144 = x^2 + 10x$

$\Rightarrow$ $x^2 + 10x - 144 = 0$

$\Rightarrow$ $x^2 + 18x - 8x - 144 = 0$

$\Rightarrow$ $x\,(x + 18) - 8\,(x + 18) = 0$

$\Rightarrow$ $(x + 18)\,(x - 8) = 0$

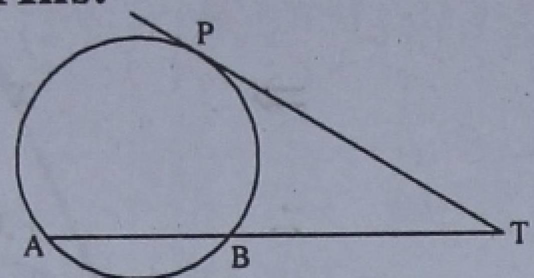
(Zero Product Rule)

Either $x + 18 = 0$, then $x = -18$ which is not possible.

or $x - 8 = 0$, then $x = 8$

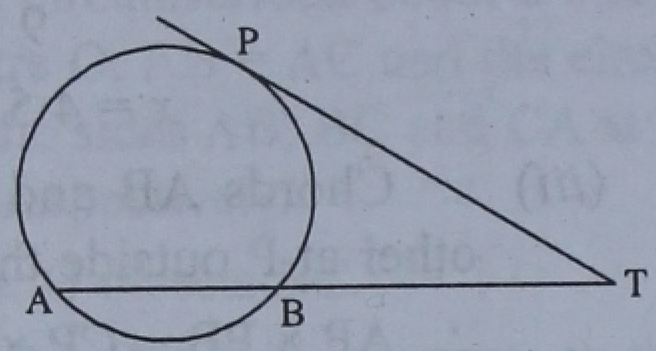
Hence, $x = 8$ cm. **Ans.**

Q. 2. (*i*) In the figure given below, PT is a tangent to the circle. Find PT if AT = 16 cm and AB = 12 cm. **(2007)**



(*ii*) Two chords AB and CD of a circle intersect at a point P inside the circle such that AB = 12 cm, AP = 2·4 cm and PD = 7·2 cm. Find CD.

Sol. (*i*) Since PT is a tangent and TAB is the secant.



$\therefore PT^2 = TA \times TB$

$= 16 \times 4 = 64$

$\therefore PT = \sqrt{64} = 8$

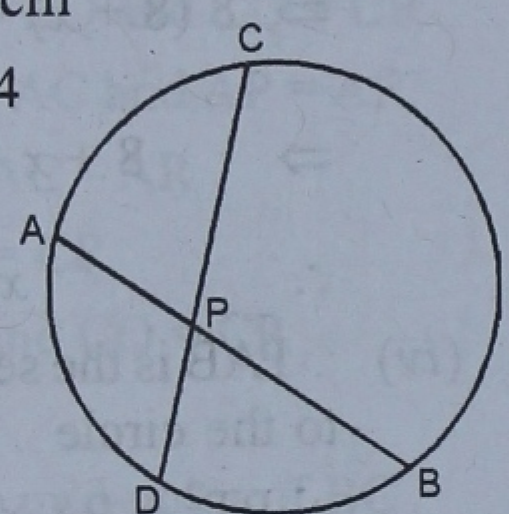
$[\because TA = 16, AB = 12,$

$\therefore TB = TA - AB = 16 - 12 = 4]$

(*ii*) AB = 12 cm, AP = 2·4 cm

$\therefore PB = AB - AP = 12 - 2{\cdot}4$

$= 9{\cdot}6$ cm.



Let, CP = x

$\because$ Chords AB and CD intersect each other at P inside the circle

$\therefore AP \times PB = CP \times PD$

$\Rightarrow 2{\cdot}4 \times 9{\cdot}6 = x \times 7{\cdot}2$

$\Rightarrow \quad x = \dfrac{2{\cdot}4 \times 9{\cdot}6}{7{\cdot}2} = 3{\cdot}2$ cm.

$\Rightarrow \quad CP = 3{\cdot}2$ cm

Hence, CD = CP + PD = 3·2 + 7·2

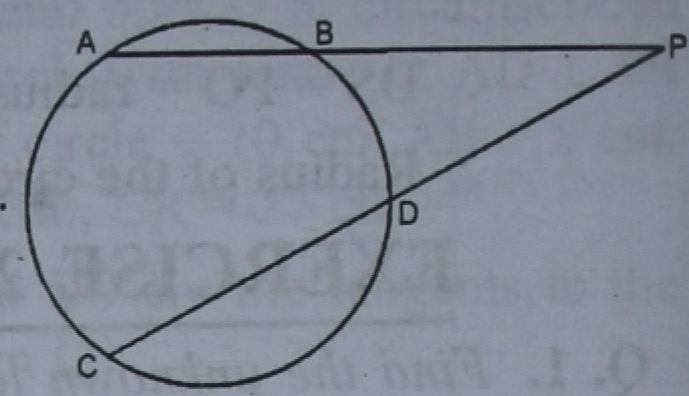
= 10·4 cm. **Ans.**

Q. 3. If AB and CD are two chords of a circle which when produced meet at a point P outside the circle such that PA = 12 cm, AB = 4 cm and CD = 10 cm, find PD.

Sol. PA = 12 cm, AB = 4 cm

$\therefore BP = AP - AB = 12 - 4 = 8$ cm.

CD = 10 cm.



Let, PD = x

$\therefore CP = (10 + x)$ cm.

$\because$ Two chords AB and CD intersect each other at P outside the circle.

$\therefore PA \times PB = PC \times PD$

$\Rightarrow \quad 12 \times 8 = (10 + x) \times x$

$\Rightarrow \quad 96 = 10x + x^2$

$\Rightarrow \quad x^2 + 10x - 96 = 0$

$\Rightarrow \quad x^2 + 16x - 6x - 96 = 0$

$\Rightarrow x(x + 16) - 6(x + 16) = 0$

$\Rightarrow \quad (x + 16)(x - 6) = 0$

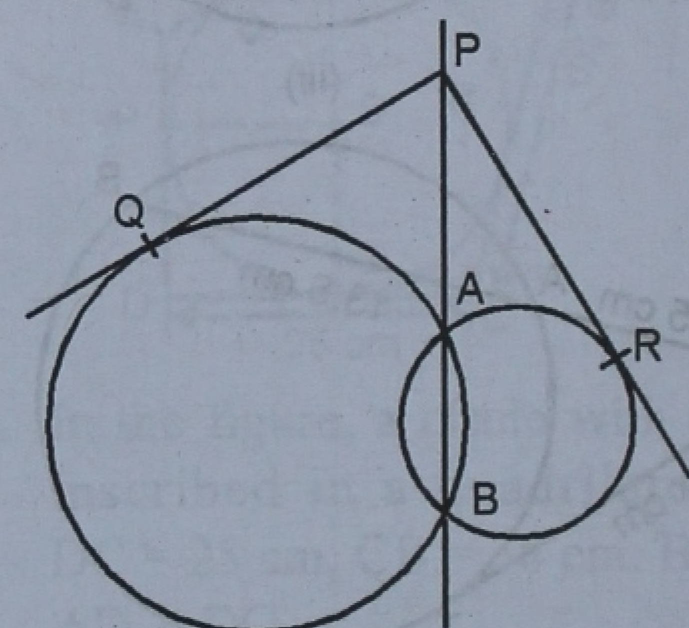
(Zero Product Rule)

Either $x + 16 = 0$, then $x = -16$ which is not possible

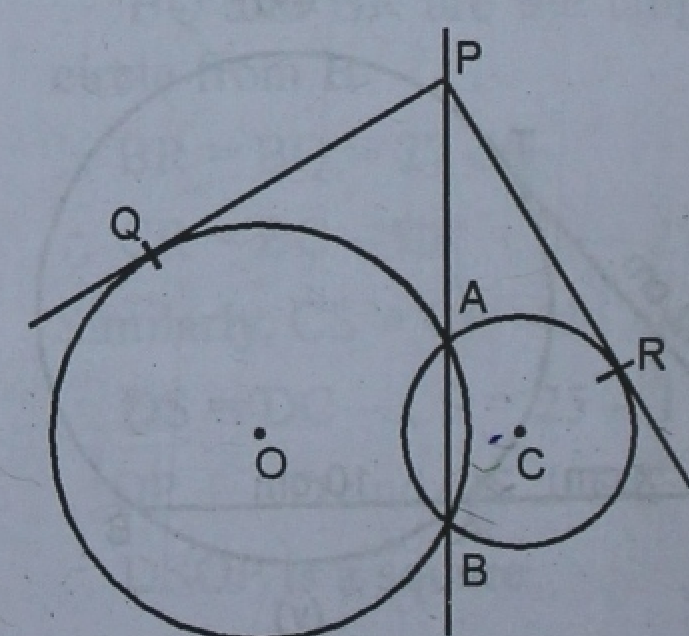
or $x - 6 = 0$, then $x = 6$

Hence, PD = 6 cm. **Ans.**

Q. 4. In the given figure, two circles intersect each other at the points A and B. If PQ and PR are tangents to these circles from a point P on BA produced, show that PQ = PR.



Sol.



:iven. Two circles with centre O and C intersect ach other at A and B. P is a point on BA produced nd from P, PQ and PR are tangents to these circles.

'o prove. PQ = PR

'roof. ∵ PQ is the tangent and PAB is the secant f the circle with centre O

. $PA \times PB = PQ^2$ ...(*i*)

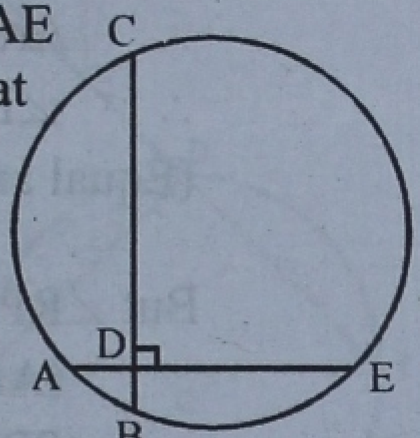
imilarly, PR is the tangent and PAB is the secant f the circle with centre C.

. $PA \times PB = PR^2$ ...(*ii*)

rom (*i*) and (*ii*),

$Q^2 = PR^2 \Rightarrow PQ = PR$ Hence proved.

Q. 5. In the given figure, AE nd BC interesect each other at oint D. If ∠CDE = 90°, B = 5 cm, BD = 4 cm and D = 9 cm, find DE. (2008)



ol. In the given figure, hords AE and BC intersect ach other at right angles in ne circle. AB is joined

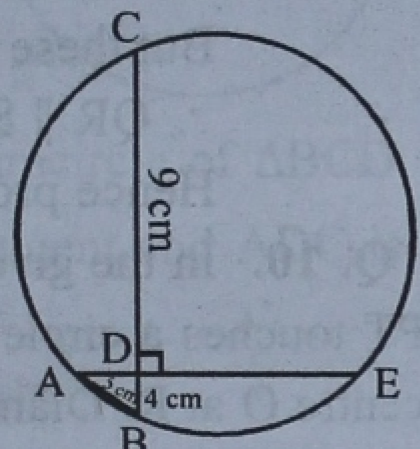
B = 5 cm, BD = 4 cm,

D = 9 cm

n right ΔADB,

$B^2 = AD^2 + BD^2$

(Pythagoras theorem)



$5)^2 = AD^2 + (4)^2$

$\Rightarrow 25 = AD^2 + 16 \Rightarrow AD^2 = 25 - 16 = 9 = (3)^2$

∴ AD = 3 cm

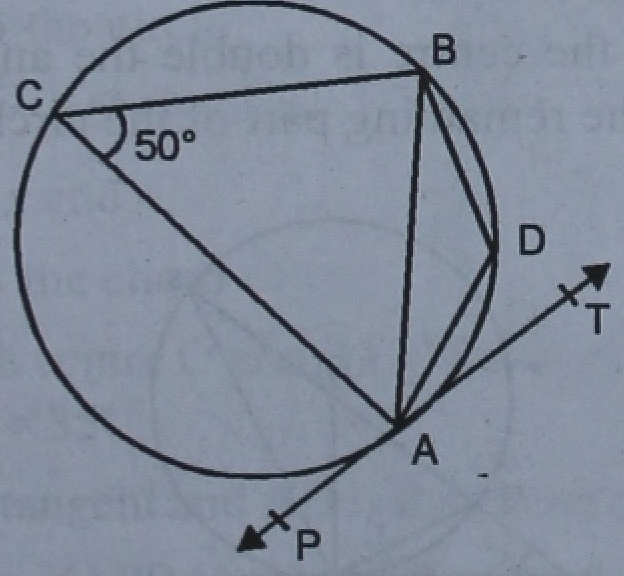
∵ Chords AE and BC intersect each other in the circle at D

∴ AD × DE = BD × DC

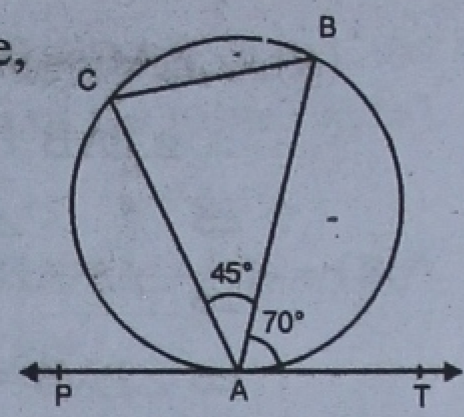
$\Rightarrow 3 \times DE = 4 \times 9 \Rightarrow DE = \frac{4 \times 9}{3} = 12$ cm

Hence DE = 12 cm

Q. 6. **(*i*)** In the given figure, PAT is tangent at . If ∠ACB = 50°. Find : (*a*) ∠TAB (*b*) ∠ADB.



(*ii*) In the given figure, PAT is tangent at A. If ∠TAB = 70° and ∠BAC = 45°, find ∠ABC.



Sol. (*i*) In the figure,

PAT is tangent to the circle at A

ΔABC is inscribed in the circle and ∠ACB = 50°

(*a*) ∵ PAT is the tangent and AB is the chord of the circle

∴ ∠ACB = ∠BAT

(Angles in the alternate segment)

∴ ∠TAB = 50°

(*b*) ADBC is a cyclic quadrilateral

∴ ∠ADB + ∠ACB = 180°

⇒ ∠ADB + 50° = 180°

⇒ ∠ADB = 180° – 50° = 130°. **Ans.**

(*ii*) ∵ PTA is the tangent and BA is the chord of the circle

∴ ∠ACB = ∠BAT = 70°

(Angles in the alternate segment)

Now in ΔABC,

∠ABC + ∠BCA + ∠BAC = 180°

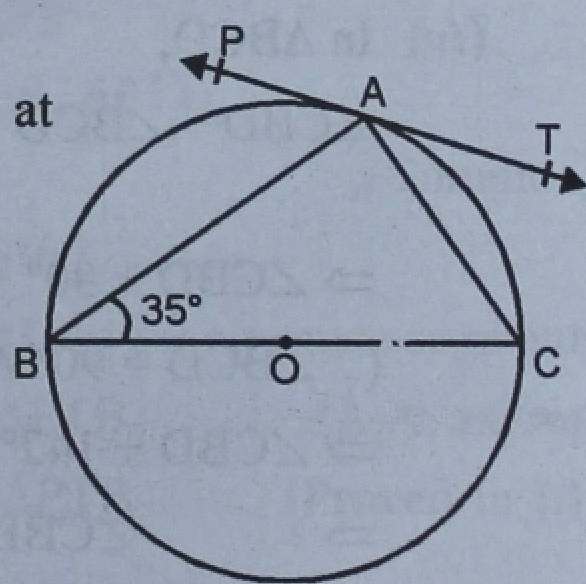
(Angles of a triangle)

⇒ ∠ABC + 70° + 45° = 180°

⇒ ∠ABC + 115° = 180°

⇒ ∠ABC = 180° – 115° = 65° **Ans.**

Q. 7. In the given figure, PAT is tangent at A, to the circle with centre O. If ∠ABC = 35°, find : (*i*) ∠TAC (*ii*) ∠PAB.



Sol.(*i*) ∵ PAT is the tangent and AC is the chord of the circle

∴ ∠TAC = ∠ABC

(Angles in the alternate segment)

= 35° (∵ ∠ABC = 35°)

(*ii*) $\angle BAC = 90^\circ$ (Angle in a semi-circle)

$\therefore \angle PAB + \angle BCA + \angle TAC = 180^\circ$

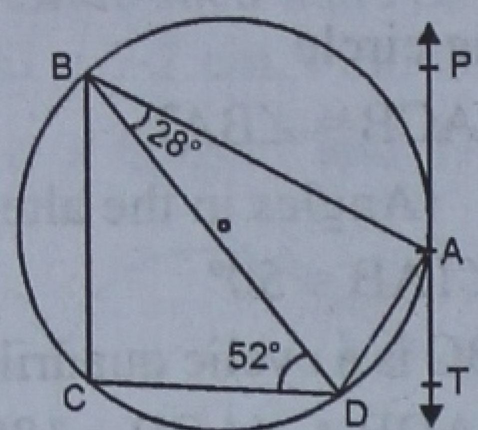
$\Rightarrow \angle PAB + 90^\circ + 35^\circ = 180^\circ$

$\Rightarrow \angle PAB + 125^\circ = 180^\circ$

$\Rightarrow \angle PAB = 180^\circ - 125^\circ = 55^\circ$ **Ans.**

Q. 8. In the given figure, PAT is tangent at A and BD is a diameter of the circle. If $\angle ABD = 28^\circ$ and $\angle BDC = 52^\circ$, find :

(*i*) $\angle TAD$ (*ii*) $\angle BAD$ (*iii*) $\angle PAB$ (*iv*) $\angle CBD$.



Sol. (*i*) PAT is the tangent and AD is the chord of the circle

$\therefore \angle TAD = \angle ABD = 28^\circ$

(Angles in the alternate segment)

(*ii*) $\because$ BD is the diameter of the circle

$\therefore \angle BAD = 90^\circ$ (Angle in a semi-circle)

(*iii*) $\angle PAB = \angle ADB$

(Angles in the alternate segment)

But, $\angle ADB = 180^\circ - (\angle ABD + \angle BAD)$

$= 180^\circ - (28^\circ + 90^\circ) \quad = 180^\circ - 118^\circ = 62^\circ$

$\therefore \angle PAB = 62^\circ$

(*iv*) In ΔBCD,

$\angle CBD + \angle BCD + \angle BDC = 180^\circ$

(Angles of a triangle)

$\Rightarrow \angle CBD + 90^\circ + 52^\circ = 180^\circ$

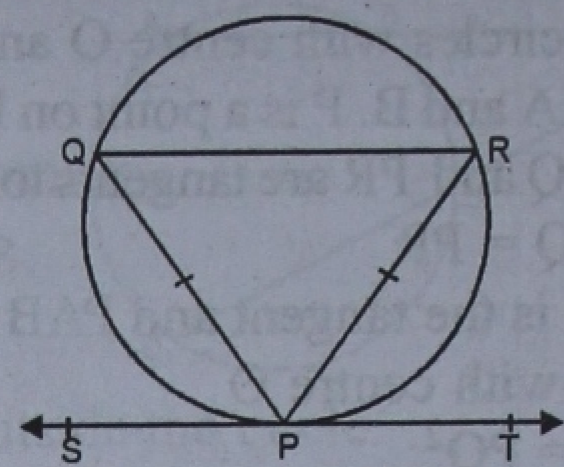
($\because \angle BCD = 90^\circ$ Angle in a semi-circle)

$\Rightarrow \angle CBD + 142^\circ = 180^\circ$

$\Rightarrow \angle CBD = 180^\circ - 142^\circ = 38^\circ$

Hence, $\angle CBD = 38^\circ$ **Ans.**

Q. 9. In the given figure, PQ and PR are two equal chords of a circle. Show that the tangent at P is parallel to QR.



Sol. Given. PQ and PR are two equal chords of the circle. QR is joined and SPT is the tangent.

To prove. QR ∥ SPT

Proof. $\because$ PQ = PR (Given)

$\therefore$ Arc PQ = arc PR

$\therefore \angle PRQ = \angle PQR$

(Equal arcs subtend equal angles at the circumference)

But $\angle RPT = \angle PQR$

(Angles in the alternate segment)

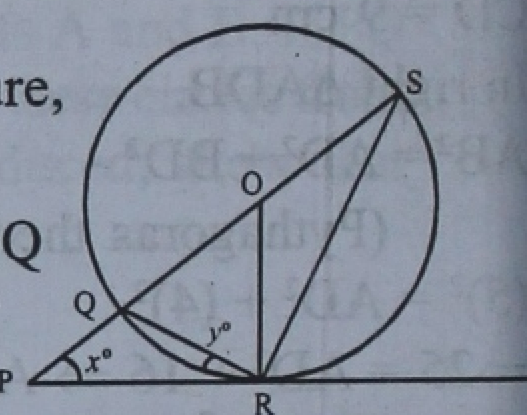
$\therefore \angle PRQ = \angle RPT$

But these are alternate angles.

$\therefore$ QR ∥ SPT.

Hence proved.

Q. 10. In the given figure, PT touches a circle with centre O at R. Diameter SQ when produced meets PT at P.



If $\angle SPR = x^\circ$ and $QRP = y^\circ$, show that $x^\circ + 2y^\circ = 90^\circ$. **(2006)**

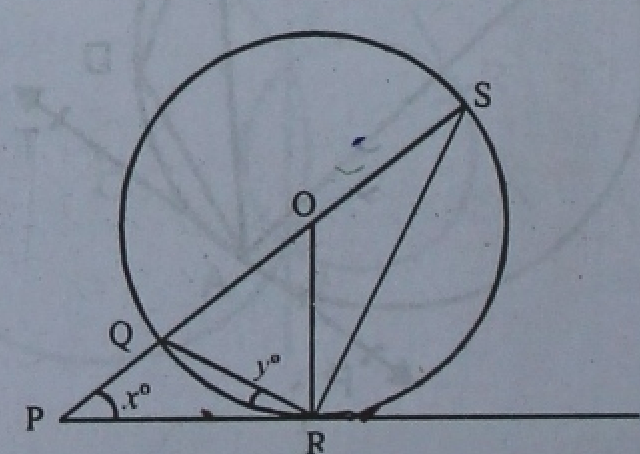
Sol. PR is a tangent to the circle at R and RQ is a chord.

$\therefore \angle PRQ = y^\circ = \angle RSQ$ .....(i)

[Angles in the alternate segments]

$\therefore \angle ROQ = 2 \angle RSQ$

[Angle at the centre is double the angle at any point on the remaining part of the circle]



⇒ $\angle ROQ = 2y^o$ ...(*ii*) [From (*i*)]

In ΔPOR, we have,

$\angle PRO = 90^o$ [Radius through the point of contact is perpendicular to the tangent]

∴ $\angle OPR + \angle ROP = 90^o$

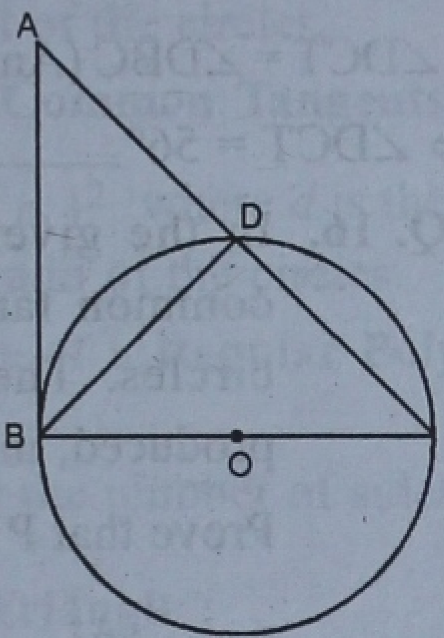
[Angle sum property of a Δ]

⇒ $x^o + 2y^o = 90^o$ [From (*ii*)] **Proved.**

Q. 11. In a right-angled Δ ABC, the perpendicular BD on hypotenuse AC is drawn.

Prove that : (*i*) $AC \times AD = AB^2$

(*ii*) $AC \times CD = BC^2$.



Sol. Given. ΔABC is a right-angled triangle. BD is a perpendicular on AC.

To prove.

(*i*) $AC \times AD = AB^2$

(*ii*) $AC \times CD = BC^2$

Construction. Draw a circumcircle of ΔBCD.

Proof. (*i*) ∵ AB is the tangent and ADC is a secant of the circle.

∴ $AB^2 = AC \times AD$

(*ii*) $AC \times CD = AC \times (AC - AD)$

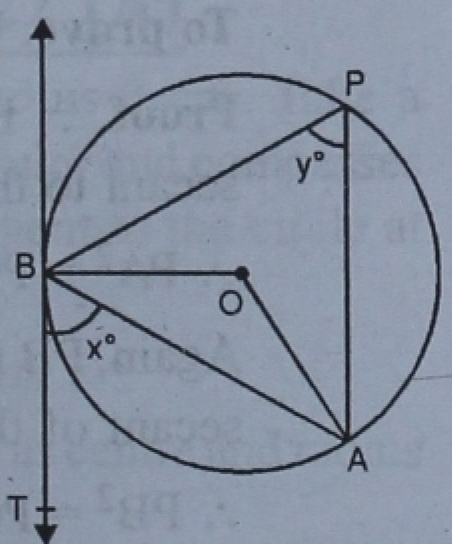
$= AC^2 - AC \times AD = AC^2 - AB^2$ [Proved in (*i*)]

But in right ΔABC,

$AC^2 = AB^2 + BC^2 \Rightarrow AC^2 - AB^2 = BC^2$

∴ $AC \times CD = BC^2$ Hence proved.

Q. 12. In the given figure, AB is a chord of the circle with centre O and BT is tangent to the circle. If $\angle OAB = 35^o$, find the values of *x* and *y*.



Sol. AB is the chord of the circle with centre O and BT is the tangent. $\angle OAB = 35^o$

BT is the tangent and AB is the chord of the circle

∴ $\angle ABT = \angle APB$ (Angles in the alternate segment)

∴ $x = y$

In ΔOAB, OA = OB (radii of the same circle)

∴ $\angle OAB = \angle OBA = 35^o$

But $\angle OAB + \angle OBA + \angle AOB = 180^o$

⇒ $35^o + 35^o + \angle AOB = 180^o$

⇒ $70^o + \angle AOB = 180^o \Rightarrow \angle AOB = 180^o - 70^o = 110^o$

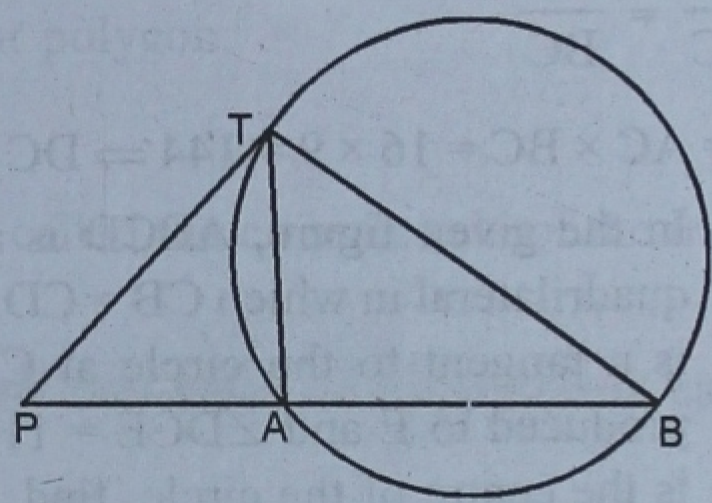
Now, arc AB subtends ∠AOB at the centre and ∠APB at the remaining part of the circle

∴ $\angle AOB = 2\angle APB$

⇒ $110^o = 2y^o \Rightarrow y = \frac{110^o}{2} = 55^o$

Hence, $x^o = 55^o, y^o = 55^o$ **Ans.**

Q. 13. In the given figure, PAB is a secant to a circle and PT is a tangent at T. Prove that:



(*i*) ΔPAT ~ ΔPTB (*ii*) $PA \times PB = PT^2$.

Sol. Given. PAB is the secant to a circle and PT is the tangent. AT is joined.

To prove. (*i*) ΔPAT ~ ΔPTB

(*ii*) $PA \times PB = PT^2$

Proof. (*i*) In ΔPAT and ΔPTB,

∠P = ∠P (Common)

∠PTA = ∠ABT or ∠PBT

(Angle in the alternate segment)

∴ ΔPAT ~ ΔPTB. (A.A. axiom)

(*ii*) ∵ ΔPAT ~ ΔPTB [Proved in (*i*)]

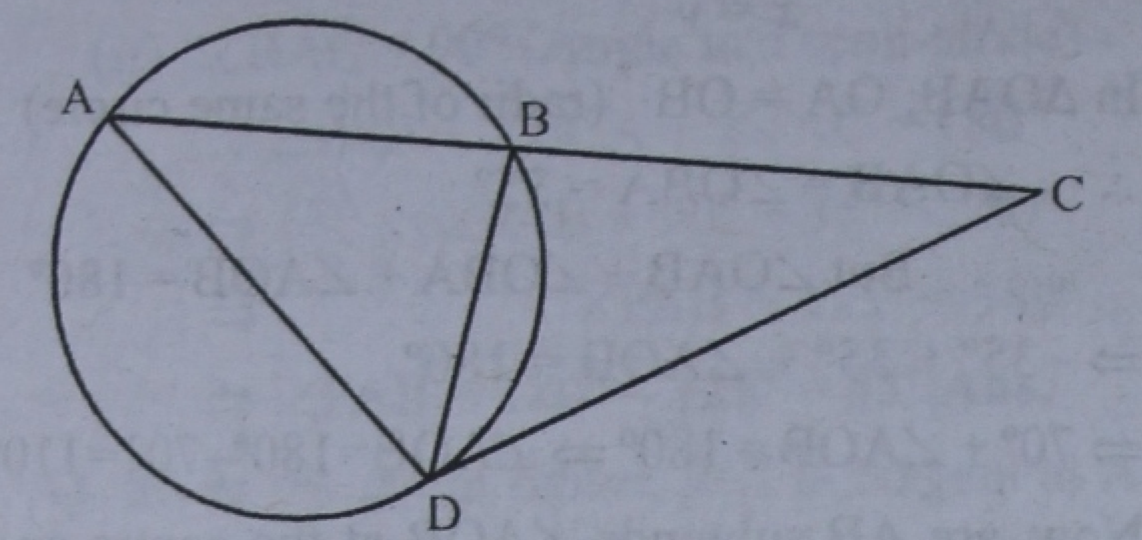
∴ $\frac{PT}{PB} = \frac{PA}{PT} \Rightarrow PT \times PT = PA \times PB$

⇒ $PT^2 = PA \times PB$ Hence proved.

Q. 14. In the figure AB = 7 cm and BC = 9 cm

(*i*) Prove ΔACD ~ ΔDCB

(*ii*) Find the length of CD **(2009)**



Sol. In ΔACD and ΔDCB

$\angle C = \angle C$ (common)

$\angle CAD = \angle CDB$

[Angle between chord and tangent is equal to angle made by chord in alternate segment.]

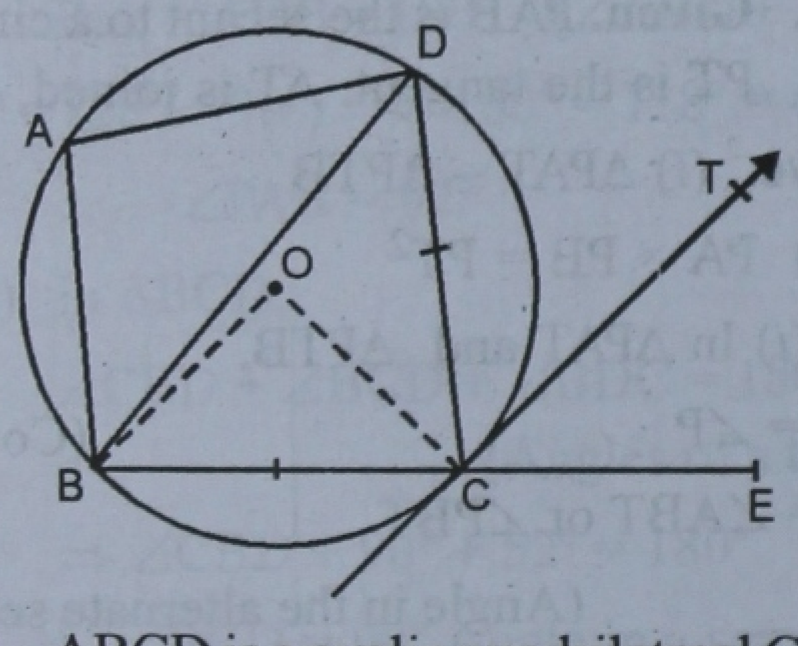
$\therefore \Delta ACD \sim \Delta DCB$

$$\therefore \frac{AC}{DC} = \frac{DC}{BC}$$

$\Rightarrow DC^2 = AC \times BC = 16 \times 9 = 144 \Rightarrow DC = 12cm$

Q. 15. In the given figure, ABCD is a cyclic quadrilateral in which CB = CD and TC is a tangent to the circle at C, BC is produced to E and $\angle DCE = 112^\circ$. If O is the centre of the circle, find

(*i*) $\angle DCT$ (*ii*) $\angle BOC$.



Sol. ABCD is a cyclic quadrilateral CB = CD And TC is the tangent to the circle at C. BC is produced to E

$\angle DCE = 112^\circ$

Join BD, OB and OC

$\because$ BCE is a straight line

$\therefore \angle BCD + \angle DCE = 180^\circ$ (Linear pair)

$\Rightarrow \angle BCD + 112^\circ = 180^\circ$

$\Rightarrow \angle BCD = 180^\circ - 112^\circ \Rightarrow \angle BCD = 68^\circ$

Now, in ΔBCD,

$\because BC = CD$ (given)

$\therefore \angle BDC = \angle DBC$ (Angles opposite to equal sides)

$$\therefore \angle BDC = \angle DBC = \frac{112^\circ}{2} = 56^\circ$$

Arc, BC subtends $\angle BOC$ at the centre and $\angle BDC$ at the remaining part of the circle.

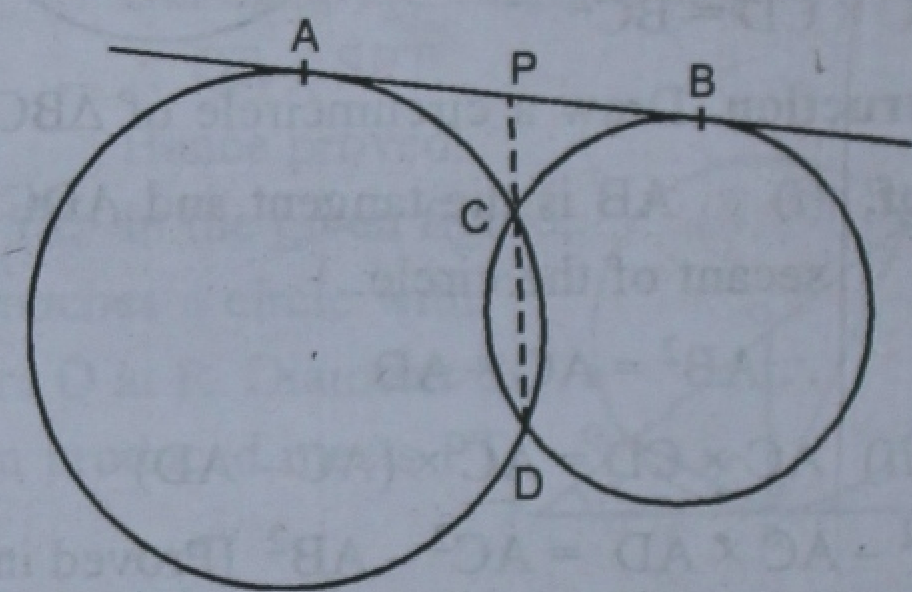
$\therefore \angle BOC = 2\,\angle BDC = 2 \times 56^\circ = 112^\circ$

$\angle DCT = \angle DBC$ (Angle in the alternate segment)

$\Rightarrow \angle DCT = 56^\circ$ ($\because \angle DBC = 56^\circ$)

Q. 16. In the given figure, AB is a direct common tangent to two intersecting circles. Their common chord when produced, intersects AB at P.

Prove that P is the mid-point of AB.



Sol. Given. AB is the direct common tangent to the circles which intersect each other at C and D. DC is produced to meet AB at P.

To prove. P is mid-point of AB.

Proof. $\because$ PA is tangent and PCD is the secant to the first circle

$\therefore PA^2 = PC \times PD$ ...(*i*)

Again, PB is the tangent and PCD is the secant of the second circle.

$\therefore PB^2 = PC \times PD$ ...(*ii*)

From (*i*) and (*ii*),

$$PA^2 = PB^2 \Rightarrow PA = PB$$

Hence, P is the mid-point of AB, Hence proved.