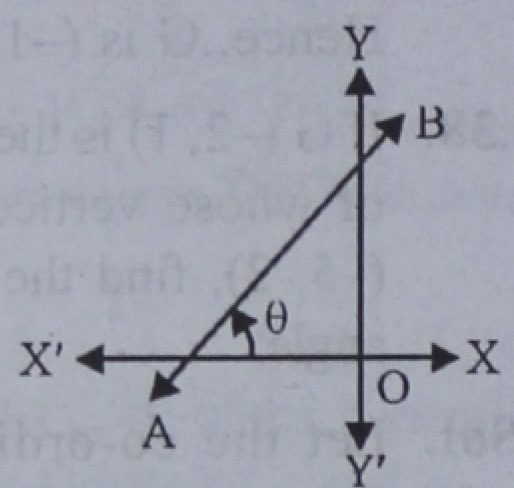
Chapter 14

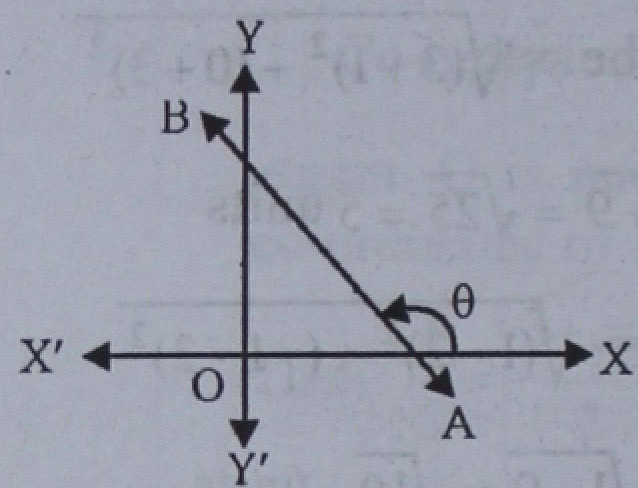
Equation of a Straight Line

POINTS TO REMEMBER

1 Inclination of a Line

The angle of inclination or simply an inclination of a line is the angle θ which the part of the line above x-axis makes with the positive direction of x-axis and measured in anticlockwise direction.





Remarks :

(*i*) The inclination of x-axis is 0°.

(*ii*) The inclination of every line parallel to x-axis is 0°.

(*iii*) The inclination of y-axis is 90°.

(*iv*) The inclination of every line parallel to y-axis is 90°.

Horizontal Line : Any line parallel to x-axis, is called a horizontal line.

Vertical Line : Any line parallel to y-axis, is called a vertical line.

Oblique Line : A line which is neither parallel to x-axis nor parallel to y-axis, is called an oblique line.

2. Slope or Gradient of a Line

If θ is the inclination of a line, then the value of tan θ is called the slope of that line and it is denoted by m. *i.e.* $m = \tan\theta$.

3. Some Results on Slope of a Line

Theorem 1. (*i*) The slope of a line parallel to x-axis is 0.

(*ii*) The slope of x-axis is 0.

Proof. (*i*) We know that the inclination of a line parallel to x-axis is 0°.

∴ Slope of a line parallel to x-axis = tan 0° = 0.

(*ii*) We know that the inclination of x-axis is 0°.

∴ Slope of x-axis = tan 0° = 0.

Remark : The slope of a horizontal line is 0.

Theorem 2. (*i*) The slope of y-axis is not defined.

(*ii*) The slope of any line parallel to y-axis, is not defined.

Proof. (*i*) We know that the inclination of y-axis is 90°.

$\therefore$ Slope of y-axis = $\tan 90° = \infty$, which is not defined.

(*ii*) We know that the inclination of a line parallel to y-axis is 90°.

$\therefore$ Slope of a line parallel to y-axis = $\tan 90° = \infty$, which is not defined.

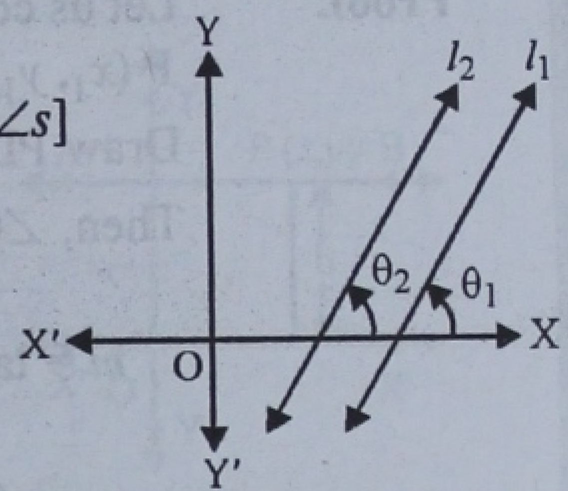
Remark : The slope of a vertical line is not defined.

Theorem 3. Two non-vertical lines are parallel if and only if their slopes are equal.

Proof. Let l_1 and l_2 be two non-vertical lines with slopes m_1 and m_2 respectively and inclinations θ_1 and θ_2 respectively.

Then $m_1 = \tan\theta_1$ and $m_2 = \tan\theta_2$.

Let $l_1 \parallel l_2$. Then,



$l_1 \parallel l_2 \quad \Rightarrow \quad \theta_1 = \theta_2$ [Corresponding $\angle s$]

$\Rightarrow \tan\theta_1 = \tan\theta_2$

$\Rightarrow \quad m_1 = m_2.$

Conversely, let $m_1 = m_2$. Then,

$m_1 = m_2 \quad \Rightarrow \tan\theta_1 = \tan\theta_2$

$\Rightarrow \quad \theta_1 = \theta_2$

$\Rightarrow \quad l_1 \parallel l_2$ [$\because$ θ_1 and θ_2 are corresponding $\angle s$]

$\therefore l_1 \parallel l_2 \Leftrightarrow m_1 = m_2.$

Hence, two lines are parallel if and only if their slopes are equal.

Theorem 4. Two non-vertical lines with slopes m_1 and m_2 are perpendicular to each other, if and only if $m_1 m_2 = -1$.

Proof. Let l_1 and l_2 be two non-vertical lines with slopes m_1 and m_2 respectively and inclinations θ_1 and θ_2 respectively. Then,

$m_1 = \tan\theta_1$ and $m_2 = \tan\theta_2$.

Let $l_1 \perp l_2$. Then,

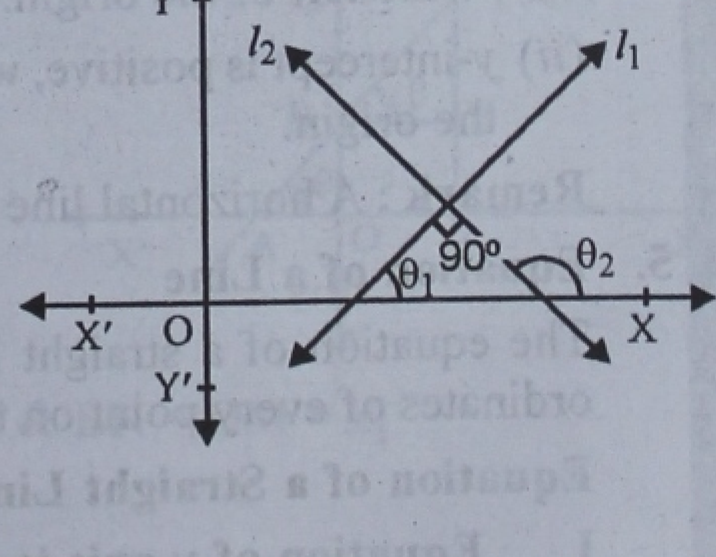
$l_1 \perp l_2 \quad \Rightarrow \quad \theta_2 = (90° + \theta_1)$

$\Rightarrow \tan\theta_2 = \tan(90° + \theta_1) = -\cot\theta_1$

$\Rightarrow \tan\theta_2 = -\dfrac{1}{\tan\theta_1}$

$\Rightarrow \tan\theta_1 \cdot \tan\theta_2 = -1$

$\Rightarrow \quad m_1 m_2 = -1.$



Conversely, let $m_1 m_2 = -1$. Then,

$m_1 m_2 = -1 \quad \Rightarrow \tan\theta_1 \cdot \tan\theta_2 = -1$

$\Rightarrow \quad \tan\theta_2 = -\dfrac{1}{\tan\theta_1}$

$\Rightarrow \quad \tan\theta_2 = -\cot\theta_1 = \tan(90° + \theta_1)$

$\Rightarrow \quad \theta_2 = 90° + \theta_1$

$\Rightarrow \quad l_1 \perp l_2.$

Hence, two lines are perpendicular to each other, if and only if the product of their slopes is– 1.

Remark : Slope of a line perpendicular a line $AB = \frac{-1}{\text{Slope of AB}}$.

Theorem 5. The slope of a line passing through two given points P (x_1, y_1) and Q (x_2, y_2) is given by $m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$.

Proof. Let us consider a line passing through the points P (x_1, y_1) and Q (x_2, y_2). Let θ be its inclination.

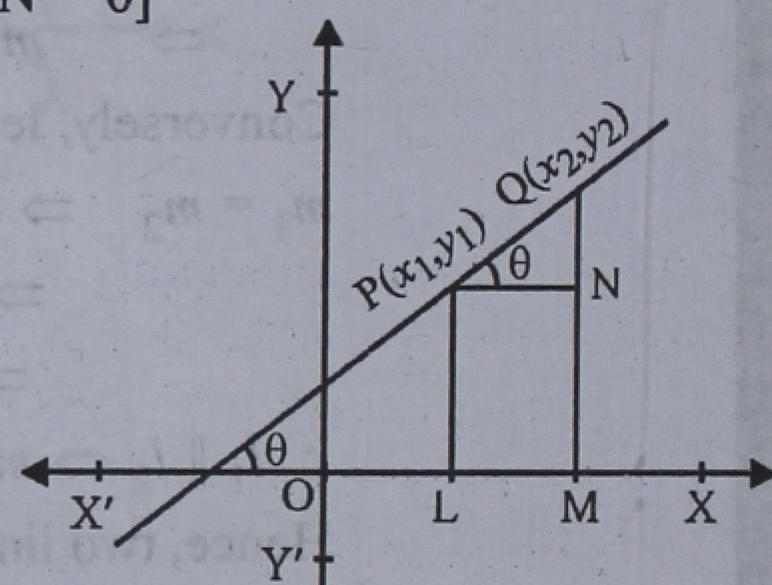
Draw PL ⊥ x-axis, QM ⊥ x-axis and PN ⊥QM.

Then, ∠QPN = θ [∵ PN ∥ x-axis ⇒ ∠QPN = θ]

$\therefore m = \tan\theta = \frac{QN}{PN} = \frac{QM - NM}{LM}$

$= \frac{QM - PL}{OM - OL} = \frac{y_2 - y_1}{x_2 - x_1}$.

Hence, slope $m = \frac{y_2 - y_1}{x_2 - x_1}$.

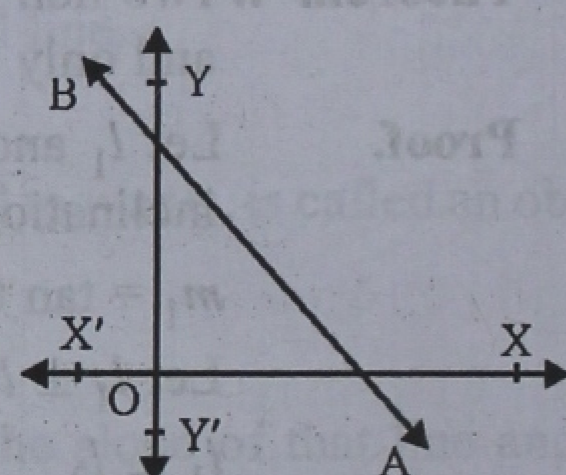


4. Intercepts Made By a Line On the Axes :

If a straight line AB meets x-axis in A and y-axis in B, then we define :

(*i*) **x-intercept** = intercept made by line AB on x-axis = **OA.**

(*ii*) **y-intercept** = intercept made by line AB on y-axis = **OB.**



Conventions For The Signs of Intercepts

(*i*) x-intercept is positive, when measured to the right of the origin and negtive, when measured to the left of the origin.

(*ii*) y-intercept is positive, when measured above the origin and negative, when measured below the origin.

Remark : A horizontal line has no x-intercept and a vertical line has no y-intercept.

5. Equation of a Line

The equation of a straight line is a linear equation in x and y, which is satisfied by the co-ordinates of every point on the line and not by any point outside this line.

Equation of a Straight Line in Various Forms

I. Equation of x-axis is $y = 0$.

We know that, the ordinate of each point on x-axis is 0.

Thus, if P (x, y) is any point on x-axis, then $y = 0$.

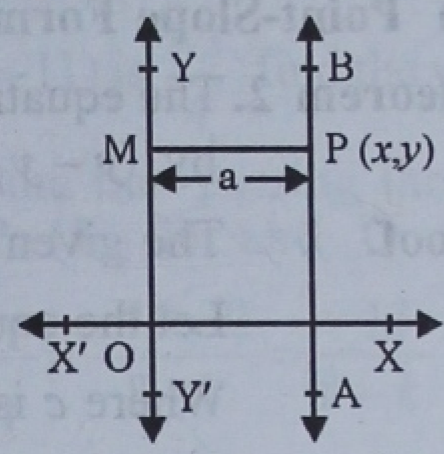
$\therefore$ The equation of x-axis is $y = 0$.

II. Equation of y-axis is $x = 0$.

We know that, the abscissa of every point on y-axis is 0. Thus, if P (x, y) is any point on y-axis, then $x = 0$.

$\therefore$ The equation of y-axis is $x = 0$.

III. Equation of a line parallel to y-axis, and at a distance a from it, is $x = a$.



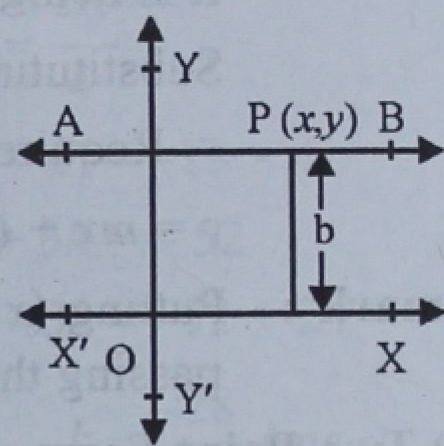
Let AB be a line parallel to y-axis, at a distance a from it.

Then, the ordinate of every point on AB is clearly a.

Thus, if P (x, y) is any point on this line, then $x = a$.

Hence, the equation of line AB is $x = a$.

IV. Equation of a line parallel to x-axis, at a distance b from x-axis is $y = b$.



Let AB be a line parallel to x-axis at a distance b from it.

Then, the abscissa of every point on AB is clearly b.

Thus, if P (x, y) is any point on this line, then $y = b$.

$\therefore$ The equation of line AB is $y = b$.

6. Equation of An Oblique Line in Various Forms

(*i*) Slope-intercept Form :

Theorem 1. The equation of a line with slope m and y-intercept c is given by, $y = mx + c$.

Proof. Let AB be the line with slope m, inclination θ and making an intercept OB = c on y-axis.

Then, $m = \tan\theta$

Let P (x, y) be any point on AB.

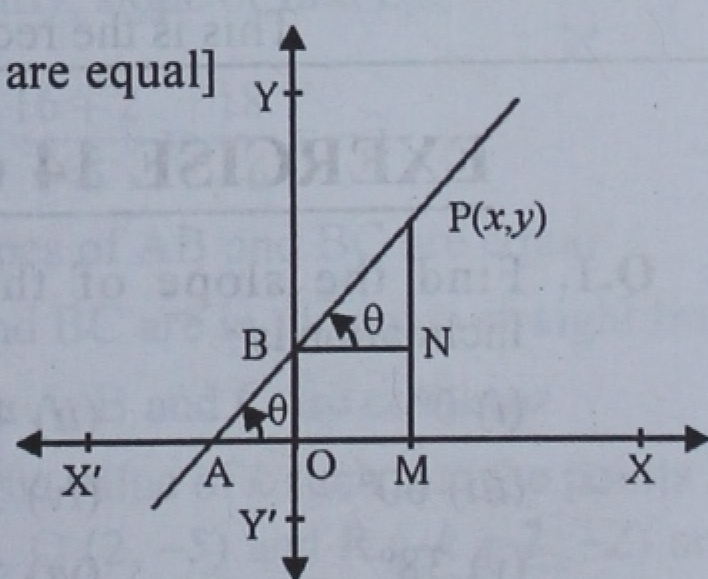
Draw PM ⊥ x-axis and BN ⊥ PM.

Then, clearly BN ∥ x-axis.

$\therefore \angle PBN = \angle BAO = \theta$ [Corresponding ∠s are equal]

Now, BN = OM = x, MN = OB = c and PM = y.

From right-angled ΔBNP, we have



$$\frac{PN}{BN} = \tan\theta$$

$$\Rightarrow \frac{PM - MN}{BN} = m \qquad [\because \tan\theta = m]$$

$$\Rightarrow \frac{y - c}{x} = m \qquad [\because PM = y, MN = OB = c \ \& \ BN = OM = x]$$

$$\Rightarrow y = mx + c.$$

Hence, the equation of the given line is $y = mx + c$.

Geometrical Interpretation of m and c in the Equation, $y = mx + c$: Let us consider a line AB whose equation is $y = mx + c$.

Then, it would mean that :

(*a*) $m = \tan\theta$, where θ is the inclination of line AB.

(*b*) The line AB makes intercept c on y-axis.

(*ii*) Point-Slope Form :

Theorem 2. The equation of a line with slope m and passing through a point A (x_1, y_1) is given by, $(y - y_1) = m(x - x_1)$.

Proof. The given line has slope m and passes through A (x_1, y_1).

Let the equation of the line be $y = mx + c$ ...(*i*)

Where c is a constant.

It is being given that the line (i) passes through the point (x_1, y_1).

Substituting $x = x_1$ and $y = y_1$ in (*i*), we get $c = y_1 - mx_1$.

$\therefore$ Required equation of the line is

$y = mx + (y_1 - mx_1)$, *i.e.*, $(y - y_1) = m(x - x_1)$.

Remark : Putting $(x_1, y_1) = (0, 0)$, we find that the equation of a line with slope m and passing through (0, 0) is $y = mx$.

(*iii*) Two-Point Form :

Theorem 3. The equation of a line passing through two points A (x_1, y_1) and B (x_2, y_2) is given by, $\frac{(y-y_1)}{(x-x_1)} = \frac{(y_2-y_1)}{(x_2-x_1)}$.

Proof. Let AB be the given line passing through the points A (x_1, y_1) and B (x_2, y_2).

Let P (x, y) be any point on AB.

Then, slope of AP = slope of AB [$\because$ A, P, B are collinear]

$$\Rightarrow \frac{(y-y_1)}{(x-x_1)} = \frac{(y_2-y_1)}{(x_2-x_1)}.$$

This is the required equation of the line.

EXERCISE 14 (A)

Q.1. Find the slope of the line whose inclination is :

(*i*) 0° (*ii*) 45°
(*iii*) 60° (*iv*) 30°
(*v*) 38° (*vi*) 56°
(*vii*) 63°30′ (*viii*) 76°

Sol. (*i*) Slope of line = $\tan 0° = 0$

(*ii*) Slope of line = $\tan 45° = 1$

(*iii*) Slope of line = $\tan 60° = \sqrt{3}$

(*iv*) Slope of line = $\tan 30° = \frac{1}{\sqrt{3}}$

(*v*) Slope of line = $\tan 38° = 0{\cdot}7813$ (Using tables)

(*vi*) Slope of line = $\tan 56° = 1{\cdot}4826$ (Using tables)

(*vii*) Slope of line = $\tan 63°30' = 2{\cdot}0057$ (Using tables)

(*viii*) Slope of line = $\tan 76° = 4{\cdot}0108$ (Using tables)

Q. 2. Find the inclination of the line whose slope is :

(i) 1 (ii) $\frac{1}{\sqrt{3}}$

(iii) 1·732 (iv) 3·487

(v) 0·364 (vi) 1·1303

Sol. Using tables, we can say that

(i) $1 = \tan 45^\circ \Rightarrow \theta = 45^\circ$

(ii) $\frac{1}{\sqrt{3}} = \tan 30^\circ \Rightarrow \theta = 30^\circ$

(iii) $1{\cdot}732 = \sqrt{3} = \tan 60^\circ \Rightarrow \theta = 60^\circ$

(iv) $3{\cdot}487 = \tan 74^\circ \Rightarrow \theta = 74^\circ$

(v) $0{\cdot}364 = \tan 20^\circ, \Rightarrow \theta = 20^\circ$

(vi) $1{\cdot}1303 = \tan 48^\circ, 30' \Rightarrow \theta = 48^\circ.30'$ **Ans.**

Q. 3. Find the slope of the line passing through the points :

(i) A (–2, 1) and B (3, –4)

(ii) A (0, –3) and B (2, 1)

(iii) A (4, –9) and B (–2, – 1)

(iv) A (2, 5) and B (–4, –4)

Sol. We know that slope of a line $= \frac{y_2 - y_1}{x_2 - x_1}$ where the line passes through the point (x_1, y_1) and (x_2, y_2)

(i) ∴ Slope of the line passing through A (–2, 1) and B (3, –4)

$$= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-1}{3-(-2)} = \frac{-5}{3+2} = \frac{-5}{5} = -1 \text{ Ans.}$$

(ii) Slope of the line passing through A (0, –3) and B (2, 1)

$$= \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-(-3)}{2-0} = \frac{1+3}{2} = \frac{4}{2} = 2 \text{ Ans.}$$

(iii) Slope of the line passing through A (4, –9) and B (–2, –1)

$$= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-(-9)}{-2-4}$$

$$= \frac{-1+9}{-6} = \frac{8}{-6} = -\frac{4}{3} \text{ Ans.}$$

(iv) Slope of the line passing through A (2, 5) and B (–4, –4)

$$= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-5}{-4-2} = \frac{-9}{-6} = \frac{3}{2} \text{ Ans.}$$

Q. 4. If the slope of the line joining P (k, 2) and Q (8, –11) is $\frac{-3}{4}$, find the value of k.

Sol. Slope of the line passing through two points P (k, 2) and Q (8, –11)

$$= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-11-2}{8-k} = \frac{-13}{8-k}$$

$$\therefore \frac{-13}{8-k} = \frac{-3}{4}$$

$$\Rightarrow -3(8-k) = -13 \times 4$$

$$\Rightarrow -24 + 3k = -52$$

$$\Rightarrow 3k = -52 + 24 = -28$$

$$\Rightarrow k = \frac{-28}{3} \text{ Ans.}$$

Q. 5. Without using distance formula, prove that the points A (1, 4), B (3, –2) and C (–3, 16) are collinear.

Sol. Slope of line AB $= \frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{-2-4}{3-1} = \frac{-6}{2} = -3$$

Similarly, slope of line BC $= \frac{16-(-2)}{-3-3}$

$$= \frac{16+2}{-6} = \frac{18}{-6} = -3$$

∵ Slopes of AB and BC are equal

∴ AB and BC are in the same straight line

Hence A, B and C are collinear.

Q. 6. Find the value of k such that the points P (k, 3), Q (2, –5) and R (–k + 2, –2) are collinear.

Sol. Slope of line PQ $= \frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{-5-3}{2-k} = \frac{-8}{2-k}$$

and, slope of line QR = $\frac{-2-(-5)}{-k+2-2}$

$= \frac{-2+5}{-k} = \frac{3}{-k}$

$\because$ Points are collinear

$\therefore \quad \frac{-8}{2-k} = \frac{3}{-k} \Rightarrow 8k = 6 - 3k$

$\Rightarrow 8k + 3k = 6 \quad \Rightarrow 11k = 6$

$\Rightarrow \quad k = \frac{6}{11}$

Hence, $k = \frac{6}{11}$ **Ans.**

Q. 7. Find the equation of a line parallel to x-axis and passing through the points (–3, 2).

Ans. $\because$ Slope of a line parallel is x-axis is 0

$\therefore$ Equation of a line passing through a given point (x_1, y_1) and having slope m is

$y - y_1 = m(x - x_1)$

Hence, equation of the line parallel to x-axis and passing through (–3, 2) will be

$y - 2 = 0\,[x - (-3)] \quad \Rightarrow y - 2 = 0$

$\therefore y = 2$ or $y - 2 = 0$ is the equation of line **Ans.**

Q. 8. Find the equation of a line parallel to y-axis and passing through the point (–7, 5).

Sol. We know that a line is parallel to y-axis

$\therefore$ Its equation will be $x = a$

But, it passes through the point (–7, 5)

Here, $x = -7$, and $y = 5$

$\therefore \quad a = -7$

Hence, equation of the line will be $x + 7 = 0$. **Ans.**

Q. 9. Find the equation of a line whose inclination is 30° and whose y–intercept is –2.

Sol. We know that equation of a line will be $y = mx + c$

Here, $m = \tan 30° = \frac{1}{\sqrt{3}}$ and $c = -2$

$\therefore$ Equation $\quad y = \frac{1}{\sqrt{3}}x + (-2)$

$\Rightarrow \quad \sqrt{3}y = x - 2\sqrt{3}$

$\therefore \sqrt{3}y - x + 2\sqrt{3} = 0$ **Ans.**

Q. 10. Find the equation of a line whose :

(*i*) Slope = $\frac{3}{4}$ and y-intercept = – 4 ;

(*ii*) Slope = – 1·2 and y-intercept = 3·8 ;

(*iii*) Gradient = $\sqrt{3}$ and y-intercept = $-\frac{2}{3}$;

(*iv*) Gradient = 0 and y-intercept = 2.

Sol. (*i*) Slope (m) = $\frac{3}{4}$ and y-intercepts = – 4

$\therefore$ Equation of the line will be

$y = mx + c \Rightarrow \quad y = \frac{3}{4}x - 4$

$\Rightarrow \quad 4y = 3x - 16$

$\Rightarrow \quad 3x - 4y - 16 = 0$ **Ans.**

(*ii*) Slope = – 1·2 and y-intercept = 3·8

$\therefore$ Equation of the line will be

$y = mx + c \quad \Rightarrow y = -1{\cdot}2x + 3{\cdot}8$

$\Rightarrow \; y = -\frac{12}{10}x + \frac{38}{10}$

$\Rightarrow y = -\frac{6}{5}x + \frac{19}{5}$ [Multiplying by 2]

$5y = -6x + 19$

$\Rightarrow 6x + 5y = 19$ **Ans.**

(*iii*) Gradient = $\sqrt{3}$ and y-intercept = $-\frac{2}{3}$

$\therefore$ Equation of the line will be

$y = mx + c \quad \Rightarrow y = \sqrt{3}x - \frac{2}{3}$

$\Rightarrow 3y = 3\sqrt{3}\,x - 2$

$\Rightarrow 3\sqrt{3}x - 3y - 2 = 0$ **Ans.**

(*iv*) Gradient = 0 and y-intercept = 2

$\therefore$ Equation of the line will be

$y = mx + c \quad \Rightarrow \quad y = 0x + 2$

$\Rightarrow y = 2 \quad \Rightarrow y - 2 = 0$ **Ans.**

Q. 11. Find the equation of the line which makes an angle of 60° with the positive direction of x-axis and passes through the point (0, –3).

Sol. Slope of the line = $\tan 60° = \sqrt{3}$

And passes through the point (0, –3)

$\therefore$ Equation of the line

$y - y_1 = m(x - x_1)$

$\Rightarrow y - (-3) = \tan 60° (x - 0) = \tan 60° x$

$\Rightarrow \ y + 3 = \sqrt{3}x \ \Rightarrow y - \sqrt{3}x + 3 = 0$

$\Rightarrow \sqrt{3}x - y = 3$ **Ans.**

Q. 12. Find the equation of a line :

(*i*) whose slope is 4 and which passes through the point (5, – 7) ;

(*ii*) whose slope is – 3 and which passes through the point (– 2, 3) ;

(*iii*) whose slope is $\frac{-5}{3}$ and which passes through the point (– 3, 5).

Sol. (*i*) Slope (m) = 4 and passes through the point = (5, – 7)

$\therefore$ Equation of the line will be

$y - y_1 = m(x - x_1)$

$\Rightarrow \quad y - (-7) = 4(x - 5)$

$\Rightarrow \quad y + 7 = 4x - 20$

$\Rightarrow y - 4x + 7 + 20 = 0$

$\Rightarrow \quad y - 4x + 27 = 0$ **Ans.**

(*ii*) Slope = – 3 and passes through the point (– 2, 3)

$\therefore$ Equation of the line will be

$y - y_1 = m(x - x_1)$

$\Rightarrow \quad y - 3 = -3[x - (-2)]$

$\Rightarrow \quad y - 3 = -3(x + 2)$

$\Rightarrow \quad y - 3 = -3x - 6$

$\Rightarrow 3x + y - 3 + 6 = 0$

$\Rightarrow \quad 3x + y + 3 = 0$ **Ans.**

(*iii*) Slope = $\frac{-5}{3}$ and passes through the point (– 3, 5)

$\therefore$ Equation of the line will be

$y - y_1 = m(x - x_1)$

$\Rightarrow \quad y - 5 = \frac{-5}{3}[x - (-3)]$

$\Rightarrow \quad y - 5 = \frac{-5}{3}(x + 3)$

$\Rightarrow \quad 3(y - 5) = -5(x + 3)$

$\Rightarrow \quad 3y - 15 = -5x - 15$

$\Rightarrow 5x + 3y - 15 + 15 = 0$

$\Rightarrow \quad 5x + 3y = 0$ **Ans.**

Q. 13. Find the equation of a line whose y-intercept is $\left(\frac{-3}{2}\right)$ and which passes through the point (– 1, – 3).

Sol. The y-intercept = $\frac{-3}{2}$

$\therefore \quad y = \frac{-3}{2}$ and, its $x = 0$

Now, the line is passing through two points $\left(0, \frac{-3}{2}\right)$ and (– 1, – 3)

$\therefore$ Equation of line will be

$$\frac{y - y_1}{x - x_1} = \frac{y - y_2}{x - x_2}$$

$$\Rightarrow \frac{y + \frac{3}{2}}{x - 0} = \frac{y + 3}{x + 1} \Rightarrow \frac{2y + 3}{2x} = \frac{y + 3}{x + 1}$$

$\Rightarrow 2xy + 2y + 3x + 3 = 2xy + 6x$

$\Rightarrow 2xy + 2y + 3x + 3 - 2xy - 6x = 0$

$\Rightarrow 2y - 3x + 3 = 0$ **Ans.**

Q. 14. (*i*) If the point (k, 2) lies on the line $3x - 8y = 2$, find the value of k.

(*ii*) If the line $x - 2y + p = 0$ passes through the point (– 2, – 1·5), find the value of p.

(*iii*) Find the value of p, given that the line $\frac{y}{2} = x - p$ passes through the point $(-4, 4)$.

Sol. (*i*) $\because$ point $(k, 2)$ lies on the line $3x - 8y = 2$

Then, it will satisfy it

$\therefore \quad 3(k) - 8(2) = 2$

$\Rightarrow \quad 3k - 16 = 2$

$\Rightarrow \quad 3k = 2 + 16 = 18$

$\Rightarrow \quad k = \frac{18}{3} = 6$

Hence, $k = 6$ **Ans.**

(*ii*) $\because$ The line $x - 2y + p = 0$ passes through the point $(-2, -1{\cdot}5)$

$\therefore$ It will satify the equation

$\therefore -2 - 2(-1{\cdot}5) + p = 0$

$\Rightarrow -2 + 3 + p = 0 \Rightarrow 1 + p = 0$

$\Rightarrow p = -1$

Hence, $p = -1$ **Ans.**

(*iii*) $\because$ The line $\frac{y}{2} = x - p$ passes through $(-4, 4)$.

$\therefore$ It will satisfy the equation

$\therefore \quad \frac{4}{2} = -4 - p \Rightarrow \quad 2 = -4 - p$

$\Rightarrow \quad 2 + 4 = -p \Rightarrow \quad p = -6$

Hence, $p = -6$ **Ans.**

Q. 15. Show that the line $3x = y + 1$ bisects the line segment joining A $(-2, 3)$ and B $(4, 1)$.

Sol. Co-ordinate of mid-point of line joining the points A $(-2, 3)$ and B $(4, 1)$ will be

$$\left[\frac{-2+4}{2}, \frac{3+1}{2}\right] \text{ or } \left[\frac{2}{2}, \frac{4}{2}\right] \text{ or } (1, 2)$$

If mid-point $(1, 2)$ lies on the line $3x = y + 1$ then it will satisfy it.

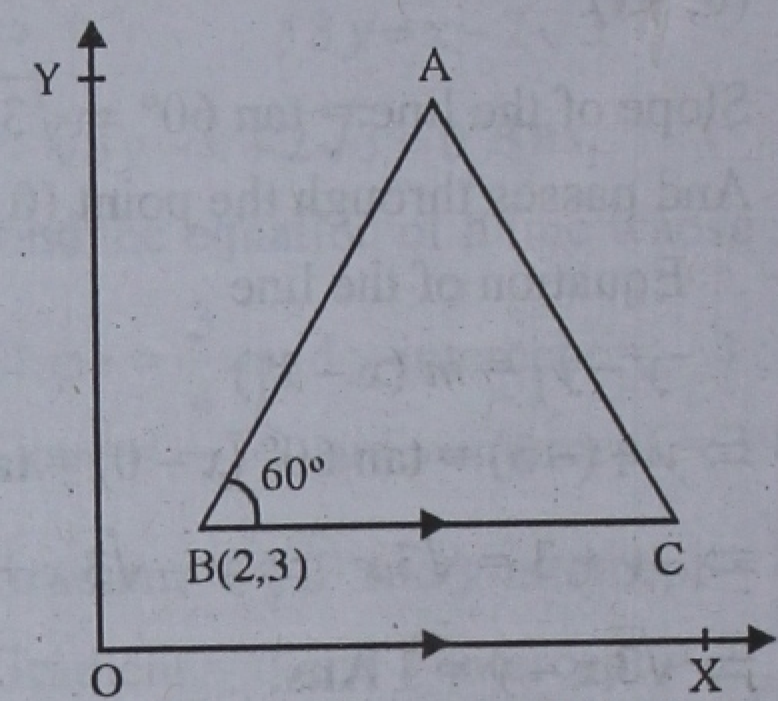
Now, substituting the value of x and y is

$3x = y + 1 \Rightarrow 3(1) = 2 + 1$

$\Rightarrow \quad 3 = 3$ which is true.

Hence, the line $3x = y + 1$ bisects the line joining the points A $(-2, 3)$, B $(4, 1)$ **Ans.**

Q. 16. The side BC of equilateral Δ ABC is parallel to x-axis. If the co-ordinates of B are B $(2, 3)$, find the equations of sides



(*i*) BC (*ii*) AB.

Sol. (*i*) $\because$ BC is parallel to x-axis and co-ordinates of B are $(2, 3)$

$\therefore$ Equation of BC will be $y = 3$

(*ii*) Slope of AB $= \tan 60°$ $\quad (\because \angle B = 60°)$

$= \sqrt{3}$

And AB passes through B $(2, 3)$

$\therefore$ Equation of line AB will be

$y - y_1 = m(x - x_1)$

$\Rightarrow y - 3 = \sqrt{3}(x - 2)$

$\Rightarrow y - 3 = \sqrt{3}x - 2\sqrt{3}$

$\Rightarrow \quad y = \sqrt{3}x - 2\sqrt{3} + 3$

$\Rightarrow \quad y = \sqrt{3}x - 2\sqrt{3} + 3$

$\Rightarrow \quad y = \sqrt{3}x + 3 - 2\sqrt{3}$ **Ans.**

Q. 17. Find the inclination of the line $2y - 2x + 5 = 0$.

Sol. Equation of line is

$2y - 2x + 5 = 0$

$\Rightarrow 2y = 2x - 5 \Rightarrow y = x - \frac{5}{2}$

$(\because y = mx + c$

$\therefore$ Slope $(m) = 1$

i.e. $\tan\theta = 1 = \tan 45°$

$\Rightarrow \quad \theta = 45°$ **Ans.**

Q. 18. Find the inclination of the line $y = \sqrt{3}x + 2$.

Sol. In the equation of line $y = \sqrt{3}x + 2$ $\quad (\because y = mx + c)$

Slope $(m) = \sqrt{3}$

i.e. $\tan\theta = \sqrt{3} = \tan 60°$

$\therefore \theta = 60°$ **Ans.**

Q.19. Find the gradient and the y–intercept of each of the following lines :

(*i*) $6x + 4y - 9 = 0$ (*ii*) $5(x - 2y) = 3$

(*iii*) $\frac{x}{6} + \frac{y}{9} = 1$ (*iv*) $3y + 2x = 6$

(*v*) $x + 5 = 0$ (*vi*) $y - 7 = 0$.

Sol. (*i*) In the equation of the line

$6x + 4y - 9 = 0, 4y = -6x + 9$

$\Rightarrow y = -\frac{6}{4}x + \frac{9}{4}$ [Dividing by 4]

$\Rightarrow y = -\frac{3}{2}x + \frac{9}{4}$ $\quad (\because y = mx + c)$

$\therefore$ Slope $(m) = \frac{-3}{2}$ and y–intercept $= \frac{9}{4}$

$\therefore$ Gradient $= \frac{-3}{2}$ and y–intercept $= \frac{9}{4}$ **Ans.**

(*ii*) In the equation of the line : $5(x - 2y) = 3$

$\Rightarrow 5x - 10y = 3 \Rightarrow -10y = -5x + 3$

$\Rightarrow \quad y = \frac{-5}{-10}x + \frac{3}{-10}$

$\Rightarrow \quad y = \frac{1}{2}x - \frac{3}{10}$

$\therefore$ Slope $(m) = \frac{1}{2}$ and $c = \frac{-3}{10}$ $\quad (\because y = mx + c)$

$\therefore$ Gradient $= \frac{1}{2}$ and y-intercept $= \frac{-3}{10}$ **Ans.**

(*iii*) In the equation of line $\frac{x}{6} + \frac{y}{9} = 1$

$\Rightarrow \frac{x}{6} \times 18 + \frac{y}{9} \times 18 = 1 \times 18$

[Multiplying by 18, LCM of 6, 9]

$\Rightarrow 3x + 2y = 18 \quad \Rightarrow 2y = -3x + 18$

$\Rightarrow \quad y = \frac{-3}{2}x + 9$ [Dividing by 2]

Here, $m = \frac{-3}{2}$ and $c = 9$ $(\because y = mx + c)$

Hence, gradient $= \frac{-3}{2}$ and y–intercept $= 9$ **Ans.**

(*iv*) In the equation of line $3y + 2x = 6$

$3y = -2x + 6$

$\Rightarrow y = \frac{-2}{3}x + \frac{6}{3}$ [Dividing by 3]

$\Rightarrow y = \frac{-2}{3}x + 2$

Hence, $m = \frac{-2}{3}$ and $c = 2$ $(\because y = mx + c)$

Hence, gradient $= \frac{-2}{3}$ and y-intercept $= 2$ **Ans.**

(*v*) In the equation of the line $x + 5 = 0$
Here, the gradient is not defined and the line has not y–intercept as it is a line parallel to y–axis.

(*vi*) In the equation of line $y - 7 = 0$

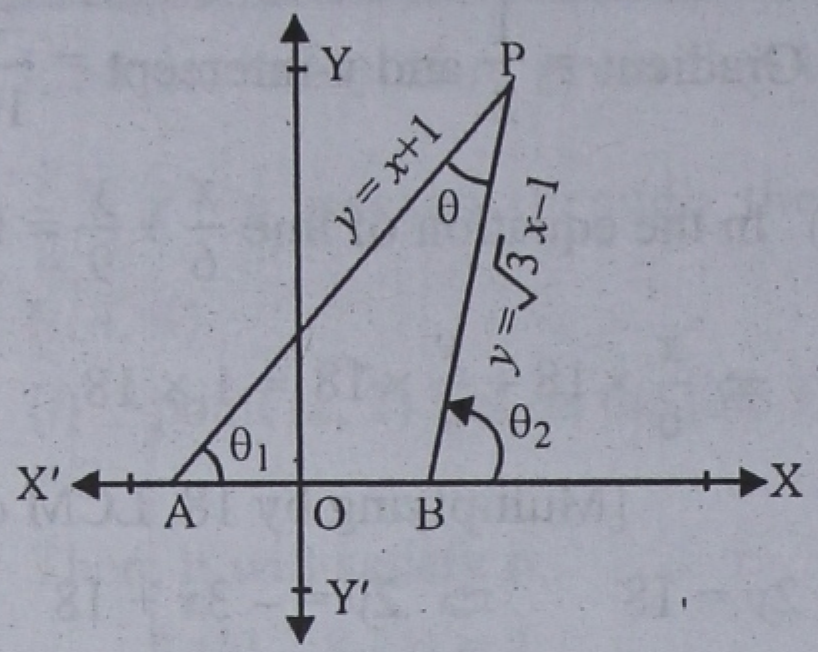
$\Rightarrow \quad y = 0\,x + 7$

Here, $m = 0$ and $c = 7$

Hence, gradient $= 0$

and y–intercept $= 7$ **Ans.**

Q.20. The given figure represents the lines $y = x + 1$ and $y = \sqrt{3}x - 1$. Write down the angles which the lines make with the positive direction of x-axis. Hence, determine θ.



Sol. Let the line $y = x + 1$ makes an angle θ_1 with the x-axis and line $y = \sqrt{3}x -$ makes an angle θ_2 wth postive direction of x-axis. Let there lines meet each other at P. Let $\angle$ APB = θ.

Now, in the equation of line $y = x + 1$

$m_1 = 1 \Rightarrow \tan\theta_1 = 1 = \tan 45^\circ$

$\therefore \theta_1 = 45^\circ$

And in the equation of the line

$y = \sqrt{3}x - 1,\ m_2 = \sqrt{3}$

$\Rightarrow \theta_2 = \sqrt{3} = \tan 60^\circ$

$\therefore \theta_2 = 60^\circ$

Now, in ΔAPB,

Ext. $\angle$PBX = Interior $\angle$A + $\angle$P

$\Rightarrow \quad \theta_2 = \theta_1 + \theta \quad \Rightarrow \theta = \theta_2 - \theta_1$

$\Rightarrow \quad \theta = 60^\circ - 45^\circ = 15^\circ$

Hence, $\theta = 15^\circ$ **Ans.**

Q. 21. Find the gradient and the equation of the line passing through the points :

(*i*) A (– 2, 1) and B (3, – 4) ;

(*ii*) A (– 6, – 2) and B (2, – 3) ;

(*iii*) A (2, – 5) and B (– 3, 7).

Sol. (*i*) Points are A (– 2, 1) and B (3, – 4)

$\therefore$ Gradient $(m) = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{-4-1}{3-(-2)} = \dfrac{-5}{3+2} = \dfrac{-5}{5} = -1$

And equation of the line will be

$y - y_1 = m(x - x_1)$

$\Rightarrow \quad y - 1 = -1\,[x - (-2)]$

$\Rightarrow \quad y - 1 = -(x + 2)$

$\Rightarrow \quad y - 1 = -x - 2$

$\Rightarrow x + y - 1 + 2 = 0$

$\therefore \quad x + y + 1 = 0$ **Ans.**

(*ii*) Points are A (– 6, – 2) and B (2, – 3)

$\therefore$ Gradient $(m) = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{-3-(-2)}{2-(-6)} = \dfrac{-3+2}{2+6} = -\dfrac{1}{8}$

And equation of the line will be

$y - y_1 = m(x - x_1)$

$\Rightarrow y - (-2) = -\dfrac{1}{8}\,[x - (-6)]$

$y + 2 = -\dfrac{1}{8}(x + 6)$

$\Rightarrow \quad 8y + 16 = -x - 6$

$\Rightarrow 8y + x + 16 + 6 = 0$

$\Rightarrow \quad x + 8y + 22 = 0$ **Ans.**

(*iii*) Points are A (2, – 5) and B (– 3, 7)

$\therefore$ Gradient $(m) = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{7-(-5)}{-3-2} = \dfrac{7+5}{-5} = \dfrac{12}{-5} = -\dfrac{12}{5}$

And equation of the line will be

$y - y_1 = m(x - x_1)$

$\Rightarrow y - (-5) = \dfrac{-12}{5}(x - 2)$

$\Rightarrow \quad y + 5 = \dfrac{-12}{5}(x - 2)$

$\Rightarrow \quad 5y + 25 = -12x + 24$

$\Rightarrow \quad 12x + 5y + 25 - 24 = 0$

$\therefore \ 12x + 5y + 1 = 0$ **Ans.**

Q. 22. A (2, 7) and B (– 3, 5) are two given points. Find :

(*i*) the gradient of AB ;

(*ii*) the equation of AB ;

(*iii*) the co-ordinates of the point, where AB intersects x–axis.

Sol. Two points are given : A (2, 7) and B (– 3, 5) (*i*) Gradient of AB (m)

$$= \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-7}{-3-2} = \frac{-2}{-5} = \frac{2}{5}$$

(*ii*) Equation of AB will be

$y - y_1 = m(x - x_1)$

$\Rightarrow \quad y - 7 = \frac{2}{5}(x - 2)$

$\Rightarrow \quad 5y - 35 = 2x - 4$

$\Rightarrow 2x - 5y - 4 + 35 = 0$

$\Rightarrow \quad 2x - 5y + 31 = 0 \quad \ldots(i)$

(*iii*) $\because$ The line intersects x–axis

$\therefore$ its $y = 0$

Let, the point of intersection be $(x_1, 0)$

Substituting, the value of x, y in (*i*)

$2x - 5 \times 0 + 31 = 0$

$\Rightarrow 2x + 31 = 0 \;\Rightarrow 2x = -31$

$\Rightarrow \quad x = \frac{-31}{2}$

$\therefore$ Co-ordinates of the points are $\left(\frac{-31}{2}, 0\right)$ **Ans.**

Q. 23. A straight line passes through the points A (2, – 5) and B (4, 3).

Find : (*i*) the slope of the line AB ;

(*ii*) the equation of the line AB ;

(*iii*) the value of k, if AB passes through the point $(k - 1, k + 4)$.

Sol. Given points are A (2, – 5) and B (4, 3).

(*i*) The slope of the line AB $= \frac{3-(-5)}{4-2}$

$= \frac{8}{2} = 4. \quad \left[m = \frac{y_2 - y_1}{x_2 - x_1}\right]$

(*ii*) The line AB passes through the point A (2, – 5) and has slope 4, therefore, its equation is

$y - (-5) = 4(x - 2)$

$[y - y_1 = m(x - x_1)]$

$\Rightarrow \quad y + 5 = 4x - 8$

$\Rightarrow \quad 4x - y - 13 = 0$

(*iii*) As the line AB passes through the line $(k - 1, k + 4)$, we get :

$4(k - 1) - (k + 4) - 13 = 0$

$\Rightarrow \quad 4k - 4 - k - 4 - 13 = 0$

$\Rightarrow \quad 3k - 21 = 0 \Rightarrow k = 7$ **Ans.**

Q. 24. Find the equation of the line with x-intercept = 5 and passing through the point (4, –7).

Sol. $\because$ The x–intercept of the line = 5 units

$\therefore$ its $y = 0$

$\therefore$ Line passes through point (5, 0)

$\therefore$ The line passes through (5, 0) and (4, – 7) and equation of the line

$$\frac{y - y_1}{x - x_1} = \frac{y - y_2}{x - x_2}$$

$\Rightarrow \quad \frac{y - (-7)}{x - 4} = \frac{y - 0}{x - 5}$

$\Rightarrow \quad \frac{y + 7}{x - 4} = \frac{y - 0}{x - 5}$

$\Rightarrow \quad (y + 7)(x - 5) = (y - 0)(x - 4)$

$\Rightarrow xy - 5y + 7x - 35 = xy - 4y$

$\Rightarrow xy - 5y + 7x - 35 - xy + 4y = 0$

$\Rightarrow \quad 7x - y - 35 = 0$

equation of the line $7x - y - 35 = 0$ **Ans.**

Q. 25. Find the equation of a line passing through the point (2, 3) and intersecting the line $2x - 3y = 6$ on the y-axis.

Sol. $\because$ The given line intersects $2x - 3y = 6$ on the y–axis

i.e. $x = 0$, then $0 - 3y = 6$

$\Rightarrow y = \frac{6}{-3} = -2$

$\therefore$ The line will pass through (0, – 2)

Now, the line passes through (0, – 2) and (2, 3)

$\therefore$ Equation of the line will be

$$\frac{y - y_1}{x - x_1} = \frac{y - y_2}{x - x_2}$$

$\Rightarrow \quad \dfrac{y-3}{x-2} = \dfrac{y-(-2)}{x-0}$

$\Rightarrow \quad \dfrac{y-3}{x-2} = \dfrac{y+2}{x}$

$\Rightarrow xy - 3x = xy + 2x - 2y - 4$

$\Rightarrow xy - 3x - xy - 2x + 2y + 4 = 0$

$\Rightarrow \quad -5x + 2y + 4 = 0$

$\Rightarrow \quad 5x - 2y - 4 = 0$ **Ans.**

Q. 26. The vertices of a Δ ABC are A (2, – 11), B (2, 13) and C (– 12, 1). Find the equations of its sides.

Sol. The co-ordinates of vertices of a Δ ABC are A (2, – 11), B (2, 13) and C (– 12, 1)

Equation of line AB will be

$\dfrac{y-y_1}{x-x_1} = \dfrac{y-y_2}{x-x_2} \Rightarrow \dfrac{y+11}{x-2} = \dfrac{y-13}{x-2}$

$\Rightarrow (y+11)(x-2) = (y-13)(x-2)$

$\Rightarrow xy - 2y + 11x - 22$

$= xy - 2y - 13x + 26$

$\Rightarrow xy - 2y + 11x - xy + 2y + 13x$

$= 26 + 22$

$\Rightarrow 24x = 48$

$\Rightarrow \quad x = \dfrac{48}{24} = 2$

$\therefore \quad x = 2$

Equation of BC will be :

$\dfrac{y-13}{x-2} = \dfrac{y-1}{x+12}$

$\Rightarrow (y-13)(x+12) = (y-1)(x-2)$

$\Rightarrow xy + 12y - 13x - 156$

$= xy - 2y - x + 2$

$\Rightarrow xy + 12y - 13x - xy + 2y + x$

$= 2 + 156$

$\Rightarrow 14y - 12x = 158$

$\Rightarrow 7y - 6x = 79$ (Dividing by 2)

Equation of CA will be :

$\dfrac{y-1}{x+12} = \dfrac{y+11}{x-2}$

$\Rightarrow (y-1)(x-2) = (y+11)(x+12)$

$\Rightarrow xy - 2y - x + 2$

$= xy + 12y + 11x + 132$

$\Rightarrow xy - 2y - x - xy - 12y - 11x = 132 - 2$

$\Rightarrow -14y - 12x = 130$

$\Rightarrow 7y + 6x = -65$

$\Rightarrow 7y + 6x + 65 = 0$ **Ans.**

Q. 27. Find the equations of the diagonals of a rectangle whose sides are $x = -1$, $x = 4$, $y = -1$ and $y = 2$.

Sol. The equations of the sides of a rectangle are $x = -1$, $x = 4$, $y = -1$ and $y = 2$.

Let, ABCD be the rectangle formed by these lines as shown in the adjoining figure, clearly its vertices are A (– 1, – 1), B (4, – 1), C (4, 2) and D (– 1, 2).

Slope of the diagonal AC $= \dfrac{2-(-1)}{4-(-1)} = \dfrac{3}{5}$

∴ The equation of AC is

$y - (-1) = \dfrac{3}{5}(x - (-1))$

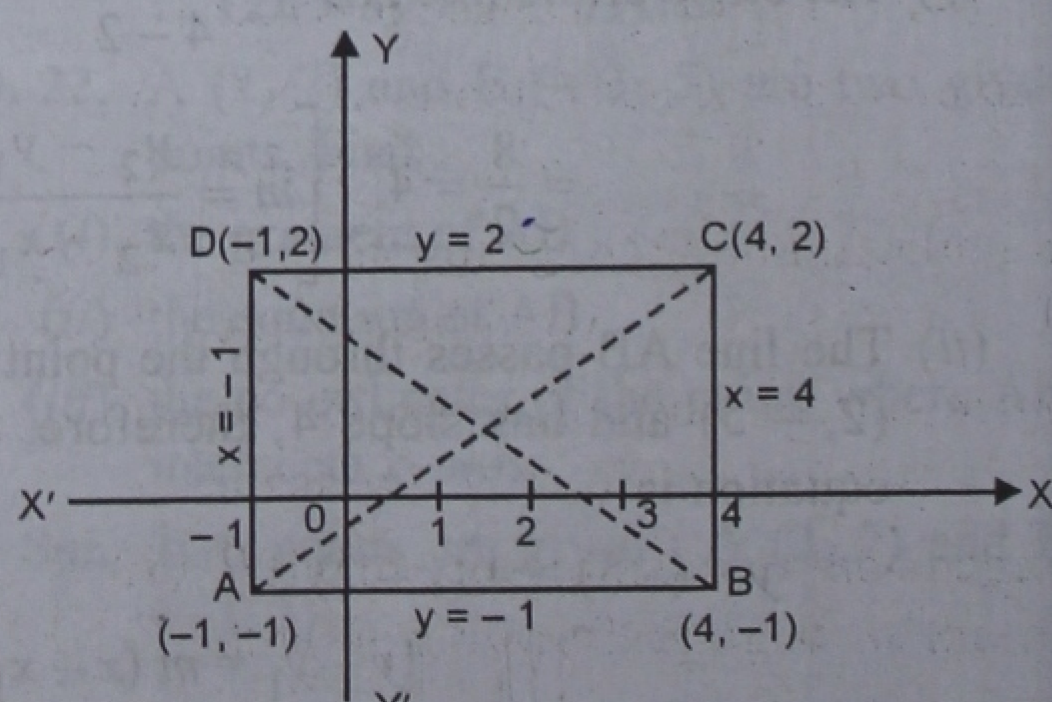
$\Rightarrow \quad y + 1 = \dfrac{3}{5}(x+1)$

$\Rightarrow \quad 5y + 5 = 3x + 3$

$\Rightarrow \quad 3x - 5y - 2 = 0$ **Ans.**

Slope of the diagonal BD

$= \dfrac{2-(-1)}{-1-4} = -\dfrac{3}{5},$



∴ the equation of BD is

$$y-(-1)=-\frac{3}{5}(x-4)$$

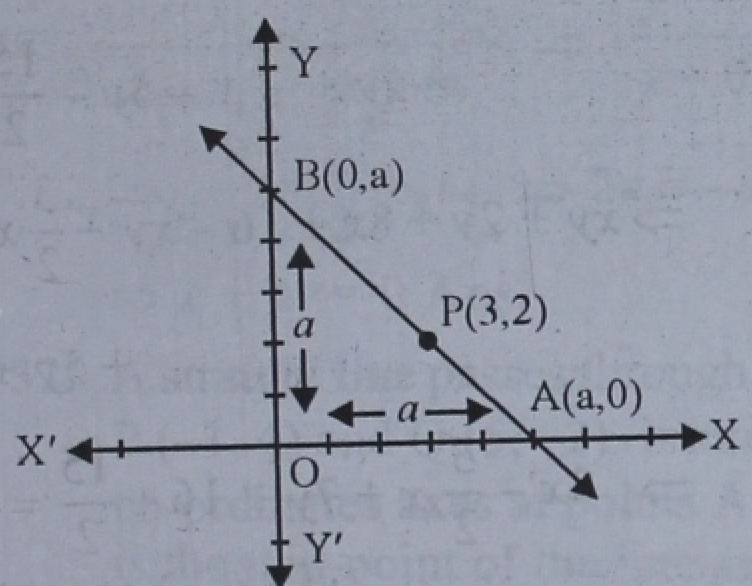
$$\Rightarrow \quad y+1=-\frac{3}{5}(x-4)$$

$$\Rightarrow \quad 5y+5=-3x+12$$

$$\Rightarrow 3x+5y-7=0$$ **Ans.**

Q. 28. Find the equation of the line passing through the point (3, 2) and making positive equal intercepts on axes. Find the length of each intercept.

Sol. Let the line passing through the point P (3, 2) makes equal intercepts with the axis, each equal to a units.



∴ The line will passes through A (a, 0) and B (0, a)

Now, equation of the line AB will be

$$\frac{y-y_1}{x-x_1}=\frac{y_2-y_1}{x_2-x_1} \Rightarrow \frac{y-0}{x-a}=\frac{a-0}{0-a}$$

$$\Rightarrow \frac{y}{x-a}=\frac{a}{-a} \qquad \Rightarrow \frac{y}{x-a}=-1$$

$$\Rightarrow \quad y=-x+a \Rightarrow x+y=a$$

∵ It passes through the point P (3, 2)

$$\therefore 3+2=a \qquad \Rightarrow a=5$$

Hence, the required equation of the line will be

$$x+y=5$$

The length of each intercept = 5 units

Ans.

Q. 29. In what ratio does the line $x-5y+15=0$ divide the join of A (2, 1) and B (–3, 6) ? Also, find the co-ordinates of their point of intersection.

Sol. Let point P (x, y) divides the line segment AB in the ratio k : 1

$$\therefore \quad x=\frac{k\times(-3)+1\times 2}{k+1}=\frac{-3k+2}{k+1}$$

$$\text{and } y=\frac{k(6)+1\times 1}{k+1}=\frac{6k+1}{k+1}$$

∵ P lies on the line $x-5y+15=0$

∴ Point P will satisfy it

$$=\frac{-3k+2}{k+1}-5\left(\frac{6k+1}{k+1}\right)+15=0$$

$$\Rightarrow (-3k+2)-5(6k+1)+15(k+1)=0$$

$$\Rightarrow -3k+2-30k-5+15k+15=0$$

$$\Rightarrow \quad -33k+15k-5+17=0$$

$$\Rightarrow -18k=-12$$

$$\Rightarrow \quad k=\frac{-12}{-18}=\frac{2}{3}$$

$$\therefore \text{Ratio } k:1 \Rightarrow \frac{2}{3}:1 \quad \Rightarrow 2:3$$

And co-ordinates of the point P are

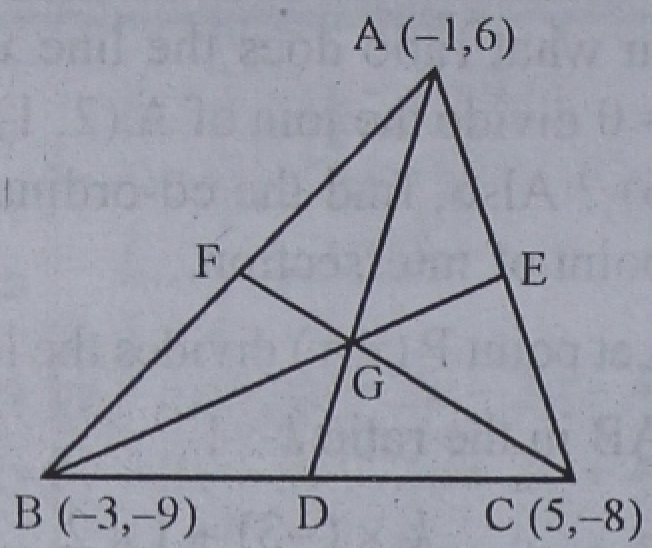
$$\left(\frac{-3k+2}{k+1}, \frac{6k+1}{k+1}\right) \text{ or } \left[\frac{-3\times\frac{2}{3}+2}{\frac{2}{3}+1}, \frac{6\times\frac{2}{3}+1}{\frac{2}{3}+1}\right]$$

$$\text{or } \left[\frac{-2+2}{\frac{5}{3}}, \frac{4+1}{\frac{5}{3}}\right] \text{ or } \left(\frac{0}{\frac{5}{3}}, \frac{5\times 3}{5}\right)$$

or (0, 3) **Ans.**

Q. 30. Find the equations of the medians of ΔABC whose vertices are A (–1, 6), B (– 3, – 9) and C (5, – 8). Hence, find the co-ordinates of the centriod of ΔABC.

Sol. Co-ordinates of the vertices of a Δ ABC are A (– 1, 6), B (– 3, – 9) and C (5, – 8)



Let D, E and F are the mid-points of the sides BC, CA and AB respectively. The medians AD, BE and CF intersect each other at point G which is the centroid of the Δ ABC.

(*i*) D is the mid-point of BC

∴ Co-ordinates of mid-point D are

$\left(\frac{-3+5}{2}, \frac{-9-8}{2}\right)$ or $\left(\frac{2}{2}, \frac{-17}{2}\right)$

or $\left(1, \frac{-17}{2}\right)$

∴ Equations of line AD will be

$$\frac{y-6}{x+1} = \frac{y+\frac{17}{2}}{x-1}$$

$$\left\{\because \frac{y-y_1}{x-x_1} = \frac{y-y_2}{x-x_2}\right\}$$

$\Rightarrow (x-1)(y-6) = (x+1)\left(y+\frac{17}{2}\right)$

$\Rightarrow xy - 6x - y + 6 = xy + \frac{17}{2}x + y + \frac{17}{2}$

$\Rightarrow xy - 6x - y - xy - \frac{17}{2}x - y = \frac{17}{2} - \frac{6}{1}$

$\Rightarrow \frac{-29}{2}x - 2y = \frac{5}{2} \Rightarrow -29x - 4y = 5$

$\Rightarrow$ $29x + 4y = 5$ **Ans.**

(*ii*) E is the mid-point of AC

∵ Co-ordinates of E are

$\left(\frac{5-1}{2}, \frac{-8+6}{2}\right)$ or $\left(\frac{4}{2}, \frac{-2}{2}\right)$ or $(2, -1)$

∴ Equation of BE will be $\frac{y+9}{x+3} = \frac{y+1}{x-2}$

$\Rightarrow (y+9)(x-2) = (y+1)(x+3)$

$\Rightarrow xy - 2y + 9x - 18 = xy + 3y + x + 3$

$\Rightarrow xy - 2y + 9x - 18 - xy - 3y - x - 3 = 0$

$\Rightarrow 8x - 5y - 21 = 0$

(*iii*) F is the mid-point of AB

∴ Co-ordinates of F are $\left(\frac{-1-3}{2}, \frac{6-9}{2}\right)$

or $\left(\frac{-4}{2}, \frac{-3}{2}\right)$ or $\left(-2, \frac{-3}{2}\right)$

Now, equation of CF will be

$$\frac{y+8}{x-5} = \frac{y+\frac{3}{2}}{x+2}$$

$\Rightarrow (y+8)(x+2) = (x-5)\left(y+\frac{3}{2}\right)$

$\Rightarrow xy + 2y + 8x + 16$

$= xy + \frac{3}{2}x - 5y - \frac{15}{2}$

$\Rightarrow xy + 2y + 8x + 16 - xy - \frac{3}{2}x$

$+ 5y + \frac{15}{2} = 0$

$\Rightarrow$ $8x - \frac{3}{2}x + 7y + 16 + \frac{15}{2} = 0$

$\Rightarrow \frac{16x-3x}{2} + 7y + \frac{32+15}{2} = 0$

$\Rightarrow$ $\frac{13}{2}x + 7y + \frac{47}{2} = 0$

$\Rightarrow$ $13x + 14y + 47 = 0$ **Ans.**

(*iv*) Now, co-ordinates of centroid 3 will be

$\left(\frac{-1-3+5}{3}, \frac{6-9-8}{3}\right)$ or $\left(\frac{1}{3}, \frac{-11}{3}\right)$ **Ans.**

Q. 31. Find the equation of the line passing through the origin and the point of intersection of the lines $5x + 7y = 3$ and $2x - 3y = 7$.

Sol. In order to find the point of the intersection of the lines $5x + 7y = 3$ and $2x - 3y = 7$, we will solve them

Now, $5x + 7y = 3$ ...(*i*)

$2x - 3y = 7$ ...(*ii*)

Multiply (*i*) by 3 and (*ii*) by 7, we get

$$15x + 21y = 9$$
$$14x - 21y = 49$$

Adding, we get

$$29x = 58 \quad \Rightarrow x = \frac{58}{29} = 2$$

Substituting the value of x in (*i*),

$\Rightarrow \quad 5 \times 2 + 7y = 3 \quad \Rightarrow 10 + 7y = 3$

$\Rightarrow \quad 7y = 3 - 10 \quad \Rightarrow \quad 7y = -7$

$\Rightarrow \quad y = \frac{-7}{7} = -1$

$\therefore$ Point of intersection is (2, –1)

Now, equation of the line passing through the origin (0, 0) and point (2, –1), will be

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{y - 0}{x - 0} = \frac{-1 - 0}{2 - 0}$$

$\Rightarrow \frac{y}{x} = \frac{-1}{2} \quad \Rightarrow 2y = -x$

$\Rightarrow x + 2y = 0$ **Ans.**

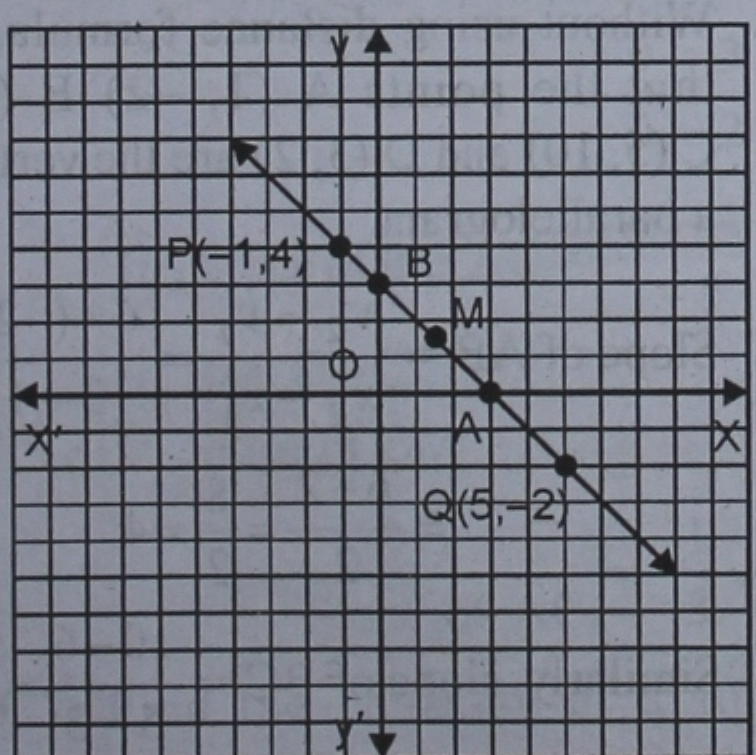
Q. 32. A straight line passes through the points P (–1, 4) and Q (5, –2). It intersects the co-ordinates axes at points A and B. M is the mid-point of the line segment AB. Find

(*i*) the equation of the line ;

(*ii*) the co-ordinates of A and B ;

(*iii*) the co-ordinates of M.

Sol. (*i*) The equation of the line passing through P (–1, 4) and Q (5, –2) will be



$$\frac{y - y_1}{x - x_1} = \frac{y - y_2}{x - x_2} \Rightarrow \frac{y+2}{x-5} = \frac{y-4}{x+1}$$

$\Rightarrow xy + 2x + y + 2 = xy - 5y - 4x + 20$

$\Rightarrow 2x + 4x + y + 5y = 20 - 2$

$\Rightarrow \quad 6x + 6y = 18$

$\Rightarrow \quad x + y = 3$

(*ii*) $\because$ This line intersects x-axis at A and y-axis at B

$\because$ A lies on x-axis

$\therefore$ Its $y = 0$

Substituting the value of y in $x + y = 3$,

$\Rightarrow x + 0 = 3 \quad \Rightarrow x = 3$

$\therefore$ Co-ordinates of A are (3, 0)

Again $\because$ B lies on y-axis

$\therefore$ its $x = 0$

Substituting the value of x is $x + y = 3$,

$0 + y = 3 \quad \Rightarrow y = 3$

$\therefore$ Co-ordinates of B are (0, 3)

(*iii*) Now, M is the mid-point of AB

$\therefore$ Co-ordinates of M will be

$$\left(\frac{0+3}{2}, \frac{3+0}{2}\right) \text{or} \left(\frac{3}{2}, \frac{3}{2}\right) \quad \textbf{Ans.}$$

EXERCISE 14 (B)

Q.1. Prove that the line through A (–2, 6) and B (4, 8) is perpendicular to the line through C (8, 12) and D (4, 24).

Sol. Slope (m_1) of the line joining the points A (–2, 6) and B (4, 8) will be

$$= \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 6}{4 - (-2)} = \frac{2}{4+2} = \frac{2}{6} = \frac{1}{3}$$

And slope (m_2) of the line joining the points C (8, 12) and D (4, 24)

$$= \frac{24 - 12}{4 - 8} = \frac{12}{-4} = -3$$

$$\because \quad m_1.m_2 = \frac{1}{3} \times (-3) = -1$$

$\therefore$ These lines are perpendicular to each other. Hence proved.

Q. 2. If A (2, – 3), B (– 5, 1), C (7, – 1) and D (0, k) be four points such that AB is parallel to CD, find the value of k.

Sol. Slope of AB (m_1)

$$= \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-(-3)}{-5-2} = \frac{1+3}{-7} = \frac{4}{-7}$$

and slope of CD $(m_2) = \frac{k-(-1)}{0-7} = \frac{k+1}{-7}$

$\because$ AB || CD

$\therefore$ Their slopes are equal.

$\Rightarrow \quad m_2 = m_1$

$\Rightarrow \quad \frac{k+1}{-7} = \frac{4}{-7}$

$\Rightarrow \quad -7(k+1) = 4(-7)$

$\Rightarrow \quad -7k - 7 = -28$

$\Rightarrow \quad -7k = -28 + 7$

$\Rightarrow \quad -7k = -21$

$\Rightarrow \quad k = \frac{-21}{-7} = 3$

Hence, $k = 3$.

Q. 3. If A (2, –5), B (–2, 5), C (k, 3) and D (1, 1) be four points such that AB and CD are perpendicular to each other, find the value of k.

Sol. Slope of the line joining A, B

$$= \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-(-5)}{-2-2}$$

$$= \frac{5+5}{-4} = \frac{10}{-4} = \frac{-5}{2}$$

And slope of the line joining C and D

$$= \frac{1-3}{1-k} = \frac{-2}{1-k}$$

$\because$ Line AB ⊥ CD

$\therefore$ Product of their slopes = –1

$\Rightarrow \frac{-5}{2} \times \frac{-2}{1-k} = -1 \Rightarrow \frac{10}{2(1-k)} = -1$

$\Rightarrow \quad \frac{5}{1-k} = -1 \quad \Rightarrow -1(1-k) = 5$

$\Rightarrow \quad -1 + k = 5 \quad \Rightarrow k = 5 + 1 = 6$ **Ans.**

Q. 4. Without using Pythagoros Theorem, prove that the points P (1, 3), Q (3, –1) and R (–5, –5) are the vertices of a right-angled triangle.

Sol. Slope of line PQ $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-3}{3-1}$

$$= \frac{-4}{2} = -2$$

Slope of QR $= \frac{-5-(-1)}{-5-3} = \frac{-5+1}{-8}$

$$= \frac{-4}{-8} = \frac{1}{2}$$

and slope of RP $= \frac{3-(-5)}{1-(-5)} = \frac{3+5}{1+5}$

$$= \frac{8}{6} = \frac{4}{3}$$

$\because$ Slope of PQ × slope of QR

$$= -2 \times \left(\frac{1}{2}\right) = -1$$

$\therefore$ PQ and QR are perpendicular to each other

Hence ΔPQR is a right triangle.

Q. 5. Without using distance formula, show that the points A (1, –2) B (3, 6), C (5, 10) and D (3, 2) are the vertices of a parallelogram.

Sol. Slope of AB $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-(-2)}{3-1}$

$$= \frac{6+2}{2} = \frac{8}{2} = 4$$

Similarly, slope of BC $= \frac{10-6}{5-3} = \frac{4}{2} = 2$

Slope of CD $= \frac{2-10}{3-5} = \frac{-8}{-2} = 4$

And slope of DA $= \frac{-2-2}{1-3} = \frac{-4}{-2} = 2$

$\because$ Slopes of AB and CD are equal

$\therefore$ AB || CD

Similarly, slopes of BC and DA are equal

$\therefore$ BC || DA

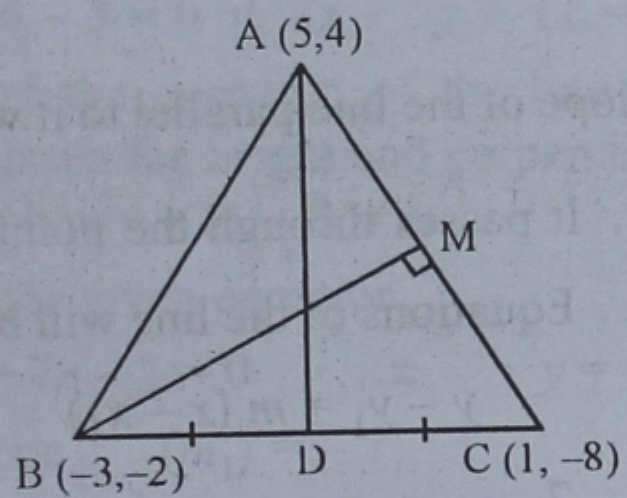
Hence, ABCD is a parallelogram.

Q. 6. A (5, 4), B (–3, –2) and C (1, –8) are the vertices of ΔABC. Find :

(*i*) the slope of median AD,

(*ii*) the slope of altitude BM.

Sol. (*i*) $\because$ D is the mid–point of BC



$\therefore$ Co-ordinates of D will be

$\left(\frac{1+(-3)}{2}, \frac{-2-8}{2}\right)$ or $\left(\frac{1-3}{2}, \frac{-10}{2}\right)$

or $\left(\frac{-2}{2}, \frac{-10}{2}\right)$ or (–1, –5)

Now, slope of AD $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5-4}{-1-5}$

$= \frac{-9}{-6} = \frac{3}{2}$

(*ii*) Slope of line AC $= \frac{-8-4}{1-5} = \frac{-12}{-4} = 3$

$\because$ BM ⊥ AC

$\therefore$ Slope of BM

$= -\frac{1}{\text{Slope of AC}} = -\frac{1}{3}$ **Ans.**

Q. 7. Find the equation of a straight line perpendicular to the line $2x + 5y + 7 = 0$ and with y-intercept – 3 units.

Sol. In the line $2x + 5y + 7 = 0$

$\Rightarrow \quad 5y = -2x - 7$

$\Rightarrow \quad y = \frac{-2}{5}x - \frac{7}{5}$

Here, slope $(m_1) = -\frac{2}{5}$

Let the slope of the line perpendicular to the given line $= m_2$

$\therefore \quad m_1 m_2 = -1$

$\Rightarrow \quad -\frac{2}{5}m_2 = -1$

$\therefore \quad m_2 = -1 \times \frac{-5}{2} = \frac{5}{2}$

$\because$ It makes y-intercept – 3 units

$\therefore$ The point where it passes = (0, – 3)

$\therefore$ Equations of the new line

$y - y_1 = m(x - x_1)$

$\Rightarrow \quad y - (-3) = \frac{5}{2}(x - 0)$

$\Rightarrow \quad y + 3 = \frac{5}{2}x$

$\Rightarrow \quad 2y + 6 = 5x$

$\Rightarrow \quad 5x - 2y - 6 = 0$ **Ans.**

Q. 8. Prove that :

(*i*) the lines $x + 2y - 5 = 0$ and $2x + 4y + 9 = 0$ are parallel ;

(*ii*) the lines $2x + 3y + 8 = 0$ and $27x - 18y + 10 = 0$ are perpendicular to each other.

Sol. (*i*) Slope of the line $x + 2y - 5 = 0$

$2y = -x + 5$

$\Rightarrow y = -\frac{1}{2}x + \frac{5}{2}$

Here, $m_1 = \frac{-1}{2}$

And slope of the line $2x + 4y + 9 = 0$

$\Rightarrow \quad 4y = -2x - 9$

$\Rightarrow \quad y = \frac{-2}{4}x - \frac{9}{4}$

$\Rightarrow \quad y = \frac{-1}{2}x - \frac{9}{4}$

Here, $m_2 = \frac{-1}{2}$

$\because \quad m_1 = m_2$

$\therefore$ The lines are parallel.

(*ii*) Slope of the line $2x + 3y + 8 = 0$

$\Rightarrow \quad 3y = -2x - 8 \Rightarrow \quad y = -\frac{2}{3}x - \frac{8}{3}$

Here, $\quad m_1 = \frac{2}{3}$

and slope of the line $27x - 18y + 10 = 0$

$\Rightarrow -18y = -27x - 10 \Rightarrow \quad 18y = 27x + 10$

$\Rightarrow \quad y = \frac{27}{18}x + \frac{10}{18} \Rightarrow \quad y = \frac{3}{2}x + \frac{5}{9}$

Here, $\quad m_2 = \frac{3}{2}$

$\because m_1 \times m_2 = \frac{-2}{3} \times \frac{3}{2} = -1$

$\therefore$ The lines are perpendicular to each other.

Q. 9. (*i*) Find the equation of the line parallel to the line $3x + 2y = 8$ and passing through the point (0, 1). **(2007)**

(*ii*) Find the equation of the line passing through (5, 1) and parallel to the line $7x - 2y + 8 = 0$.

Sol. (*i*) Given

$3x + 2y = 8 \quad$ ....(i)

$\Rightarrow 2y = -3x + 8$

$\Rightarrow y = -\frac{3}{2}x + 4 \quad (y = mx + c \text{ form})$

$\therefore$ The slope of the line $(l) = -\frac{3}{2}$

$\therefore$ The slope of a line parallel to line $(l) = -\frac{3}{2}$ is

$y - 1 = -\frac{3}{2}(x - 0) \; [(y - y_1 = m(x - x_1)]$

$\Rightarrow 2y - 2 = -3x \Rightarrow \quad 3x + 2y - 2 = 0$

Which is the required equation.

(*ii*) Slope of the line $7x - 2y + 8 = 0$ will be as $7x - 2y + 8 = 0$

$\Rightarrow \quad -2y = -7x - 8 \Rightarrow 2y = 7x + 8$

$\Rightarrow \quad y = \frac{7}{2}x + 4$

$\because$ Slope $(m) = \frac{7}{2}$

$\therefore$ Slope of the line parallel to it will be $= \frac{7}{2}$

$\because$ It passes through the point (5, 1)

$\therefore$ Equations of the line will be

$y - y_1 = m(x - x_1)$

$\Rightarrow \quad y - 1 = \frac{7}{2}(x - 5) \quad \Rightarrow 2y - 2 = 7x - 35$

$\Rightarrow 7x - 2y + 2 - 35 = 0 \Rightarrow \quad 7x - 2y - 33 = 0$

$\Rightarrow \quad -2y + 7x - 33 = 0 \Rightarrow 2y - 7x + 33 = 0$ **Ans.**

Q. 10. Find the equation of the line passing through the origin and parallel to the line $3x - 2y + 1 = 0$.

Sol. In the given equation $3x - 2y + 1 = 0$

$\Rightarrow -2y = -3x - 1 \Rightarrow 2y = 3x + 1 \Rightarrow \quad y = \frac{3}{2}x + \frac{1}{2}$

Slope $(m) = \frac{3}{2}$

Now, equation of the line which is parallel to the given line and passes through the origin (0, 0) will be $y - y_1 = m(x - x_1)$

$= y - 0 = \frac{3}{2}(x - 0)$

$\Rightarrow \quad y = \frac{3}{2}x \Rightarrow 2y = 3x$

$\Rightarrow \quad 3x - 2y = 0$ or $2y - 3x = 0$ **Ans.**

Q. 11. Find the equation of the line through the point P(– 2, 1) and parallel to the line joining the points A(4, –3) and B(– 1, 5).

Sol. Slope of the line joining A (4, – 3) and B(– 1, 5)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-(-3)}{-1-4} = \frac{5+3}{-5} = \frac{8}{-5}$$

∴ Slope of the line parallel to AB = $\frac{8}{-5}$

and equation of the line passing through P(– 2, 1) will be

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \frac{8}{-5}(x + 2) \Rightarrow -5y + 5 = 8x + 16$$

$8x + 5y + 16 - 5 = 0 \Rightarrow 8x + 5y + 11 = 0$ Ans.

Q.12. Find the equation of the line passing through the origin and perpendicular to the line $y + 7x - 3 = 0$.

Sol. In the given equation,

$y + 7x - 3 = 0 \Rightarrow y = -7x + 3$

Slope $(m_1) = -7$

∴ Slope (m_2) of the line which is perpendicular to it, will be $= -\frac{1}{-7} = \frac{1}{7}$

$(\because m_1 m_2 = -1)$

∵ The line passes through the origin (0, 0)

∴ Equation of the line will be

$$y - y_1 = m(x - x_1) \Rightarrow y - 0 = \frac{1}{7}(x - 0)$$

$$\Rightarrow y = \frac{1}{7}x \Rightarrow 7y = x$$

$\Rightarrow 7y - x = 0$ **Ans.**

Q. 13. Find the equation of the line parallel to $2x + 5y - 9 = 0$ and passing through the mid-point of the line segment joining A (2, 7) and B (– 4, 1).

Sol. The equation of the line is

$2x + 5y - 9 = 0$

$$\Rightarrow 5y = -2x + 9 \Rightarrow y = \frac{-2}{5}x + \frac{9}{5}$$

∴ Slope $(m) = \frac{-2}{5}$

Now, the slope of the line parallel to the given line will be $= \frac{-2}{5}$

Now, co-ordinates of the mid-point of the line joining A (2, 7) and B (– 4, 1)

$$= \left(\frac{2-4}{2}, \frac{7+1}{2}\right) \text{ or } \left(\frac{-2}{2}, \frac{8}{2}\right) \text{ or } (-1, 4)$$

∴ Equation of the line passing through the mid-point (– 1, 4) and parallel to the given line will be

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow y - 4 = \frac{-2}{5}(x + 1) \Rightarrow 5y - 20 = -2x - 2$$

$\Rightarrow 2x + 5y - 20 + 2 = 0$

$\Rightarrow 2x + 5y - 18 = 0$ **Ans.**

Q. 14. The line $4x - 3y + 12 = 0$ meets the x-axis at A.

(*i*) Write down the co-ordinates of A.

(*ii*) Determine the equation of the line passing through A and perpendicular to $4x - 3y + 12 = 0$. **(1997)**

Sol. Line $4x - 3y + 12 = 0$ meets x-axis at A.

(*i*) Let co-ordinates of A be (x, 0)

Substituting the value of (x, 0) in the equation

$4x - 3 \times 0 + 12 = 0 \Rightarrow 4x + 12 = 0$

$\Rightarrow 4x = -12 \Rightarrow x = -3$

∴ Co-ordinates of point A will be (– 3, 0)

(*ii*) Now in equation

$4x - 3y + 12 = 0 \Rightarrow -3y = -4x - 12$

$\Rightarrow y = \frac{4}{3}x + 4$ (Dividing by – 3)

Slope $(m_1) = \frac{4}{3}$

∴ Slope of the line perpendicular to the given line

$(m_2) = \frac{-4}{3}$ $(\because m_1 \times m_2 = -1)$

As it passes through A

∴ Equation of the line will be

$$y - y_1 = m_2(x - x_1)$$

$\Rightarrow \quad y - 0 = -\frac{3}{4}\{x - (-3)\}$

$\Rightarrow \quad y = \frac{-3}{4}\{x + 3\}$

$\Rightarrow \quad 4y = -3x - 9$

$\Rightarrow \quad 3x + 4y + 9 = 0$ **Ans.**

Q. 15. (*i*) Write down the equation of the line AB through (3, 2), perpendicular to the line $2y = 3x + 5$.

(*ii*) AB meets the *x*-axis at A and the *y*- axis at B. Write down the co-ordinates of A and B. Calculate the area of ΔOAB, where O is the origin. **(1995)**

Sol. Equation of the line given is

$2y = 3x + 5$

$\Rightarrow \quad y = \frac{3}{2}x + \frac{5}{2}$

$\therefore$ Its slope $(m_1) = \frac{3}{2}$

$\because$ AB is perpendicular to the given line.

$\therefore$ Slope of B $(m_2) = \frac{-2}{3}$

$(\because m_1 m_2 = -1)$

$\therefore$ It passes through the points (3, 2)

$\therefore$ Equation of AB will be

$y - y_1 = m(x - x_1)$

$\Rightarrow \quad y - 2 = \frac{-2}{3}(x - 3)$

$\Rightarrow \quad 3y - 6 = -2x + 6$

$\Rightarrow \quad 2x + 3y = 6 + 6$

$\Rightarrow \quad 2x + 3y = 12$

$\Rightarrow \quad 2x + 3y - 12 = 0$

(*ii*) $\because$ Line AB meets *x*-axis at A and *y*-axis at B.

$\therefore$ Ordinates of A = 0 *i.e.* $y = 0$

and abscissa of B = 0 *i.e.* $x = 0$

$\therefore$ If $y = 0$, then

$2x + 3 \times 0 = 12$

$\Rightarrow \quad 2x = 12$

$\Rightarrow \quad x = 6$

and if $x = 0$, then

$2 \times 0 + 3y = 12$

$\Rightarrow \quad 3y = 12$

$\Rightarrow \quad y = 4$

$\therefore$ Co-ordinates of A are (6, 0) and of B are (0, 4)

Now, area of $\Delta OAB = \frac{1}{2} OA \times OB$

$= \frac{1}{2} \times 6 \times 4$ sq. units

$= 12$ sq. units **Ans.**

Q. 16. Find the equation of the line that has *x*-intercept –3 and is perpendicular to the line $3x + 5y = 1$.

Sol. $\because$ *x*-intercept = –3,

$\therefore$ The required line will pass through this point (–3, 0)

In the equation $3x + 5y = 1$

$\Rightarrow \quad 5y = -3x + 1 \quad \Rightarrow y = \frac{-3}{5}x + \frac{1}{5}$

Slope $(m_1) = \frac{-3}{5}$

$\therefore$ Slope of the line which is perpendicular to it

$(m_2) = \frac{5}{3} \qquad (\because m_1 m_2 = -1)$

$\therefore$ Equation of the line perpendicular to the given line and passing through (–3, 0) will be

$y - y_1 = m(x - x_1)$

$\Rightarrow y - 0 = \frac{5}{3}(x + 3) \quad \Rightarrow 3y = 5x + 15$

$\Rightarrow 3y - 5x - 15 = 0$

$\Rightarrow 5x - 3y + 15 = 0$ **Ans.**

Q. 17. Find the equation of the line passing through (2, 4) and perpendicular to *x*-axis.

Sol. $\because$ The required line is perpendicular to x-axis.

$\therefore$ It will be parallel to y-axis

$\because$ It passes through the point (2, 4)

$\therefore$ The equation of this line will be $x = 2$.

or $x - 2 = 0$. **Ans.**

Q. 18. Find the value of m, if the lines represented by $2mx - 3y = 1$ and $y = 1 - 2x$ are perpendicular to each other. **(1994)**

Sol. Lines $2mx - 3y = 1$ and $y = 1 - 2x$ are perpendicular to each other.

Now in the line

$$2mx - 3y = 1 \Rightarrow -3y = -2mx + 1$$

$$\Rightarrow y = \frac{2}{3}mx - \frac{1}{3}$$

$\therefore$ Slope $(m_1) = \frac{2}{3}m$

and in the line, $y = 1 - 2x \Rightarrow y = -2x + 1$

Slope $(m_2) = -2$

$\because$ Lines are perpendiculars to each other is $m_1 \times m_2 = -1$

$$\Rightarrow \frac{2}{3}m \times (-2) = -1 \Rightarrow -\frac{4}{3}m = -1$$

$$\Rightarrow m = -1 \times \frac{3}{-4} = \frac{3}{4}$$

Hence, $m = \frac{3}{4}$ **Ans.**

Q. 19. (*i*) If $3y - 2x = 4$ and $4y - px = 2$ are perpendicular to each other, find the value of p.

(*ii*) If the lines $y = 3x + 7$ and $2y + px = 3$ are perpendicular to each other, find the value of p. **(2006)**

Sol. (*i*) Given $3y - 2x = 4$ ...(*i*)

$$\Rightarrow 3y = 2x + 4 \Rightarrow y = \frac{2}{3}x + \frac{4}{3}.$$

[$y = mx + c$ form]

$\therefore$ The slope of the line (*i*) $= \frac{2}{3}$.

Given $4y - px = 2$

$\Rightarrow 4y = px + 2$ ...(*ii*)

$$\Rightarrow y = \frac{p}{4}x + \frac{1}{2}. \quad [y = mx + c \text{ form}]$$

$\therefore$ The slope of the line (*ii*) $= \frac{p}{4}$.

Since, the given lines are perpendicular to each other, we get

$$\frac{2}{3}.\frac{p}{4} = -1 \qquad | m_1m_2 = -1$$

$\Rightarrow$ $p = -6$ **Ans.**

(*ii*) Gradient m_1 of the line $y = 3x + 7$ is 3

$$2y + px = 3 \Rightarrow y = \frac{-px}{2} + \frac{3}{2}$$

Gradient m^2 of this line is $-\frac{p}{2}$.

Since, the given lines are perpendicular to each other.

$\therefore m_1 \times m_2 = -1$

$$\Rightarrow 3 \times \left(-\frac{p}{2}\right) = -1 \quad p = \frac{2}{3} \text{ Ans.}$$

Q. 20. Lines $2x - by + 5 = 0$ and $ax + 3y = 2$ are parallel. Find the relation connecting a and b.

Sol. In equation $2x - by + 5 = 0$

$$\Rightarrow -by = -2x - 5 \Rightarrow y = \frac{2}{b} + \frac{5}{b}$$

Slope $(m) = \frac{2}{b}$

and in equation $ax + 3y = 2$

$$\Rightarrow 3y = -ax + 2 \Rightarrow y = -\frac{a}{3}x + \frac{2}{3}$$

$\therefore$ Slope $(m_2) = -\frac{a}{3}$

$\because$ Lines are parallel

$$\therefore m_1 = m_2 \Rightarrow \frac{2}{b} = -\frac{a}{3}$$

$\Rightarrow -ab = 6 \Rightarrow ab = -6$ **Ans.**

Q. 21. Given that the line $\frac{y}{2} = x - p$ and the line $ax + 5 = 3y$ are parallel, find the value of a.

Sol. In equation $\frac{y}{2} = x - p \Rightarrow y = 2x - 2p$

Slope $(m_1) = 2$

In equation $ax + 5 = 3y \Rightarrow y = \frac{a}{3}x + \frac{5}{3}$

$\because$ Slope $(m_2) = \frac{a}{3}$

$\Rightarrow$ Lines are paralle

$m_1 = m_2 \Rightarrow \frac{a}{3} = 2 \Rightarrow a = 6$ **Ans.**

Q.22. (*i*) Find the values of k for which the lines $kx + 2y + 3 = 0$ and $8x + ky - 1 = 0$ are parallel.

(*ii*) Find the value of p for which the lines $2x + 3y - 7 = 0$ and $4y - px - 12 = 0$ are perpendicular to each other. (2009)

Sol. (*i*) In line $kx + 2y + 3 = 0$

$\Rightarrow 2y = -kx - 3 \Rightarrow y = \frac{-k}{2}x - \frac{3}{2}$

Slope $(m_1) = -\frac{k}{2}$

And in line $8x + ky - 1 = 0$

$\Rightarrow ky = -8x + 1 \Rightarrow y = \frac{-8}{k}x + \frac{1}{k}$

Its slope $(m_2) = \frac{-8}{k}$

$\because$ The two lines are parallel to each other

$\therefore m_1 = m_2 \Rightarrow \frac{-k}{2} = \frac{-8}{k}$

$\Rightarrow -k^2 = -16 \Rightarrow k^2 = 16 \Rightarrow k = \pm\sqrt{16} = \pm 4$

(*ii*) **Given :** $2x + 3y - 7 = 0$ and $4y - px - 12 = 0$

Slope of first line $= -\frac{2}{3}$

and Slope of second line $= \frac{p}{4}$

Lines are $\perp$,

$\therefore$ Product of slopes $= -1$

$\therefore -\frac{2}{3} \times \frac{p}{4} = -1$

$-\frac{2p}{12} = -1 \Rightarrow -2p = -12 \Rightarrow p = 6,$

Q.23. Find the equation of the line passing through the point of intersection of the lines $5x - 8y + 23 = 0$ and $7x + 6y - 71 = 0$ and perpendicular to the line $4x - 2y = 3$.

Sol. $5x - 8y + 23 = 0$ ...(*i*)

$7x + 6y - 71 = 0$ ...(*ii*)

Multiplying (*i*) by 3 and (*ii*) by 4, we get

$15x - 24x + 69 = 0$

$28x + 24y - 284 = 0$

Adding, we get

$43x - 215 = 0 \Rightarrow 43x = 215 \Rightarrow x = \frac{215}{43} = 5$

Substituting the value of x in (*i*),

$5 \times 5 - 8y + 23 = 0$

$\Rightarrow 25 - 8y + 23 = 0$

$-8y + 48 = 0$

$\Rightarrow 8y = 48 \Rightarrow y = \frac{48}{8} = 6$

$\therefore$ Point of intersection of the given two lines is (5, 6)

In the equation of the line $4x - 2y = 3$

$\Rightarrow -2y = -4x + 3 \Rightarrow 2y = 4x - 3$

$\Rightarrow y = \frac{4}{2}x - \frac{3}{2} \Rightarrow y = 2x - \frac{3}{2}$

Slope $(m_1) = 2$

$\therefore$ Slope of the line perpendicular to it

$(m_2) = -\frac{1}{2}$ $(\because m_1 m_2 = -1)$

Now equation of the line perpendicular to the given line and passing through the point of intersection *i.e.* (5, 6), will be

$y - y_1 = m(x - x_1)$

$\Rightarrow y - 6 = -\frac{1}{2}(x - 5) \Rightarrow 2y - 12 = -x + 5$

$x + 2y = 5 + 12 \Rightarrow x + 2y = 17$ **Ans.**

Q. 24. Find the equation of the perpendicular dropped from the point (–1, 2) onto the line joining (1, 4) and (2, 3).

Sol. Slope of the line joining the points A(1, 4) and B (2, 3)

$= \frac{3-4}{2-1}$ | $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= -1$

∴ The slope of a line perpendicular to AB = 1. The equation of the line through (– 1, 2) and having slope 1 is

$y - 2 = 1\,(x - (-1))$

$\Rightarrow \quad y - 2 = x + 1$

$\Rightarrow \quad x - y + 3 = 0,$

which is the required equation. **Ans.**

25. Points A and B have coordinates (7, –3) and (1, 9) respectively. Find

(*i*) the slope of AB.

(*ii*) the equation of the perpendicular bisector of the line segment AB.

(*iii*) the value of 'p' if (–2, p) lies on it.

Sol. Coordinates of A are (7, –3), of B = (1, 9)

(*i*) ∴ slope $(m) = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{9-(-3)}{1-7}$

$= \dfrac{9+3}{1-7} = \dfrac{12}{-6} = -2$

(*ii*) Let PQ is the perpendicular bisector of AB intersecting it at M.

∴ Co-ordinates of M will be

$= \dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2} = \dfrac{7+1}{2}, \dfrac{-3+9}{2} = \dfrac{8}{2}, \dfrac{6}{2}$

or (4, 3)

∴ Slope of PQ $= \dfrac{1}{2}\,(m_1, m_2 = -1)$

∴ Equation of PQ $= y - y_1 = m\,(x - x_1)$

$\Rightarrow y - 3 = \dfrac{1}{2}\,(x-4) \quad \Rightarrow 2y - 6 = x - 4$

$\Rightarrow x - 2y + 6 - 4 = 0 \quad \Rightarrow x - 2y + 2 = 0$

(iii) ∵ Point $(-2, p)$ lies on it

∴ $-2 - 2p + 2 = 0 \quad \Rightarrow -2p + 0 = 0$

$\Rightarrow -2p = 0$

∴ $p = 0$

Q. 26. Is the line through (– 2, 3) and (4, 1) perpendicular to the line 3 *x* = *y* + 1 ? Does the line 3 *x* = *y* + 1 bisect the join of (– 2, 3) and (4, 1) ? **(1993)**

Sol. Slope of the line passing through the points

$(-2, 3)$ and $(4, 1) = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{1-3}{4+2} = \dfrac{-2}{6} = \dfrac{-1}{3}$

Slope of line 3 *x* = *y* + 1

$\Rightarrow \quad y = 3x - 1 = 3$

$\because m_1 \times m_2 = -\dfrac{1}{3} \times 3 = -1$

∴ These lines are perpendicular to each other.

(*ii*) Co-ordinates of mid-point of the line joining the points (– 2, 3) and (4, 1) will be

$\left[\dfrac{-2+4}{2}, \dfrac{3+1}{2}\right]$ or $\left[\dfrac{2}{2}, \dfrac{4}{2}\right]$ or (1, 2)

If mid-point (1, 2) lies on the line 3 *x* = *y* + 1 then it will satisfy it.

Now, substituting the value of *x* and *y* is

$3x = y + 1$

$\Rightarrow \quad 3\,(1) = 2 + 1$

$\Rightarrow \quad 3 = 3$ which is true.

Hence, the line 3 *x* = *y* + 1 bisects the line joining the points (– 2, 3), (4, 1) **Ans.**

Q. 27. A line through origin meets the line 2*x* = 3*y* + 13 at right angles at point Q. Find the co-ordinates of Q.

Sol. In the given line 2*x* = 3*y* + 13

$\Rightarrow 3y = 2x - 13 \quad \Rightarrow y = \dfrac{2}{3}x - \dfrac{13}{3}$

Slope $(m_1) = \dfrac{2}{3}$

Slope (m_2) of the line which is perpendicular to it

$= -\dfrac{3}{2} \qquad (\because m_1 m_2 = -1)$

∵ It passes through the origin O (0, 0)

∴ Equation of that line will be

$y - y_1 = m\,(x - x_1) \Rightarrow y - 0 = \dfrac{-3}{2}(x - 0)$

$\Rightarrow \quad 2y = -3x \qquad \Rightarrow 3x + 2y = 0$

Now, solving the equations

$$2x = 3y + 13$$

$\Rightarrow \quad 2x - 3y = 13 \qquad ...(i)$

And $3x + 2y = 0 \qquad ...(ii)$

From (*ii*) $3x = -2y \quad \Rightarrow x = \frac{-2}{3}y$

Substituting the value of x in (*i*),

$$2\left(\frac{-2}{3}y\right) - 3y = 13 \Rightarrow \frac{-4}{3}y - 3y = 13$$

$$\Rightarrow \quad \frac{-13}{3}y = 13 \Rightarrow y = \frac{13 \times 3}{-13} = -3$$

Substituting the value of y in (*ii*),

$3x + 2(-3) = 0 \quad \Rightarrow 3x - 6 = 0$

$\Rightarrow \quad 3x = 6$

$\therefore \quad x = \frac{6}{3} = 2$

Hence, co-ordinates of Q are (2, –3) **Ans.**

Q. 28. Find the equation of the perpendicular from the point P (1, – 2) on the line $4x - 3y - 5 = 0$. Also, find the co-ordinates of the foot of the perpendicular.

Sol. In the equation $4x - 3y - 5 = 0$,

$\Rightarrow 3y = 4x - 5$

$\Rightarrow \quad y = \frac{4}{3}x - \frac{5}{3}$

Slope $(m_1) = \frac{4}{3}$

Let the slope of the perpendicular $= m_2$

$$m_1 \times m_2 = -1$$

$\Rightarrow \quad \frac{4}{3} \times m_2 = -1$

$\therefore \quad m_2 = -\frac{3}{4}$

$\therefore$ Equation of the perpendicular where slope is $-\frac{3}{4}$ and drawn through the point (1, – 2),

$$y - y_1 = m(x - x_1)$$

$\Rightarrow \quad y - 2 = \frac{-3}{4}(x - 1)$

$\Rightarrow \quad 4y + 8 = -3x + 3$

$\Rightarrow \quad 3x + 4y + 8 - 3 = 0$

$\Rightarrow \quad 3x + 4y + 5 = 0$

For finding the co-ordinates of the foot of the perpendicular.

We have to solve the equations

$4x - 3y - 5 = 0 \qquad ...(i)$

and $3x + 4y + 5 = 0 \qquad ...(ii)$

Multiplying (*i*) by 4 and (*ii*) by 3, we get :

$$16x - 12y - 20 = 0$$

$$9x + 12y + 15 = 0$$

Adding, we get :

$$25x - 5 = 0 \Rightarrow 25x = 5$$

$\therefore \quad x = \frac{5}{25} = \frac{1}{5}$

Substituting the value of x in (*i*),

$$4 \times \left[\frac{1}{5}\right] - 3y - 5 = 0$$

$\Rightarrow \quad \frac{4}{5} - 3y - 5 = 0$

$\Rightarrow \quad 3y = \frac{4}{5} - 5 = \frac{4 - 25}{5} = \frac{-21}{5}$

$\Rightarrow \quad y = \frac{-21}{5 \times 3} = \frac{-7}{5}$

$\therefore$ Co-ordinates are $\left[\frac{1}{5}, \frac{-7}{5}\right]$ **Ans.**

Q. 29. Find the equation of the line which is perpendicular to the line $\frac{x}{a} - \frac{y}{b} = 1$ at the point where the given line meets y-axis.

Sol. Let the lines $\frac{x}{a} - \frac{y}{b} = 1$ meets y-axis at P

$\therefore$ its $x = 0$

Now, $\frac{0}{a} - \frac{y}{b} = 1 \quad \Rightarrow \frac{-y}{b} = 1$

$\Rightarrow \quad -y = b \quad \Rightarrow \quad y = -b.$

$\therefore$ Co-ordinates of point P are $(0, -b)$

Now in the line $\frac{x}{a} - \frac{y}{b} = 1$

$\Rightarrow bx - ay = ab$

$\Rightarrow \quad ay = bx - ab$

$\Rightarrow \quad y = \frac{b}{a}x - b$

Slope $(m_1) = \frac{b}{a}$

Now, slope (m_2) of the line which is perpendicular to it $= -\frac{a}{b}$

$(\because m_1 m_2 = -1)$

∴ Equation of the line whose slope is $-\frac{a}{b}$ and passes through P $(0, -b)$, will be

$y - y_1 = m(x - x_1)$

$\Rightarrow \quad y + b = \frac{-a}{b}(x - 0)$

$\Rightarrow \quad y + b = \frac{-a}{b}x$

$\Rightarrow \quad by + b^2 = -ax$

$\Rightarrow ax + by + b^2 = 0$ **Ans.**

Q. 30. The points B (7, 3) and D (0, – 4) are two opposite vertices of a rhombus ABCD. Find the equation of the diagonal AC.

Sol. Slope of the diagonal BD (m_1)

$$= \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-(-4)}{7-0} = \frac{3+4}{7} = \frac{7}{7} = 1$$

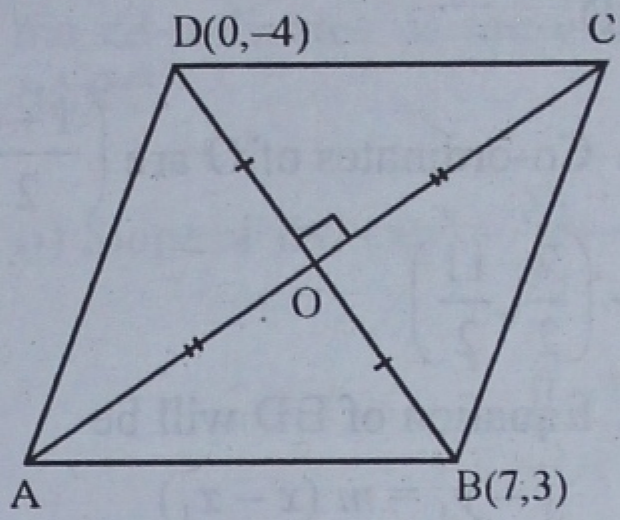
∵ Diagonals of a rhombus bisect each other at right angles

∴ AC is the perpendicular of BD

∴ Slope of AC $(m_2) = -1$

$(\because m_1 \times m_2 = -1)$

∵ O is the mid-point of AC as well as BD



∴ Coordinates of O will be

$$\left(\frac{7+0}{2}, \frac{3-4}{2}\right) \text{ or } \left(\frac{7}{2}, \frac{-1}{2}\right)$$

∴ Equation of the line AC

$y - y_1 = m(x - x_1)$

$\Rightarrow y + \frac{1}{2} = -1\left(x - \frac{7}{2}\right)$

$\Rightarrow \frac{2y+1}{2} = -\left(\frac{2x-7}{2}\right)$

$\Rightarrow 2y + 1 = -(2x - 7)$

$\Rightarrow 2y + 1 = -2x + 7 \Rightarrow 2y + 2x = 7 - 1$

$\Rightarrow 2(y + x) = 6 \quad \Rightarrow \quad x + y = 3$ **Ans.**

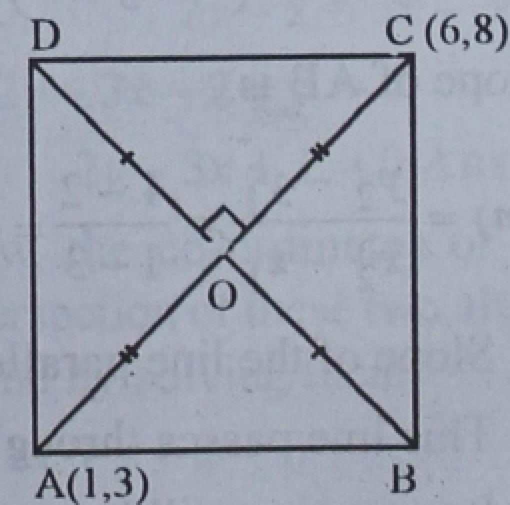
Q. 31. The points A (1, 3) and C (6, 8) are two opposite vertices of a square ABCD. Find the equation of the diagonal BD.

Sol. Co-ordinates of A and C of a square ABCD are (1, 3), (6, 8)

∴ Slope of the diagonal AC (m_1)

$$= \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-3}{6-1} = \frac{5}{5} = 1$$

∵ Diagonals of a square are perpendicular bisectors of each other



∴ Slope of BD $(m_2) = -1$

$(\because m_1 m_2 = -1)$

$\because$ O is the mid-point of AC as well as of BD

$\therefore$ Co-ordinates of O are $\left(\frac{1+6}{2}, \frac{3+8}{2}\right)$

or $\left(\frac{7}{2}, \frac{11}{2}\right)$

$\therefore$ Equation of BD will be

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow \quad y - \frac{11}{2} = -1\left(x - \frac{7}{2}\right)$$

$$\Rightarrow \frac{2y-11}{2} = -\frac{2x-7}{2}$$

$$\Rightarrow \quad 2y - 11 = -2x + 7$$

$$\Rightarrow 2x + 2y - 11 - 7 = 0$$

$$\Rightarrow \quad 2x + 2y - 18 = 0$$

$$\Rightarrow \quad x + y - 9 = 0 \text{ Ans.}$$

Q. 32. A (1, 4), B (3, 2) and C (7, 5) are the vertcies of a Δ ABC. Find :

(*i*) the co-ordinates of the centroid G of Δ ABC. **(2002)**

(*ii*) the equation of a line through G, and parallel to AB.

Sol. (*i*) Vertices of a ΔABC are A (1, 4), B (3, 2) and C (7, 5) and G is its centroid of the ΔABC

$\therefore$ Co-ordinates of G will be

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{2}\right) \text{ or}$$

$$\left[\left(\frac{1+3+7}{3}\right), \left(\frac{4+2+5}{3}\right)\right] \text{ or } \left(\frac{11}{3}, \frac{11}{3}\right)$$

(*ii*) Slope of AB is

$$(m) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-2}{1-3} = \frac{2}{-2} = -1$$

$\therefore$ Slope of the line parallel to AB = –1

$\because$ This line passes through G

$\therefore$ Its equation will be

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow \quad y - \frac{11}{3} = -1\left(x - \frac{11}{3}\right)$$

$$\Rightarrow \quad y - \frac{11}{3} = -x + \frac{11}{3}$$

$$\Rightarrow x + y - \frac{11}{3} - \frac{11}{3} = 0$$

$$\Rightarrow \quad x + y - \frac{22}{3} = 0$$

$$\Rightarrow \quad 3x + 3y - 22 = 0$$

$$\Rightarrow \quad 3x + 3y = 22 \text{ Ans.}$$

Q. 33. A (– 4, 2), B (6, 4) and C (2, –2) are the vertices of Δ ABC. Find :

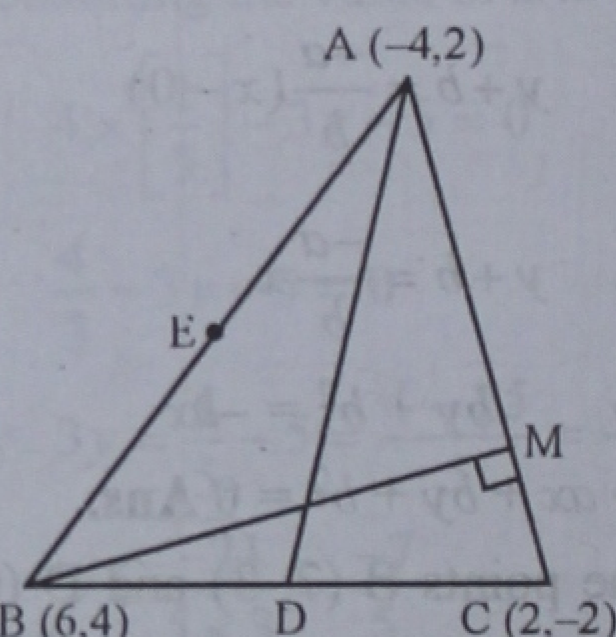
(*i*) the equation of median AD ;

(*ii*) the equation of altitude BM ;

(*iii*) the equation of right bisector of AB ;

(*iv*) the co-ordinates of centroid of ΔABC.

Sol. (*i*) $\because$ D is the mid-point of BC



$\therefore$ Co-ordinates of D will be

$$\left(\frac{6+2}{2}, \frac{4-2}{2}\right) \text{ or } \left(\frac{8}{2}, \frac{2}{2}\right) \text{ or } (4, 1)$$

$\therefore$ Equation of AB

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1} \Rightarrow \frac{y-2}{x+4} = \frac{1-2}{4+4}$$

$$\Rightarrow \frac{y-2}{x+4} = \frac{-1}{8}$$

$$\Rightarrow \quad 8y - 16 = -x - 4$$

$$\Rightarrow x + 8y - 16 + 4 = 0$$

$$\Rightarrow \quad x + 8y - 12 = 0$$

(*ii*) Slope of AC $(m_1) = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-2-2}{2+4} = \frac{-4}{6} = \frac{-2}{3}$

$\therefore$ Slope of the its altitude $(m_2) = \frac{3}{2}$

$(\because m_1 m_2 = -1)$

and equation of BM will be

$y - y_1 = m(x - x_1)$

$\Rightarrow \quad y - 4 = \frac{3}{2}(x-6)$

$\Rightarrow \quad 2y - 8 = 3x - 18$

$\Rightarrow 2y - 3x - 8 + 18 = 0$

$\Rightarrow \quad 2y - 3x + 10 = 0$

$\Rightarrow \quad 3x - 2y - 10 = 0$

(*iii*) Slope of AB $= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{2-4}{-4-6} = \frac{-2}{-10} = \frac{1}{5}$

$\therefore$ Slope of right bisector of AB = – 5

$(\because m_1 m_2 = -1)$

Co-ordinates of the mid-point E of AB

$= \left(\frac{-4+6}{2}, \frac{2+4}{2}\right)$ or $\left(\frac{2}{2}, \frac{6}{2}\right)$ or (1, 3)

And equation of right bisector drawn from E, the mid-point of AB will be

$y - y_1 = m(x - x_1)$

$\Rightarrow y - 3 = -5(x-1) \Rightarrow y - 3 = -5x + 5$

$\Rightarrow 5x + y - 3 - 5 = 0 \Rightarrow 5x + y - 8 = 0$

(*iv*) Co-ordinates of centroid of ΔABC will be

$\left(\frac{-4+6+2}{3}, \frac{2+4-2}{3}\right)$ or $\left(\frac{4}{3}, \frac{4}{3}\right)$

Ans.

Q. 34. A (2, –2), B (1, 1) and C (–1, 0) are the vertices of ΔABC. Find :

(*i*) the equation of altitude through A ;

(*ii*) the equation of altitude through B ;

(*iii*) the co-ordinates of the orthocentre of ΔABC.

Sol. (*i*) Slope of BC $(m_1) = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{0-1}{-1-1} = \frac{-1}{-2} = \frac{1}{2}$

$\therefore$ Slope of perpendicular to BC $(m_2) = -2$

$(\because m_1 m_2 = -1)$

$\therefore$ Equation of the perpendicular passing through (2, – 2)

$y - y_1 = m(x - x_1)$

$\Rightarrow y + 2 = -2(x-2)$

$\Rightarrow y + 2 = -2x + 4$

$\Rightarrow 2x + y + 2 - 4 = 0$

$\Rightarrow \quad 2x + y - 2 = 0$

(*ii*) Slope of the line CA $(m_1) = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{0+2}{-1-2} = \frac{2}{-3} = -\frac{2}{3}$

$\therefore$ Slope of the line perpendicular to CA

$(m_2) = \frac{3}{2} \qquad (\because m_1 m_2 = -1)$

$\therefore$ Equation of the line perpendicular to CA passing through point B (1, 1) will be

$\Rightarrow \quad y - y_1 = m(x - x_1)$

$\Rightarrow \quad y - 1 = \frac{3}{2}(x-1)$

$\Rightarrow \quad 2y - 2 = 3x - 3$

$\Rightarrow 2y - 3x - 2 + 3 = 0$

$\Rightarrow \quad 2y - 3x + 1 = 0$ **Ans.**

(*iii*) Now, the co-ordinates of the point of intersection of these two altitudes can be found by solving them

$2x + y - 2 = 0 \quad ...(i)$

$-3x + 2y + 1 = 0 \quad ...(ii)$

From (*i*), $y = 2 - 2x \quad ...(iii)$

Substituting the value of y in (*ii*),

$-3x+2(2-2x)+1=0$

$\Rightarrow -3x+4-4x+1=0$

$\Rightarrow -7x+5=0$

$\Rightarrow 7x=5 \quad \Rightarrow x=\frac{5}{7}$

from (*iii*), $y=2-2\times\frac{5}{7}$

$=2-\frac{10}{7}=\frac{14-10}{7}=\frac{4}{7}$

$\therefore$ Co-ordinates of the orthocentre of ΔABC are $\left(\frac{5}{7},\frac{4}{7}\right)$ **Ans.**

Q. 35. The points A (1, 2), B (3, – 4) and C (5, – 6) are the vertices of a ΔABC. Find the equations of the right bisectors of the sides BC and CA. Hence, find the circumcentre of ΔABC.

Sol. Co-ordinates of A (1, 2), B (3, – 4) and C (5, – 6) slope of BC $(m_1)=\frac{y_2-y_1}{x_2-x_1}$

$=\frac{-6+4}{5-3}=\frac{-2}{2}=-1$

Co-ordinates of mid-points of BC circle be $\left[\frac{3+5}{2},\frac{-4-6}{2}\right]$ or $\left[\frac{8}{2},\frac{-10}{2}\right]$ or (4, – 5)

Now, slope of perpendicular bisector of BC

$m_2=1 \qquad (\because m_1m_2=-1)$

$\therefore$ Equation of the perpendicular bisector of BC

$y-y_1=m(x-x_1)$

$\Rightarrow y+5=1(x-4)$

$\Rightarrow y+5=x-4$

$\Rightarrow x-y=5+4$

$x-y=9$

$\Rightarrow x-y-9=0 \qquad ...(i)$

Again, slope of AC $=\frac{y_2-y_1}{x_2-x_1}$

$=\frac{-6-2}{5-1}=\frac{-8}{4}=-2$

and co-ordinates of mid-point (E) of AC will be

$\left[\frac{1+5}{2},\frac{2-6}{2}\right]$ or $\left[\frac{6}{2},\frac{-4}{2}\right]$ or (3, – 2)

$\therefore$ Slope of the perpendicular bisector of

AC $=\frac{1}{2} \qquad (\because m_1m_2=-1)$

$\therefore$ Equation of the perpendicular bisector will be $y-y_1=m(x-x_1)$

$\Rightarrow y+2=\frac{1}{2}(x-3)$

$\Rightarrow 2y+4=x-3$

$\Rightarrow x-2y=4+3=7$

$\Rightarrow x-2y=7$

$\Rightarrow x-2y-7=0 \qquad ...(ii)$

Solving the system of equations (*i*) and (*ii*) we get the co-ordinates of the circumcentre of the circle.

Subtracting (*ii*) from (*i*)

$y=2 \Rightarrow y=2$

Substituting the value of y in (*i*),

$x-(2)=9 \quad \Rightarrow x-2=9$

$\Rightarrow x=9+2=11$

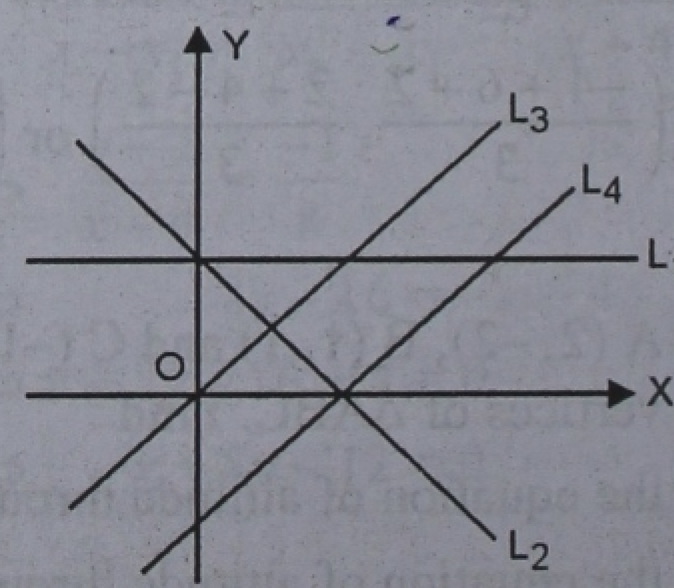
$\therefore$ Co-ordinates will be (11, 2) **Ans.**

Q. 36. Match the equations A, B, C, D with the lines L_1, L_2, L_3 and L_4 whose graphs are roughly drawn in the adjoining diagram.

$A=y=2x$, $B=y-2x+2=0$,

$C=3x+2y=6$, $D=y=2$.

(1996)

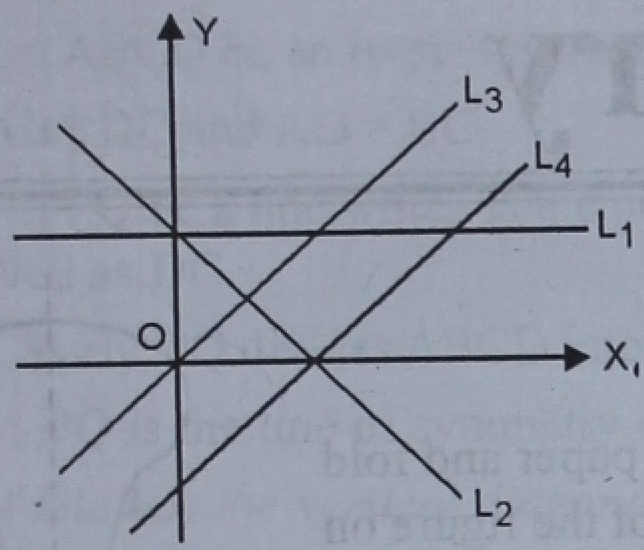


Sol. $A \equiv y = 2x$

$B \equiv y - 2x + 2 = 0$

$C \equiv 3x + 2y = 6$

$D \equiv y = 2$



(*i*) $\because L_3$ passes through O and *y*-intercept = 0

$\therefore A \equiv y = 2x \equiv L_3$

(*ii*) In $B \equiv y - 2x + 2 = 0$,

$\Rightarrow y = 2x - 2$

Slope = 2

and *y*-intercept = – 2 (negative)

$\therefore B \equiv y - 2x + 2 = 0 \equiv L_4$

(*iii*) $C \equiv 3x + 2y = 6$

$\Rightarrow 2y = -3x + 6$

$\Rightarrow y = \frac{-3}{2}x + 3$

Slope $= -\frac{3}{2}$ and *y* intercept = 3 (positive 3)

$\therefore C \equiv 3x + 2y = 6 \equiv L_2$

(*iv*) $D \equiv y = 2$

$\because y = 2$ is parallel to *x*-axis at $y = 2$

$\therefore D \equiv y = 2 \equiv L_1$ **Ans.**