Chapter 13

Distance and Section Formulae

POINTS TO REMEMBER

1. Distance Formula

Theorem 1. Show that the distance between the points P (x_1, y_1) and Q (x_2, y_2) is given by the formula :

$$PQ=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Proof. Let X′OX and YOY′ be the co-ordinate axes. Let P (x_1, y_1) and Q (x_2, y_2) be the given points in the plane.

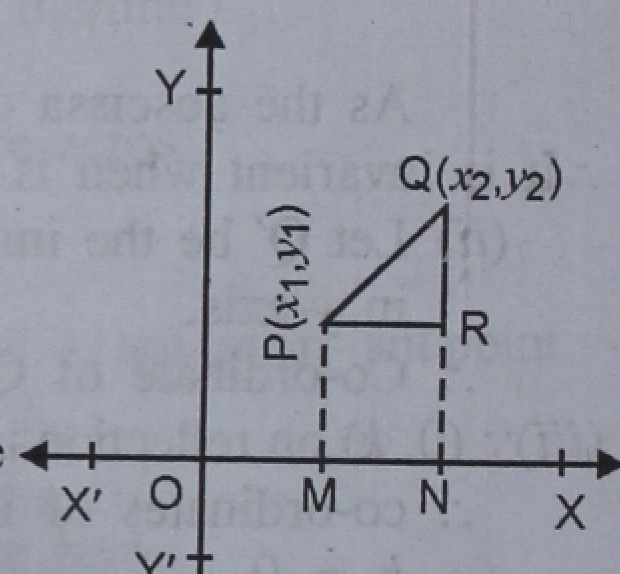
Draw PM and QN perpendicualrs on x-axis.

Also, draw PR ⊥ NQ. Then,

$OM = x_1$, $ON = x_2$, $PM = y_1$ and $QN = y_2$.

$\therefore$ $PR = MN = ON - OM = (x_2 - x_1)$.

$QR = (QN - RN) = (y_2 - y_1)$. $\quad [\because RN = PM = y_1]$



Now, from right-angled ΔPQR, by Pythagoras Theorem, we have

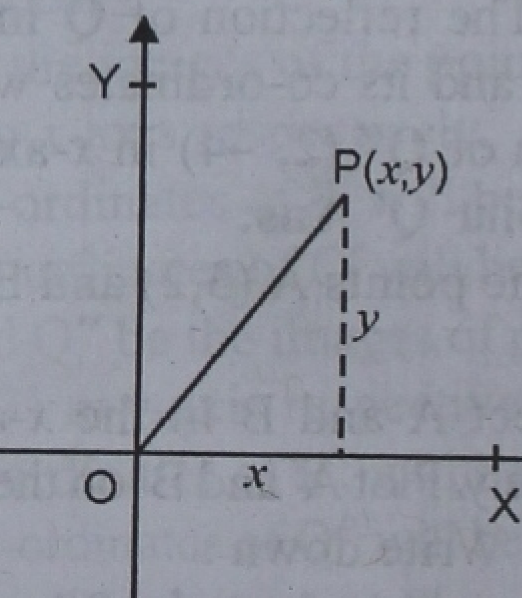
$PQ^2 = PR^2 + QR^2$

$= (x_2 - x_1)^2 + (y_2 - y_1)^2$.

$\therefore$ $PQ=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$.

Corollary : The distance of a point P (x, y) from the origin O (0, 0) is given by :

$$OP=\sqrt{(x-0)^2+(y-0)^2}=\sqrt{x^2+y^2}.$$

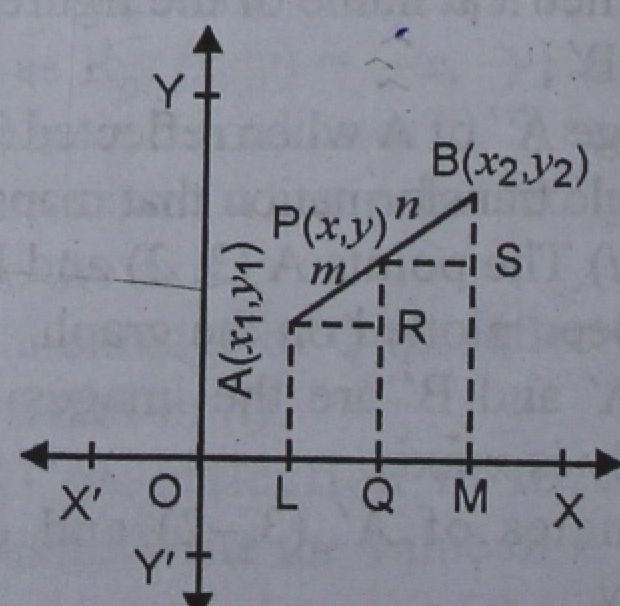


2. Section Formula

Theorem 2 : Prove that the co-ordinates of the points P (x, y) which divides the line joining A (x_1, y_1) and B (x_2, y_2) internally in the ratio $m : n$ are given by

$$x=\frac{mx_2+nx_1}{m+n},\ y=\frac{my_2+ny_1}{m+n},$$

Proof : Let A (x_1, y_1) and B (x_2, y_2) be the given points and let P (x, y) be the point which divides AB in the ratio $m : n$.



Then, $\frac{AP}{PB}=\frac{m}{n}$.

Draw AL, BM and PQ perpendicualrs on x-axis.

Also, draw AR ⊥ PQ and PS ⊥ BM. Then,

AR = LQ = OQ – OL = $(x - x_1)$.

PS = QM = OM – OQ = $(x_2 - x)$.

PR = PQ – RQ = PQ – AL = $(y - y_1)$.

BS = BM – SM = BM – PQ = $(y_2 - y)$.

Clearly, ΔARP and ΔPSB are similar and therefore, their sides are proportional.

$$\therefore \quad \frac{AP}{PB}=\frac{AR}{PS}=\frac{PR}{BS}.$$

Now, $\frac{AP}{PB}=\frac{AR}{PS} \Rightarrow \frac{m}{n}=\frac{x-x_1}{x_2-x} \Rightarrow m(x_2 - x) = n(x - x_1) \Rightarrow x=\left(\frac{mx_2+nx_1}{m+n}\right)$.

Again, $\frac{AP}{PB}=\frac{PR}{BS} \Rightarrow \frac{m}{n}=\frac{y-y_1}{y_2-y} \Rightarrow m(y_2 - y) = n(y - y_1)$

$$\Rightarrow \quad y=\left(\frac{my_2+ny_1}{m+n}\right).$$

Hence, the co-ordinates of P are $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$.

Corollary : Show that the co-ordinates of the mid-point M of a line segment with end points A (x_1, y_1) and B (x_2, y_2) are : $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

Proof. Let M be the mid-point of the line segment joining the points A (x_1, y_1) and B (x_2, y_2). Then, M divides AB in the ratio 1 : 1.

∴ Co-ordinates of M are $\left(\frac{1\cdot x_2+1\cdot x_1}{1+1}, \frac{1\cdot y_2+1\cdot y_1}{1+1}\right)$ *i.e.*, $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

Hence, the co-ordinates of the mid-point of AB are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

3. **Centroid of a Triangle :** The point of intersection of the medians of a triangle is called its centroid.

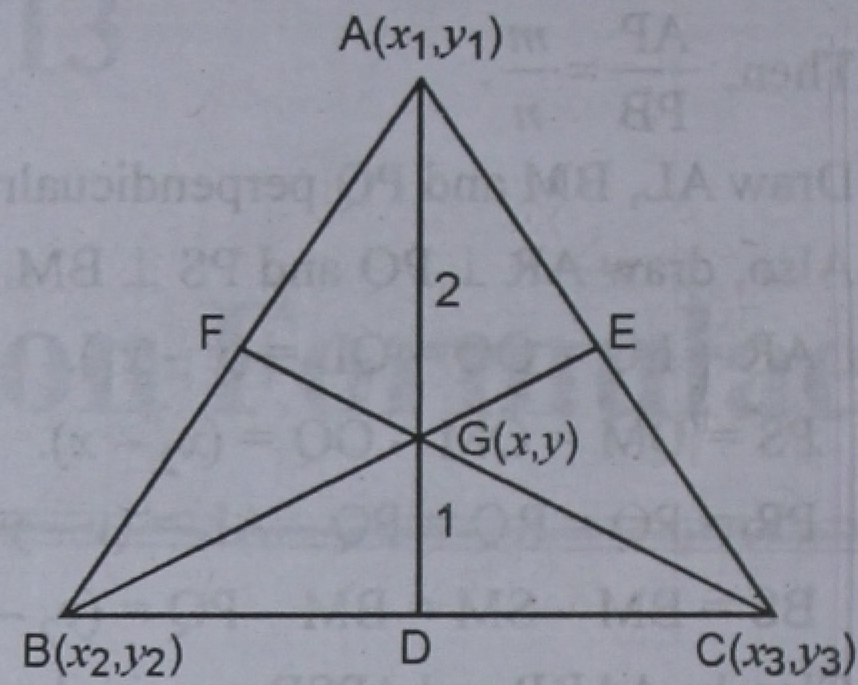
To Find the Co-ordinates of the Centroid of a Triangle.

Let A(x_1, y_1), B (x_2, y_2) and C (x_3, y_3) be the vertices of a given ΔABC. Let D be the mid-point of BC.

Then, the co-ordinates of D are $\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$

Let G (x, y) be the centroid of ΔABC.

Then, G divides AD in the ratio 2 : 1.



$$\therefore\ x = \frac{2\cdot\frac{(x_2+x_3)}{2}+1\cdot x_1}{2+1} = \left(\frac{x_1+x_2+x_3}{3}\right).$$

$$\text{and } y = \frac{2\cdot\frac{(y_2+y_3)}{2}+1\cdot y_1}{2+1} = \left(\frac{y_1+y_2+y_3}{3}\right)$$

Hence, the co-ordinates of G are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$

EXERCISE 13 (A)

Q.1. Find the distance between each of the following pairs of points :

(*i*) A (8, 3) and B (14,11)

(*ii*) A (3, –5) and B (8, 7)

(*iii*) P (2, –3) and Q (–6, 3)

(*iv*) P (–6, –4) and Q (9, 4)

(*v*) M (–8, –3) and N (– 2, –5)

(*vi*) R $(a + b, a - b)$ and S $(a - b, a + b)$,

Sol. We know that, distance between two points

A (x_1, y_1) and B (x_2, y_2)

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

(*i*) $\therefore$ Distance between A (8, 3) and B (14, 11)

$$= \sqrt{(14-8)^2 + (11-3)^2}$$

$$= \sqrt{(6)^2 + (8)^2} = \sqrt{36+64}$$

$$= \sqrt{100} = 10 \text{ units } \textbf{Ans.}$$

(*ii*) Distance between A (3, –5) and B (8, 7)

$$= \sqrt{(8-3)^2 + [7-(-5)]^2}$$

$$= \sqrt{(8-3)^2 + (7+5)^2}$$

$$= \sqrt{(5)^2 + (12)^2} = \sqrt{25+144}$$

$$= \sqrt{169} = 13 \text{ units. } \textbf{Ans.}$$

(*iii*) Distance between P (2, –3) and Q (–6, 3)

$$= \sqrt{(-6,-2)^2 + [3-(-3)]^2}$$

$$= \sqrt{(-8)^2 + (3+3)^2} = \sqrt{(-8)^2 + (6)^2}$$

$$= \sqrt{64+36} = \sqrt{100} = 10 \text{ units } \textbf{Ans.}$$

(*iv*) Distance between P (–6, –4) and Q (9, 4)

$$= \sqrt{[9-(-6)]^2 + [4-(-4)]^2}$$

$$= \sqrt{(9+6)^2 + (4+4)^2} = \sqrt{(15)^2 + (8)^2}$$

$$= \sqrt{225+64} = \sqrt{289} = 17 \text{ units } \textbf{Ans.}$$

(*v*) Distance between M (–8, –3) and N (–2, –5)

$$= \sqrt{[-2-(-8)]^2 + [-5-(-3)]^2}$$

$$= \sqrt{(-2+8)^2 + (-5+3)^2}$$

$$= \sqrt{(6)^2 + (-2)^2} = \sqrt{36+4} = \sqrt{40} = \sqrt{4\times 10}$$

$$= 2\sqrt{10} \text{ units } \textbf{Ans.}$$

(*vi*) Dis tance between R $(a + b, a - b)$ and S $(a - b, a + b)$.

$$= \sqrt{(a-b-a-b)^2 + (a+b-a+b)^2}$$

$= \sqrt{(-2b)^2 + (2b)^2} = \sqrt{4b^2 + 4b^2} = \sqrt{8b^2}$

$= \sqrt{4b^2 \times 2} = 2b\sqrt{2} = 2\sqrt{2}b$ units **Ans.**

Q. 2. Find the distance of each of the following points from the origin :

(*i*) A (12, –5) (*ii*) B (– 4, 3)

(*iii*) C (–8, –15)

Sol. We know that, distance between A (*x*, *y*) and B (*x*, *y*)

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(*i*) ∴ Distance between O (0, 0) and A (12, – 5)

$= \sqrt{(12-0)^2 + (-5-0)^2} = \sqrt{(12)^2 + (-5)^2}$

$= \sqrt{144+25} = \sqrt{169}$ = 13 units **Ans.**

(*ii*) Distance between O (0, 0) and B (– 4, 3)

$= \sqrt{(-4-0)^2 + (3-0)^2} = \sqrt{(-4)^2 + (3)^2}$

$= \sqrt{16+9} = \sqrt{25} = 5$ units. **Ans.**

(*iii*) Distance between O (0, 0) and C (–8, –15)

$= \sqrt{(-8-0)^2 + (-15-0)^2}$

$= \sqrt{(-8)^2 + (-15)^2} = \sqrt{64+225}$

$= \sqrt{289}$ = 17 units **Ans.**

Q. 3. Find the distance between the point A **(–3, 5) and the point B on *x*-axis with 9** as abscissa.

Sol. ∵ B lies on *x*-axis and its abscissa = 9

∴ Co-ordinates of B (9, 0)

∴ Distance between A (–3, 5) and B (9, 0)

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{[9-(-3)]^2 + (0-5)^2}$

$= \sqrt{(9+3)^2 + (-5)^2} = \sqrt{(12)^2 + (-5)^2}$

$= \sqrt{144+25} = \sqrt{169}$ = 13 units. **Ans.**

Q. 4. A is the point on the *y*-axis with 6 as ordinate and B (–3, 2) is the other point. Find the length of AB.

Sol. ∵ Point A lies on *y*-axis and 6 is its ordinate

∴ Co-ordinates of A are (0, 6) and B (–3, 2)

∴ Distance between AB

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-3-0)^2 + (2-6)^2} = \sqrt{(-3)^2 + (-4)^2}$

$= \sqrt{9+16} = \sqrt{25} = 5$units.

Q. 5. A is a point on *x*-axis with abscissa – 8 and B is a point on *y*-axis with ordinate 15. Find the distance AB.

Sol. ∵ A lies on *x*-axis and abscissa is – 8

∴ Co-ordinates of A will be (–8,0)

Again, B lies on *y*-axis and its ordinate is 15

∴ Co-ordinates of B will be (0, 15)

Now, distance AB

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{[0-(-8)]^2 + (15-0)^2}$

$= \sqrt{(8)^2 + (15)^2} = \sqrt{64+225}$

$= \sqrt{289} = 17$units.

Q. 6. Points A (5, – 1) on reflection in *x*-axis is mapped as A′. Also, A on reflection in *y*-axis is mapped as A″. Write the coordinates of A′ and A″. Also calculate the distance AA″.

Sol. A′ is the image of A (5, – 1) reflected in *x*-axis

∴ Co-ordinates of A′ are (5, 1)

Again, A″ is the image of A reflected in *y*-axis.

∴ Co-ordinates of A″ are (– 5, – 1)

Distance between A′, A″

$= \sqrt{(-5-5)^2 + [-1-(-1)^2]}$

$= \sqrt{(-10)^2 + (-1+1)^2}$

$= \sqrt{(-10)^2 + (0)^2} = \sqrt{100+0}$

$= \sqrt{100} = 10$ units **Ans.**

Q. 7. Point A (1, – 5) is mapped as A′ on reflection in the *x*-axis. Point B (3, 2) is mapped as B′on reflection in the origin. Write the coordinates of A′ and B′. Calculate AB′. **(1994)**

Sol. Since, the point A′ is the reflection of the point A (1, – 5) in the *x*-axis, the co-ordinates of A′ are (1, 5).

Further, as the point B′ is the reflection of the point B (3, 2) in the origin, the co-ordinates of B′ are (– 3, – 2).

AB′ = distance between the points A (1, – 5) and B′ (– 3, – 2)

$= \sqrt{(-3-1)^2 + (-2-(-5))^2}$

$= \sqrt{(-4)^2 + 3^2}$

$= \sqrt{16+9} = \sqrt{25} = 5$ units **Ans.**

Q. 8. If the distance between the points A (*a*, –2) and B (5, 3) be 5 units, find the value of *a*.

Sol. Distance between A (*a*, –2) and B (5, 3)

$= \sqrt{(5-a)^2 + [3-(-2)]^2} = \sqrt{(5-a)^2 + (3+2)^2}$

$= \sqrt{(5-a)^2 + (5)^2}$

But AB = 5 units

$\therefore \sqrt{(5-a)^2 + (5)^2} = 5$

$(5 - a)^2 + 25 = 25$ (Squaring both sides)

$25 + a^2 - 10a + 25 - 25 = 0$

$a^2 - 10a + 25 = 0$

$\Rightarrow (a-5)^2 = 0$

$\therefore \quad a - 5 = 0 \Rightarrow a = 5$

Hence, $a = 5$ **Ans.**

Q. 9. If the point P (*a*, 2) is equidistant from A (8, –2) and B (2, –2), find the value of *a*.

Sol. $\because$ P (*a*, 2) is equidistant from A (8, –2) and B (2, –2)

and PA = PB.

Now, $PA = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(8-a)^2 + (-2-2)^2}$

$= \sqrt{(8-a)^2 + (-4)^2}$

And, $PB = \sqrt{(2-a)^2 + (-2-2)^2}$

$= \sqrt{(2-a)^2 + (-4)^2}$

Since PA = PB

$\therefore \sqrt{(8-a)^2 + (-4)^2} = \sqrt{(2-a)^2 + (-4)^2}$

Squaring both sides, we get

$(8 - a)^2 + (-4)^2 = (2 - a)^2 + (-4)^2$

$\Rightarrow \quad (8 - a)^2 = (2 - a)^2$

$\Rightarrow 64 - 16a + a^2 = 4 - 4a + a^2$

$\Rightarrow 4 - 4a + a^2 - 64 + 16a - a^2 = 0$

$\Rightarrow 12a - 60 = 0 \quad \Rightarrow a = \frac{60}{12} = 5$

Hence, $a = 5$ **Ans.**

Q. 10. Let P (6, –1), Q (1, 3) and R (*p*, 8) be three points given in such a way that PQ = QR. Find the value of *p*.

Sol. Co-ordinates of P (6, –1), Q (1, 3) and R (*p*, 8)

$\therefore PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(1-6)^2 + [3-(-1)]^2}$

$\sqrt{(-5)^2 + (3+1)^2} = \sqrt{25+16} = \sqrt{41}$

Similarly, $QR = \sqrt{(p-1)^2 + (8-3)^2}$

$= \sqrt{(p-1)^2 + (5)^2}$

$\because$ PQ = QR

$\therefore \sqrt{(p-1)^2 + (5)^2} = \sqrt{41}$

Squaring both sides, we get

$(p - 1)^2 + 25 = 41$

$\Rightarrow \quad (p-1)^2 = 41 - 25 = 16$

$p - 1 = \pm\sqrt{16}$

$\Rightarrow \quad p - 1 = \pm 4$

If $p - 1 = 4$, then $p = 4 + 1 = 5$

And if $p - 1 = -4$ then $p = -4 + 1 = -3$

Hence, $p = 5, -3$ **Ans.**

1. What point on the y-axis is equidistant from A (–4, 3) and B (5, 2) ?

Sol. $\because$ The points lies on y-axis

$\therefore$ its abscissa = 0

Let, the point P be $(0, a)$

Now, $PA = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-4-0)^2 + (3-a)^2}$

$= \sqrt{(-4)^2 + (3-a)^2} \quad ...(i)$

Similarly, $PB = \sqrt{(5-0)^2 + (2-a)^2}$

$= \sqrt{(5)^2 + (2-a)^2} \quad ...(i)$

$\because PA = PB$

$\therefore \sqrt{(-4)^2 + (3-a)^2} = \sqrt{(5)^2 + (2-a)^2}$

Squaring both sides, we get

$(-4)^2 + (3-a)^2 = (5)^2 + (2-a)^2$

$\Rightarrow 16 + 9 + a^2 - 6a = 25 + 4 + a^2 - 4a$

$\Rightarrow a^2 - 6a - a^2 + 4a = 25 + 4 - 16 - 9$

$\Rightarrow -2a = 4 \qquad \Rightarrow a = \dfrac{4}{-2} = -2$

$\therefore$ Co-ordinates of point P will be (0, –2) **Ans.**

12. What point on the x-axis is equidistant from A (5, 4) and B (–2, 3) ?

Sol. Let, P exists on x-axis

$\therefore$ It ordinate = 0

Let its abscissa = a

Then, co-ordinates of P will be $(a, 0)$

Co-ordinates of A and B are A (5, 4) and B (–2, 3)

Now, $PA = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(5-a)^2 + (4-0)^2}$

$= \sqrt{(5-a)^2 + (4)^2} \quad ...(i)$

Similarly, PB

$= \sqrt{(-2-a)^2 + (3-0)^2}$

$= \sqrt{(-2-a)^2 + (3)^2} \quad ...(ii)$

$\because PA = PB$

$\therefore \sqrt{(5-a)^2 + (4)^2} = \sqrt{(-2-a)^2 + (3)^2}$

Squaring both sides, we get

$(5-a)^2 + (4)^2 = (-2-a)^2 + (3)^2$

$25 + a^2 - 10a + 16 = 4 + a^2 + 4a + 9$

$a^2 - 10a - a^2 - 4a = 4 + 9 - 25 - 16$

$\Rightarrow -14a = -28$

$\Rightarrow \quad a = \dfrac{-28}{-14} = 2$

$\therefore$ Co-ordinates of point P will be (2, 0) **Ans.**

Q. 13. What points on x-axis are at a distance of 17 units from the point A (11, –8) ?

Sol. $\because$ The points lie on x-axis

$\therefore$ Their ordinates are zeros

Let, P and Q be two points on x-axis then, PA = QA = 17 units.

$\therefore \quad PA = \sqrt{(11-a)^2 + (-8-0)^2}$

$= \sqrt{(11-a)^2 + (-8)^2}$

$= \sqrt{(11-a)^2 + 64}$

But, PA = 17

$\therefore \sqrt{(11-a)^2 + 64} = 17$

Squaring both sides, we get

$(11-a)^2 + 64 = (17)^2 = 289$

$\Rightarrow \quad (11-a)^2 = 289 - 64$

$\Rightarrow \quad (11-a)^2 = 225 = (\pm 15)^2$

$\Rightarrow 11 - a = \pm 15$

(*i*) $11 - a = 15 \Rightarrow a = 11 - 15 = -4$

(*ii*) $11 - a = -15 \Rightarrow a = 11 + 15 = 26$

∴ Points will be (–4, 0) and (26,0) **Ans.**

Q. 14. What points on *y*-axis are at a distance of 10 units from the point A (–8, 4) ?

Sol. ∵ The points lie on *y*-axis

∴ Their abscissa will be zero

Let P and Q be the two points

Then, PA = PB = 10 units.

Let P be (0, *a*) but A is (–8, 4)

$$PA = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\Rightarrow \sqrt{(-8-0)^2 + (4-a)^2} = 10$$

$$\Rightarrow \sqrt{(-8)^2 + (4-a)^2} = 10$$

Squaring both sides, we get

$(-8)^2 + (4 - a)^2 = (10)^2$

$\Rightarrow 64 + (4 - a)^2 = 100$

$(4 - a)^2 = 100 - 64 = 36$

Taking square root of both sides,

$4 - a = \pm\sqrt{36} = \pm 6$

(*i*) $4 - a = 6 \Rightarrow a = 4 - 6 = -2$

(*ii*) $4 - a = -6 \Rightarrow a = 4 + 6 = 10$

∴ Points will be (0, –2) and (0, 10) **Ans.**

Q. 15. A point P is at a distance of $\sqrt{10}$ units from the point A (4, 3). Find the co-ordinates of P, it being given that its ordinate is twice its abscissa.

Sol. Let, co-ordinates of P be (*a*, 2*a*)

(∵ ordinate is twice of its abscissa)

and point A is (4, 3). PA = $\sqrt{10}$ = units

$$\therefore \quad PA = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\Rightarrow \sqrt{10} = \sqrt{(4-a)^2 + (3-2a)^2}$$

$\Rightarrow (4 - a)^2 + (3 - 2a)^2 = 10$

(Squaring both sides)

$\Rightarrow 16 - 8a + a^2 + 9 + 4a^2 - 12a = 10$

$\Rightarrow \quad 5a^2 - 20a + 25 - 10 = 0$

$\Rightarrow \quad 5a^2 - 20a + 15 = 0$

$\Rightarrow \quad a^2 - 4a + 3 = 0$ [Dividing by 5]

$\Rightarrow \quad a^2 - a - 3a + 3 = 0$

$\Rightarrow a(a - 1) - 3(a - 1) = 0$

$\Rightarrow \quad (a - 1)(a - 3) = 0$

[Zero Product Rule]

Either $a - 1 = 0$, then $a = 1$

or $a - 3 = 0$, then $a = 3$

∴ Co-ordinates of P will be (1, 2) or (3, 6) **Ans.**

Q. 16. Show that the given points are collinear

(*i*) A (–2, 3), B (1, 2) and C (7, 0).

(*ii*) A (3, – 2), B (5, 2) and C (8, 8).

(*iii*) A (1, 1), B (–2, 7) and C (3, –3).

Sol. (*i*) $AB = \sqrt{[1-(-2)]^2 + (2-3)^2}$

$= \sqrt{(1+2)^2 + (2-3)^2}$

$= \sqrt{(3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{1}$

$BC = \sqrt{(7-1)^2 + (0-2)^2}$

$= \sqrt{(6)^2 + (-2)^2} = \sqrt{36+4} = \sqrt{4}$

$= \sqrt{4 \times 10} = 2\sqrt{10}$

$CA = \sqrt{(-2-7)^2 + (3-0)^2} = \sqrt{(-9)^2 + (3)}$

$= \sqrt{81+9} = \sqrt{90} = \sqrt{9 \times 10} = 3\sqrt{10}$

$\therefore \quad AB + BC = \sqrt{10} + 2\sqrt{10}$

$= 3\sqrt{10} = CA$

∴ AB, BC and CA are collinear.

(*ii*) $AB = \sqrt{(5-3)^2 + [2-(-2)]^2}$

$= \sqrt{(2)^2 + (2+2)^2} = \sqrt{(2)^2 + (4)}$

$= \sqrt{4+16} = \sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$

$BC = \sqrt{(8-5)^2 + (8-2)^2}$

$= \sqrt{(3)^2 + (6)^2} = \sqrt{9+36}$

$= \sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}$

$CA = \sqrt{(3-8)^2 + (-2-8)^2}$

$= \sqrt{(-5)^2 + (-10)^2}$

$= \sqrt{25+100} = \sqrt{125}$

$= \sqrt{25\times5} = 5\sqrt{5}$

$\because AB + BC = 2\sqrt{5} + 3\sqrt{5} = 5\sqrt{5} = CA$

$\therefore$ AB, BC and CA are collinear.

(*iii*) $AB = \sqrt{(-2-1)^2 + (7-1)^2}$

$= \sqrt{(-3)^2 + (6)^2}$

$= \sqrt{9+36} = \sqrt{45} = \sqrt{9\times5} = 3\sqrt{5}$

$BC = \sqrt{[3-(-2)]^2 + (-3-7)^2}$

$= \sqrt{(3+2)^2 + (-10)^2}$

$= \sqrt{(5)^2 + (-10)^2} = \sqrt{25+100}$

$= \sqrt{125} = \sqrt{25\times5} = 5\sqrt{5}$

$CA = \sqrt{(1-3)^2 + [1-(-3)]^2}$

$= \sqrt{(-2)^2 + (1+3)^2}$

$= \sqrt{(-2)^2 + (4)^2}$

$= \sqrt{4+16} = \sqrt{20} = \sqrt{4\times5} = 2\sqrt{5}$

$\because AB + CA = 3\sqrt{5} + 2\sqrt{5} = 5\sqrt{5} = BC$

$\therefore$ AB, BC and CA are collinear. **Ans.**

Q. 17. A (2, 2), B (– 2, 4) and C (2, 6) are the vertices of a triangle ABC. Prove that ABC is an isosceles triangle. **(1993)**

Sol. A (2, 2), B (– 2, 4) and C (2, 6) are the vertices of a ΔABC.

$\therefore AB = \sqrt{(-2-2)^2 + (4-2)^2}$

$= \sqrt{(-4)^2 + (2)^2} = \sqrt{16+4}$

$= \sqrt{20} = \sqrt{4\times5} = 2\sqrt{5}$ units

$BC = \sqrt{[2-(-2)]^2 + (6-4)^2}$

$= \sqrt{(2+2)^2 + (2)^2}$

$= \sqrt{(4)^2 + (2)^2} = \sqrt{16+4}$

$= \sqrt{20} = \sqrt{4\times5} = 2\sqrt{5}$ units

$AC = \sqrt{(2-2)^2 + (6-2)^2}$

$= \sqrt{(0)^2 + (4)^2} = \sqrt{0+16}$

$= \sqrt{16} = 4$

$\because AB = BC = 2\sqrt{5}$ units.

$\therefore \Delta ABC$ is an isosceles triangle.

Q. 18. Show that the following points are the vertices of a right triangle.

(*i*) A (3, 5), B (–1, –1) and C(4, 4).

(*ii*) P (–2, 2), Q (–4, –3) and R (8, –2).

(*iii*) L (–2, 4), M (3, – 1), and N (6, 2).

Sol. (*i*) $AB = \sqrt{(-1-3)^2 + (-1-5)^2}$

$= \sqrt{(-4)^2 + (-6)^2} = \sqrt{16+36} = \sqrt{52}$

$BC = \sqrt{[4-(-1)]^2 + [4-(-1)]^2}$

$= \sqrt{(4+1)^2 + (4+1)^2}$

$= \sqrt{(5)^2 + (5)^2} = \sqrt{25+25} = \sqrt{50}$

$CA = \sqrt{(3-4)^2 + (5-4)^2}$

$= \sqrt{(-1)^2 + (1)^2} = \sqrt{1+1} = \sqrt{2}$

Now, $BC^2 + CA^2 = (\sqrt{50})^2 + (\sqrt{2})^2$

$= 50 + 2 = 52$

and $AB^2 = (\sqrt{52})^2 = 52$

$\because AB^2 = BC^2 + CA^2$

Hence, ΔABC is a right triangle.

(*ii*) $PQ = \sqrt{[-4-(-2)]^2 + (-3-2)^2}$

$= \sqrt{(-4+2)^2 + (-5)^2}$

$= \sqrt{(-2)^2 + (-5)^2} = \sqrt{4+25} = \sqrt{29}$

$QR = \sqrt{[8-(-4)]^2 + [(-2-(-3)]^2}$

$= \sqrt{(8+4)^2 + (-2+3)^2}$

$= \sqrt{(12)^2 + (1)^2} = \sqrt{144+1} = \sqrt{145}$ and

$RP = \sqrt{(-2-8)^2 + [2-(-2)]^2}$

$= \sqrt{(-10)^2 + (2+2)^2}$

$= \sqrt{(-10)^2 + (4)^2}$

$= \sqrt{100+16} = \sqrt{116}$

Now, $PQ^2 + RP^2 = (\sqrt{29})^2 + (\sqrt{116})^2$

$= 29 + 116 = 145$

And $QR^2 = (\sqrt{145})^2 = 145$

$\because PQ^2 + RP^2 = QR^2$

$\therefore$ ΔPQR is a right triangle.

(*iii*) $LM = \sqrt{[3-(-2)]^2 + (-1-4)^2}$

$= \sqrt{(3+2)^2 + (-5)^2}$

$= \sqrt{(5)^2 + (-5)^2} = \sqrt{25+25} = \sqrt{50}$

$MN = \sqrt{(6-3)^2 + [2-(-1)]^2}$

$= \sqrt{(3)^2 + (2+1)^2}$

$= \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18}$

$NL = \sqrt{(-2-6)^2 + (4-2)^2}$

$= \sqrt{(-8)^2 + (2)^2} = \sqrt{64+4} = \sqrt{68}$

Now, $LM^2 + MN^2 = (\sqrt{50})^2 + (\sqrt{18})^2$

$= 50 + 18 = 68$

and $NL^2 = (\sqrt{68})^2 = 68$

$\because LM^2 + MN^2 = NL^2$

$\therefore$ ΔLMN is a right triangle.

Q. 19. Show that the points A (7, 10), B (–2, 5) and C (3, – 4) are the vertices of an isosceles right-angled triangle. Also, find the area of the triangle.

Ans. $AB = \sqrt{(-2-7)^2 + (5-10)^2}$

$= \sqrt{(-9)^2 + (-5)^2}$

$= \sqrt{81+25} = \sqrt{106}$

$BC = \sqrt{[3-(-2)]^2 + (-4-5)^2}$

$= \sqrt{(3+2)^2 + (-9)^2}$

$= \sqrt{(5)^2 + (-9)^2} = \sqrt{25+81} = \sqrt{106}$

and $CA = \sqrt{(7-3)^2 + (10-(-4))^2}$

$= \sqrt{(4)^2 + (10+4)^2}$

$= \sqrt{16+196} = \sqrt{212}$

$\therefore AB = BC = \sqrt{106}$

$\therefore$ ΔABC is an isosceles triangle.

Again, $AB^2 + BC^2$

$= (\sqrt{106})^2 + (\sqrt{106})^2 = 106 + 106 = 212$

And $CA^2 = (\sqrt{212})^2 = 212$

$\therefore AB^2 + BC^2 = CA^2$

$\therefore$ Δ ABC is an isosceles right triangle.

Now, area of ΔABC

$= \frac{AB \times BC}{2} = \frac{\sqrt{106} \times \sqrt{106}}{2}$

$= \frac{106}{2} = 53$ square units. **Ans.**

Q. 20. Show that the points P (1, 1), Q (–1, –1) and R $(-\sqrt{3}, \sqrt{3})$ are the vertices of an equilateral triangle.

Sol. $PQ = \sqrt{(-1-1)^2 + (-1-1)^2}$

$= \sqrt{(-2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8}$

$QR = \sqrt{[-\sqrt{3}-(-1)]^2 + [\sqrt{3}-(-1)]^2}$

$= \sqrt{(-\sqrt{3}+1)^2 + (\sqrt{3}+1)^2}$

$= \sqrt{3+1-2\sqrt{3}+3+1+2\sqrt{3}} = \sqrt{8}$

$RP = \sqrt{[1-(-\sqrt{3})]^2 + (1-\sqrt{3})^2}$

$= \sqrt{(1+\sqrt{3})^2 + (1-\sqrt{3})^2}$

$= \sqrt{1+3+2\sqrt{3}+1+3-2\sqrt{3}} = \sqrt{8}$

$\because PQ = QR = RP = \sqrt{8}$

$\therefore$ Δ PQR is an equilateral.

Q. 21. Show that the points L (2a, 4a), M (2a, 6a) and N (2a + $\sqrt{3}a$, 5a) are the vertices of an equilateral triangle with each side 2a.

Sol. $LM = \sqrt{(2a-2a)^2+(6a-4a)^2}$

$= \sqrt{0^2+(2a)^2} = \sqrt{4a^2} = 2a$

$MN = \sqrt{(2a+\sqrt{3}a-2a)^2+(5a-6a)^2}$

$= \sqrt{(\sqrt{3}a)^2+(-a)^2}$

$= \sqrt{3a^2+a^2} = \sqrt{4a^2} = 2a$

$NL = \sqrt{(2a-2a-\sqrt{3}a)^2+(4a-5a)^2}$

$= \sqrt{(\sqrt{3}a)^2+(-a)^2}$

$= \sqrt{3a^2+a^2} = \sqrt{4a^2} = 2a$

$\because$ LM = MN = NL = 2a

$\therefore$ ΔLMN is an equilateral triangle.

Whose each side is 2a.

Q. 22. Show that the following points are the vertices of a rectangle :

(*i*) A (3, 2), B (11, 8), C (8, 12), and D (0, 6).

(*ii*) A (0, –4), B (6, 2), C (3, 5) and D (–3, –1).

(*iii*) A (0, –1), B (–2, 3), C (6, 7) and D (8, 3).

Sol. ABCD will be a rectangle if its opposite sides are equal and its diagonals are also equal.

(*i*) A (3, 2), B (11, 8), C (8, 12) and D (0, 6)

Now, $AB = \sqrt{(11-3)^2+(8-2)^2}$

$= \sqrt{(8)^2+(6)^2} = \sqrt{64+36}$

$= \sqrt{100} = 10$

$BC = \sqrt{(8-11)^2+(12-8)^2}$

$= \sqrt{(-3)^2+(4)^2}$

$= \sqrt{9+16} = \sqrt{25} = 5$

$CD = \sqrt{(0-8)^2+(6-12)^2}$

$= \sqrt{(-8)^2+(-6)^2}$

$= \sqrt{64+36} = \sqrt{100} = 10$

$DA = \sqrt{(3-0)^2+(2-6)^2}$

$= \sqrt{(3)^2+(-4)^2}$

$= \sqrt{9+16} = \sqrt{25} = 5$

And, $AC = \sqrt{(8-3)^2+(12-2)^2}$

$= \sqrt{(5)^2+(10)^2}$

$= \sqrt{25+100} = \sqrt{125}$

$BD = \sqrt{(0-11)^2+(6-8)^2}$

$= \sqrt{(-11)^2+(-2)^2} = \sqrt{121+4}$

$= \sqrt{125}$

From above we see that,

AB = CD and BC = DA and AC = BD

$\therefore$ ABCD is a rectangle.

(*ii*) A (0, – 4), B (6, 2), C (3, 5) and D (– 3, – 1)

Now $AB = \sqrt{(6-0)^2+2-(-4)^2}$

$= \sqrt{6^2+(2+4)^2} = \sqrt{6^2+6^2}$

$= \sqrt{36+36} = \sqrt{36\times 2} = 6\sqrt{2}$

$BC = \sqrt{(3-6)^2+(5-2)^2}$

$= \sqrt{(-3)^2+(3)^2} = \sqrt{9+9}$

$= \sqrt{9\times 2} = 3\sqrt{2}$

$CD = \sqrt{(-3-3)^2+(-1-5)^2}$

$= \sqrt{(-6)^2+(-6)^2}$

$= \sqrt{36+36} = \sqrt{36\times 2} = 6\sqrt{2}$

$DA = \sqrt{(0-(-3)^2+(-4-(-1)^2}$

$= \sqrt{(3)^2+(-4+1)^2} = \sqrt{(3)^2+(-3)^2}$

$= \sqrt{9\times 9} = \sqrt{9\times 2} = 3\sqrt{2}$

Diagonal $AC = \sqrt{(3-0)^2+(5-(-4)^2}$

$= \sqrt{3^2+(5+4)^2}$

$= \sqrt{3^2+9^2} = \sqrt{9+81} = \sqrt{90}$

and diagonal

$BD = \sqrt{(-3-6)^2 + (-1-2)^2}$

$= \sqrt{(-9)^2 + (-3)^2}$

$= \sqrt{81+9} = \sqrt{90}$

From above, we can say that

AB = CD, BC = DA

and AC = BD

∴ ABCD is a rectangle.

Hence proved.

(*iii*) A (0, – 1), B (– 2, 3), C (6, 7) and D (8, 3)

Now $AB = \sqrt{(-2-0)^2 + (3-(-1)^2}$

$= \sqrt{(-2)^2 + (3+1)^2}$

$= \sqrt{(-2)^2 + (4)^2}$

$= \sqrt{4+16} = \sqrt{20}$

$BC = \sqrt{(6-(-2)^2 + (7-3)^2}$

$= \sqrt{(6+2)^2 + (4)^2} = \sqrt{8^2 + 4^2}$

$= \sqrt{64+16} = \sqrt{80}$

$CD = \sqrt{(8-6)^2 + (3-7)^2}$

$= \sqrt{(2)^2 + (-4)^2}$

$= \sqrt{4+16} = \sqrt{20}$

$DA = \sqrt{(0-8)^2 + (-1-3)^2}$

$= \sqrt{(-8)^2 + (-4)^2}$

$= \sqrt{64+16} = \sqrt{80}$

Diagonal $AC = \sqrt{(6-0)^2 + (7-(-1)^2}$

$= \sqrt{(6)^2 + (7+1)^2}$

$= \sqrt{6^2 + 8^2}$

$= \sqrt{36+64} = \sqrt{100} = 10$

and diagonal

$BD = \sqrt{(8-(-2)^2 + (3-3)^2}$

$= \sqrt{10^2 + 0^2} = \sqrt{100} = 10$

From above, we see that

AB = CD, BC = DA and AC = BD

∴ ABCD is a rectangle .

Hence proved.

Q. 23. Show that the following points are the vertices of a square :

(*i*) A (3, 2), B (0, 5), C (–3, 2) and D (0, –1).

(*ii*) A (0, –1), B (2,1), C (0, 3), and D (–2, 1).

(*iii*) A (0, –2), B (3, 1), C (0, 4), and D (–3, 1).

Find the area of the square in each case.

Sol. (*i*) A (3, 2), B (0, 5), C (–3, 2) and D (0, –1)

$AB = \sqrt{(0-3)^2 + (5-2)^2}$

$= \sqrt{(-3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18}$

$BC = \sqrt{(-3-0)^2 + (2-5)^2}$

$= \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18}$

$CD = \sqrt{(0-3)^2 + (-1-2)^2}$

$= \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18}$

And, $DA = \sqrt{(3-0)^2 + [2-(-1)]^2}$

$= \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18}$

∵ $AB = BC = CD = DA = \sqrt{18}$

∴ ABCD is a square or rhombus

Now, diagonal AC

$= \sqrt{(-3-3)^2 + (2-2)^2}$

$= \sqrt{(-6)^2 + (0)^2}$

$= \sqrt{36+0} = \sqrt{36} = 6$

and diagonal $BD = \sqrt{(0-0)^2 + (-1-5)^2}$

$= \sqrt{(0)^2 + (-6)^2} = \sqrt{0+36}$

$= \sqrt{36} = 6$

$\because AC = BD$

$\therefore$ ABCD is a square

Hence proved.

Now Area = $(side)^2$

$= 6 \times 6 = 36$ sq. units. **Ans.**

(*ii*) A (0, –1), B (2, 1), C (0, 3) and D (–2, 1)

$AB = \sqrt{(2-0)^2 + [1-(-1)]^2}$

$= \sqrt{(2)^2 + (2)^2} = \sqrt{4+4} = \sqrt{8}$

$BC = \sqrt{(0-2)^2 + (3-1)^2}$

$= \sqrt{(-2)^2 + (2)^2} = \sqrt{4+4} = \sqrt{8}$

$CD = \sqrt{(-2-0)^2 + (1-3)^2}$

$= \sqrt{(-2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8}$

And, $DA = \sqrt{[0-(-2)]^2 + (-1-1)^2}$

$= \sqrt{(2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8}$

$\because AB = BC = CD = DA$

$\therefore$ ABCD is a square or rhombus.

Now, diagonal

$AC = \sqrt{(0-0)^2 + [3-(-1)]^2}$

$= \sqrt{0^2 + (4)^2} = \sqrt{(4)^2} = \sqrt{16} = 4$

and diagonal

$BD = \sqrt{(-2-2)^2 + (1-1)^2}$

$= \sqrt{(-4)^2 + (0)^2} = \sqrt{(4)^2 + (0)^2}$

$= \sqrt{16} = 4$

$\because AC = BD$

$\therefore$ ABCD is a square. Hence proved.

Area of square $= (side)^2 = (4)^2 = 16$ square units. **Ans.**

(*iii*) A (0, – 2), B (3, 1), C (0, 4) and D (– 3, 1)

$AB = \sqrt{(3-0)^2 + [1-(-2)]^2}$

$= \sqrt{(-3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18}$

$BC = \sqrt{(0-3)^2 + (4-1)^2}$

$= \sqrt{(-3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18}$

$CD = \sqrt{(-3-0)^2 + (1-4)^2}$

$= \sqrt{(-3)^2 + (-3)^2} = \sqrt{(9+9)} = \sqrt{18}$

$DA = \sqrt{(0-(-3))^2 + (-2-1)^2}$

$= \sqrt{(0+3)^2 + (-2-1)^2}$

$= \sqrt{(3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18}$

$\because AB = AC = CD = DA$

$\therefore$ ABCD is a square or rhombus.

Hence proved.

Now, diagonal

$AC = \sqrt{(0-0)^2 + [4-(-2)]^2}$

$= \sqrt{(0)^2 + (6)^2} = \sqrt{0+36} = \sqrt{36} = 6$

and diagonal

$BD = \sqrt{(-3-3)^2 + (1-1)^2}$

$= \sqrt{(-6)^2 + (0)^2} = \sqrt{36+0} = \sqrt{36} = 6$

$\because AC = BD$

$\therefore$ ABCD is a square. Hence proved.

Area of square $= (side)^2 = (6)^2 = 36$ square units. **Ans.**

Q. 24. Show that the following points are the vertices of a parallelogram :

(*i*) P (3, 1), Q (0, –2), R (1, 1) and S (4, 4).

(*ii*) P (1, –2), Q (3, 6), R (5, 10) and S (3, 2).

(*iii*) P (–1, 0), Q (0, 3), R (1, 3) and S (0, 0).

Sol. (*i*) P (3, 1), Q (0, –2), R (1, 1) and S (4, 4)

Then, $PQ = \sqrt{(0-3)^2 + (-2-1)^2}$

$= \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18}$

$QR = \sqrt{(1-0)^2 + [1-(-2)]^2}$

$= \sqrt{(1)^2 + (1+2)^2}$

$= \sqrt{(1)^2 + (3)^2}$

$= \sqrt{1+9} = \sqrt{10}$

$RS = \sqrt{(4-1)^2+(4-1)^2}$

$= \sqrt{(3)^2+(3)^2} = \sqrt{9+9} = \sqrt{18}$

$SP = \sqrt{(3-4)^2+(1-4)^2}$

$= \sqrt{(-1)^2+(-3)^2} = \sqrt{1+9} = \sqrt{10}$

$\because PQ = RS = \sqrt{18}$ and $QR = SP = \sqrt{10}$

$\therefore$ PQRS is a rectangle or parallelogram.

Now, diagonal PR

$= \sqrt{(1-3)^2+(1-1)^2}$

$= \sqrt{(-2)^2+(0)^2} = \sqrt{4+0} = \sqrt{4} = 2$

and diagonal

$QS = \sqrt{(4-0)^2+[4-(-2)]^2}$

$= \sqrt{(4)^2+(4+2)^2} = \sqrt{16+36} = \sqrt{52}$

But, $PR \neq QS$.

$\therefore$ PQRS is a parallelogram.

Hence proved.

(*ii*) P (1, –2), Q (3, 6), R (5, 10) and S (3, 2)

Then, $PQ = \sqrt{(3-1)^2+[6-(-2)]^2}$

$= \sqrt{(2)^2+(6+2)^2} = \sqrt{(2)^2+(8)^2}$

$= \sqrt{4+64} = \sqrt{68}$

$QR = \sqrt{(5-3)^2+(10-6)^2}$

$= \sqrt{(2)^2+(4)^2} = \sqrt{4+16} = \sqrt{20}$

$RS = \sqrt{(3-5)^2+(2-10)^2}$

$= \sqrt{(-2)^2+(-8)^2} = \sqrt{4+64} = \sqrt{68}$

$SP = \sqrt{(1-3)^2+(-2-2)^2}$

$= \sqrt{(-2)^2+(-4)^2} = \sqrt{4+16} = \sqrt{20}$

$\because PQ = RS = \sqrt{68}$ and $QR = SP = \sqrt{20}$

$\therefore$ PQRS is a rectangle or parallelogram.

Now, diagonal

$PR = \sqrt{(5-1)^2+[10-(-2)]^2}$

$= \sqrt{(4)^2+(10+2)^2} = \sqrt{(4)^2+(12)^2}$

$= \sqrt{16+144} = \sqrt{160}$

and diagonal $QS = \sqrt{(3-3)^2+(2-6)^2}$

$= \sqrt{(0)^2+(-4)^2} = \sqrt{0+16} = \sqrt{16}$ =

$\therefore PR \neq QS$

$\therefore$ PQRS is a parallelogram.

Hence proved.

(*iii*) P (–1, 0), Q (0, 3), R (1, 3) and S (0,

then, $PQ = \sqrt{[0-(-1)]^2+(3-0)^2}$

$= \sqrt{(1)^2+(3)^2} = \sqrt{1+9} = \sqrt{10}$

$QR = \sqrt{(1-0)^2+(3-3)^2}$

$= \sqrt{(1)^2+(0)^2} = \sqrt{1+0} = \sqrt{1}$ =

$RS = \sqrt{(0-1)^2+(0-3)^2}$

$= \sqrt{(-1)^2+(-3)^2} = \sqrt{1+9} = \sqrt{}$

and $SP = \sqrt{(-1-0)^2+(0-0)^2}$

$= \sqrt{(-1)^2+(0)^2} = \sqrt{1+0} = \sqrt{1}$ =

$\because PQ = RS = \sqrt{10}$ and QR = SP = 1

$\therefore$ PQRS is a rectangle or parallelogra

Now, diagonal

$PR = \sqrt{[1-(-1)]^2+(3-0)^2}$

$= \sqrt{(1+1)^2+(3)^2} = \sqrt{(2)^2+(3)}$

$= \sqrt{4+9} = \sqrt{13}$

and diagonal $QS = \sqrt{(0-0)^2+(0-3)}$

$= \sqrt{(0)^2+(-3)^2} = \sqrt{0+9} = \sqrt{9}$ =

$\because PR \neq QS$

$\therefore$ PQRS is a parallelogram.

Hence proved.

Q. 25. Show that the points P (–3, 2), Q (–5, –
R (2, –3) and S (4, 4) are the vertices
a rhombus. Also, find the area of t
rhombus.

Sol. P (–3, 2), Q (–5, –5), R (2, –3) and S (4,

then, $PQ = \sqrt{[-5-(-3)]^2+(-5-2)^2}$

$= \sqrt{(-5+3)^2+(-7)^2}$

$= \sqrt{(-2)^2+(-7)^2} = \sqrt{4+49} = \sqrt{53}$

$QR = \sqrt{[2-(-5)^2+[-3-(-5)]^2}$

$= \sqrt{(2+5)^2+(-3+5)^2}$

$= \sqrt{(7)^2+(2)^2} = \sqrt{49+4} = \sqrt{53}$

$RS = \sqrt{(4-2)^2+[4-(-3)]^2}$

$= \sqrt{(2)^2+(4+3)^2} = \sqrt{(2)^2+(7)^2}$

$= \sqrt{4+49} = \sqrt{53}$

$SP = \sqrt{(-3-4)^2+(2-4)^2}$

$= \sqrt{(-7)^2+(-2)^2} = \sqrt{49+4} = \sqrt{53}$

$\because PQ = QR = RS = SR = \sqrt{53}$

$\therefore$ PQRS is a square or rhombus.

Now, diagonal

$PR = \sqrt{[2-(-3)]^2+(-3-2)^2}$

$= \sqrt{(2+3)^2+(-5)^2}$

$= \sqrt{(5)^2+(-5)^2} = \sqrt{25+25} = \sqrt{50}$

and diagonal $QS = \sqrt{[4-(-5)]^2+[4-(-5)]^2}$

$= \sqrt{(4+5)^2+(4+5)^2}$

$= \sqrt{(9)^2+(9)^2} = \sqrt{81+81} = \sqrt{162}$

$\because PR \neq QS$

$\therefore$ PQRS is a rhombus.

Hence proved.

Q. 26. Let A (2, –1), B (3, 4), C (–2, 3) and D (–3, –2) be four given points. Show that ABCD is a rhombus but not a square. Also, find the area of rhombus.

Sol. A (2, –1), B (3, 4), C (–2, 3) and D (–3, –2)

Then, $AB = \sqrt{(3-2)^2+[4-(-1)]^2}$

$= \sqrt{(1)^2+(4+1)^2} = \sqrt{(1)^2+(5)^2}$

$= \sqrt{1+25} = \sqrt{26}$

$BC = \sqrt{(-2-3)^2+(3-4)^2}$

$= \sqrt{(-5)^2+(-1)^2} = \sqrt{25+1} = \sqrt{26}$

$CD = \sqrt{[-3-(-2)]^2+(-2-3)^2}$

$= \sqrt{(-3+2)^2+(-2-3)^2}$

$= \sqrt{(-1)^2+(-5)^2} = \sqrt{1+25} = \sqrt{26}$

$DA = \sqrt{[2-(-3)]^2+[-1-(-2)]^2}$

$= \sqrt{(2+3)^2+(-1+2)^2}$

$= \sqrt{(5)^2+(1)^2} = \sqrt{25+1} = \sqrt{26}$

$\because AB = BC = CD = DA = \sqrt{26}$

$\therefore$ ABCD is a square or rhombus.

Now, diagonal

$AC = \sqrt{(-2-2)^2+[3-(-1)]^2}$

$= \sqrt{(-4)^2+(3+1)^2} = \sqrt{16+16} = \sqrt{32}$

and diagonal $BD = \sqrt{(-3-3)^2+(-2-4)^2}$

$= \sqrt{(-6)^2+(-6)^2} = \sqrt{36+36} = \sqrt{72}$

$\because AC \neq BD$

$\therefore$ ABCD is a rhombus.

Hence proved.

Now, Area of rhombus ABCD

$= \frac{AC \times BD}{2} = \frac{\sqrt{32} \times \sqrt{72}}{2}$

$= \frac{\sqrt{16 \times 2 \times 2 \times 36}}{2} = \frac{4 \times 2 \times 6}{2}$

= 24 sq. units. **Ans.**

Q. 27. Find a point equidistant from the points A (6, 2), B (–1, 3) and C (–3, –1).

Sol. Let, P (x, y) be the point which is equidistant from A, B and C

$\therefore$ PA = PB = PC

Now, $PA = \sqrt{(6-x)^2+(2-y)^2}$

And $PB = \sqrt{(-1-x)^2+(3-y)^2}$

And $PC = \sqrt{(-3-x)^2+(-1-y)^2}$

According to the condition,

When, PA = PB, then

$\sqrt{(6-x)^2+(2-y)^2}$

$= \sqrt{(-1-x)^2+(3-y)^2}$

$\Rightarrow (6-x)^2+(2-y)^2=(-1-x)^2+(3-y)^2$

[Squaring both sides]

$\Rightarrow 36+x^2-12x+4+y^2-4y$

$=1+x^2+2x+9+y^2-6y$

$\Rightarrow x^2-12x+y^2-4y-x^2-2x-y^2+6y$

$=1+9-36-4$

$\Rightarrow -14x+2y=-30$

$\Rightarrow \quad 7x-y=15$ ...(*i*) [Dividing by –2]

Again, when PA = PC, then

$\sqrt{(6-x)^2+(2-y)^2}$

$= \sqrt{(-3-x)^2+(-1-y)^2}$

$\Rightarrow (6-x)^2+(2-y)^2=(-3-x)^2+(-1-y)^2$

(Squaring both sides)

$\Rightarrow 36+x^2-12x+4+y^2-4y$

$=9+x^2+6x+1+y^2+2y$

$\Rightarrow x^2-12x+y^2-4y-x^2-6x-y^2-2y$

$=9+1-36-4$

$\Rightarrow -18x-6y=-30 \Rightarrow 3x+y=5$ ...(*ii*)

[Dividing by – 6]

from (*i*), $y=7x-15$ ...(*iii*)

Substituting the value of y in (*ii*),

$3x+7x-15=5 \quad \Rightarrow \quad 10x=5+15$

$\Rightarrow \quad 10x=20 \quad \Rightarrow \quad x=\frac{20}{10}=2$

Substituting the value of x in (*iii*),

$y=7(2)-15=14-15=-1$

Hence, point P will be (2, –1) **Ans.**

Q. 28. Find the co-ordinates of the centre of a circle which passes through the points A (0, 0), B (–3, 3) and C (5, –1).

Sol. ∵ O is the centre of the circle and A, B and C are three points on the circle

∴ OA = OB = OC

[Radii of the same circle]

Let, co-ordinates of O be (x, y), then

$OA = \sqrt{(0-x)^2+(0-y)^2}$

$= \sqrt{(-x)^2+(-y)^2} = \sqrt{x^2+y^2}$

OB $\sqrt{(-3-x)^2+(3-y)^2}$ and

$OC = \sqrt{(5-x)^2+(-1-y)^2}$

(*i*) when OA = OB, then

$\sqrt{x^2+y^2} = \sqrt{(-3-x)^2+(3-y)^2}$

$\Rightarrow x^2+y^2=9+x^2+6x+9+y^2-6y$

[Squaring both sides]

$\Rightarrow x^2+6x+y^2-6y-x^2-y^2=-9-9$

$\Rightarrow 6x-6y=-18$

$\Rightarrow \quad x-y=-3$ ...(*i*) [Dividing by 6]

(*ii*) Again, when OA = OC, then

$\sqrt{x^2+y^2} = \sqrt{(5-x)^2+(-1-y)^2}$

$\Rightarrow x^2+y^2=25+x^2-10x+1+y^2+2y$

[Squaring both sides]

$\Rightarrow x^2+y^2-x^2+10x-y^2-2y=25+1$

$\Rightarrow \quad 10x-2y=26 \Rightarrow 5x-y=13$ ...(*ii*)

[Dividing by 2]

Subtracting (*i*) from (*ii*),

$4x=16 \quad \Rightarrow \quad x=\frac{16}{4}=4$

Substituting the value of x in (*i*),

$4-y=-3 \quad \Rightarrow -y=-4-3$

$\Rightarrow -y=-7 \quad \Rightarrow \quad y=7$

∴ Co-ordinates of O will be (4, 7) **Ans.**

Q. 29. Find the co-ordinates of the circumcentre of ΔABC whose vertices are A (4, 6), B (0, 4) and C (6, 2). Also, find the circumradius.

Sol. Let, O be the circumcentre of ΔABC and let co-ordinates of O be (x, y), then OA = OB = OC = radius of the circumcircle.

Points A (4, 6), B (0, 4) and C (6, 2) are the vertices of ΔABC.

$\therefore \quad OA = \sqrt{(4-x)^2+(6-y)^2}$

$OB = \sqrt{(0-x)^2+(4-y)^2}$

and, $OC = \sqrt{(6-x)^2+(2-y)^2}$

Now, when OA = OB, then

$\sqrt{(4-x)^2+(6-y)^2} = \sqrt{(0-x)^2+(4-y)^2}$

$\Rightarrow (4-x)^2 + (6-y)^2 = (-x)^2 + (4-y)^2$ [Squaring both sides]

$16 + x^2 - 8x + 36 + y^2 - 12y$

$= x^2 + 16 + y^2 - 8y$

$\Rightarrow x^2 - 8x + y^2 - 12y - x^2 - y^2 + 8y$

$= 16 - 16 - 36$

$\Rightarrow -8x - 4y = -36$

$\Rightarrow 2x + y = 9$ ...(*i*) [Dividing by 4]

Again, when OA = OC, then

$\sqrt{(4-x)^2+(6-y)^2}$

$= \sqrt{(6-x)^2+(2-y)^2}$

$\Rightarrow (4-x)^2 + (6-y)^2 = (6-x)^2 + (2-y)^2$ [Squaring both sides]

$\Rightarrow 16 + x^2 - 8x + 36 + y^2 - 12y$

$= 36 + x^2 - 12x + 4 + y^2 - 4y$

$\Rightarrow x^2 - 8x + y^2 - 12y - x^2 + 12x - y^2 + 4y$

$= 36 - 16 - 36 + 4$

$4x - 8y = -12$

$\Rightarrow x - 2y = -3$ ...(*ii*) [Dividing by 4]

From (*ii*) $x = 2y - 3$, ...(*iii*)

Substituting the value of x in (*i*),

$2(2y-3) + y = 9 \Rightarrow 4y - 6 + y = 9$

$\Rightarrow 5y = 9 + 6 = 15 \Rightarrow y = \frac{15}{5} = 3$

Substituting the value of Y in (*iii*),

$x = 2y - 3 = 2 \times 3 - 3 = 6 - 3 = 3$

$\therefore$ The co-ordinate of O are (3, 3).

And, radius $OA = \sqrt{(4-3)^2+(6-3)^2}$

$= \sqrt{(1)^2+(3)^2} = \sqrt{1+9} = \sqrt{10}$ Units. **Ans.**

EXERCISE 13 (B)

Q.1. Find the co-ordinates of the point P which divides the join of A (–2, 1) and B (7, 4) in the ratio 1 : 2.

Sol. Here, $m_1 = 1$ and $m_2 = 2$

Let P (x, y) be the points on the line segment joining the points A (–2, 1) and B (7, 4) dividing it in the ratio 1 : 2

$\therefore x = \frac{m_1x_2+m_2x_1}{m_1+m_2}$

$= \frac{1\times7+2\times(-2)}{1+2} = \frac{7-4}{3} = \frac{3}{3} = 1$

and $y = \frac{m_1y_2+m_2y_1}{m_1+m_2}$

$= \frac{1\times4+2\times(1)}{1+2} = \frac{4+2}{3} = \frac{6}{3} = 2$

$\therefore$ Co-ordinates of P will be (1, 2) **Ans.**

Q.2. Find the co-ordinates of the point C which divides the join of A (4, –3) and B (9, 7) in the ratio 3 : 2.

Sol. Let, the co-ordinates of C be (x, y) which divides the line segment joining the points A (4, –3) and B (9, 7) in the ratio $m_1 : m_2$ *i.e.*, 3 : 2. Here $m_1 = 3$, $m_2 = 2$

$\therefore x = \frac{m_1x_2+m_2x_1}{m_1+m_2} = \frac{3\times9+2\times4}{3+2}$

$= \frac{27+8}{5} = \frac{35}{5} = 7$

and $y = \frac{m_1y_2+m_2y_1}{m_1+m_2} = \frac{3\times7+2(-3)}{3+2}$

$= \frac{21-6}{5} = \frac{15}{5} = 3$

$\therefore$ Co-ordinates of C are (7, 3) **Ans.**

Q.3. Find the co-ordinates of the point R which divides the line segment joining P (–2, –5) and Q (6, –1) in the ratio 5 : 3.

Sol. Let, the co-ordinates of R be (x, y) and ratio $m_1 : m_2 = 5 : 3$

$$\therefore\ x=\frac{m_1x_2+m_2x_1}{m_1+m_2}=\frac{5\times6+3\times(-2)}{5+3}$$

$$=\frac{30-6}{8}=\frac{24}{8}=3$$

$$y=\frac{m_1y_2+m_2y_1}{m_1+m_2}=\frac{5\times(-1)+3(-5)}{5+3}$$

$$=\frac{-5-15}{8}=\frac{-20}{8}=\frac{-5}{2}$$

$\therefore$ Co-ordinates of R are $\left(3,\frac{-5}{2}\right)$ **Ans.**

Q. 4. The line segment joining the points A (4,–3) and B (4, 2) is divided by the point P such that AP : AB = 2 : 5. Find the co-ordinates of P.

Sol. $\frac{AP}{AB}=\frac{2}{5}\ \Rightarrow\ \frac{AP}{AP+PB}=\frac{2}{5}$

$\Rightarrow\quad 5AP = 2AP + 2PB$

$\Rightarrow 5AP - 2AP = 2PB \Rightarrow 3AP = 2PB$

$\Rightarrow \frac{AP}{PB}=\frac{2}{3}\quad \Rightarrow AP : PB = 2 : 3$

Let co-ordinates of P be (x, y) and $m_1 : m_2 = 2 : 3$

$$\therefore\ x=\frac{m_1x_2+m_2x_1}{m_1+m_2}=\frac{2\times4+3\times4}{2+3}$$

$$=\frac{8+12}{5}=\frac{20}{5}=4$$

$$\text{And } y=\frac{m_1y_2+m_2y_1}{m_1+m_2}=\frac{2\times2+3(-3)}{2+3}$$

$$=\frac{4-9}{5}=\frac{-5}{5}=-1$$

$\therefore$ Co-ordinates of P are (4, – 1) **Ans.**

Q. 5. Find a point P on the line segment joining A (14, – 5) and B (– 4, 4) which is twice as far from A as from B.

Sol. AP = 2PB $\Rightarrow \frac{AP}{PB}=\frac{2}{1}$

$\Rightarrow$ AP : PB = 2 : 1

Let co-ordinates of P be (x, y), then

$$x_1=\frac{m_1x_2+m_2x_1}{m_1+m_2}=\frac{2\times(-4)+1\,(14)}{2+1}$$

$$=\frac{-8+14}{3}=\frac{6}{3}=2$$

$$\text{And } y=\frac{m_1y_2+m_2y_1}{m_1+m_2}=\frac{2\times4+1\times(-5)}{2+1}$$

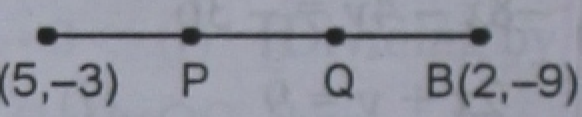
$$=\frac{8-5}{3}=\frac{3}{3}=1$$

$\therefore$ Co-ordinates of P are (2, 1) **Ans.**

Q. 6. Find the co-ordinates of the points of trisection of the line segment joining the points A (5, – 3) and B (2 – 9).

Sol. Let P and Q trisect the line segment AB and let the points be

P (x', y') and Q (x'', y'')



Now, the point P divides the line segment AB in the ratio 1 : 2

$$\therefore\ x'=\frac{m_1x_2+m_2x_1}{m_1+m_2}=\frac{1\times2+2\times5}{1+2}$$

$$=\frac{2+10}{3}=\frac{12}{3}=4$$

$$\text{and } y'=\frac{1\times(-9)+2\,(-3)}{1+2}$$

$$=\frac{-9-6}{3}=\frac{-15}{3}=-5$$

$\therefore$ Co-ordinates of P are (4, –5)

Again, Q divides the line segment AB in the ratio 2 : 1

$$\therefore\ x''=\frac{m_1x_2+m_2x_1}{m_1+m_2}=\frac{2\times2+1+5}{2+1}$$

$$=\frac{4+5}{3}=\frac{9}{3}=3$$

$$\text{and } y''=\frac{m_1y_2+m_2y_1}{m_1+m_2}$$

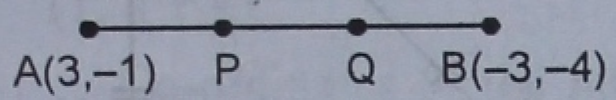
$$=\frac{2\,(-9)+1\,(-3)}{2+1}=\frac{-18-3}{3}=\frac{-21}{3}=-7$$

$\therefore$ Co-ordinates of Q are (3, – 7) **Ans.**

Q. 7. Find the co-ordinates of the points of trisection of the line segment joining the points A (3, – 1) and B (– 3, – 4).

Sol. Let P and Q be the two points which trisect the line segment joining the points A (3, –1) and B (– 3, – 4)

Let the coordinates of P be (x', y') and Q be (x'', y'').



Now, P divides AB in the ratio 1 : 2 and Q divides it in 2 : 1

$$x' = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} = \frac{1 \times (-3) + 2 \times 3}{1+2}$$

$$= \frac{-3+6}{3} = \frac{3}{3} = 1$$

$$y' = \frac{m_1y_2 + m_2y_1}{m_1 + m_2} = \frac{1 \times (-4) + 2\,(-1)}{1+2}$$

$$= \frac{-4-2}{3} = \frac{-6}{3} = -2$$

∴ Co-ordinates of P are (1, – 2)

$$\text{Again } x'' = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$$

$$= \frac{2 \times (-3) + 1\,(3)}{2+1} = \frac{-6+3}{3} = \frac{-3}{3} = -1$$

$$\text{and } y'' = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

$$= \frac{2\,(-4) + 1\,(-1)}{2+1} = \frac{-8-1}{3} = \frac{-9}{3} = -3$$

∴ Co-ordinates of Q are (– 1, – 3) **Ans.**

Q. 8. Find the co-ordinates of the mid-point of the line-segment joining :

(*i*) A (5, 7) and B (6, 3)

(*ii*) A (– 5, – 8) and B (3, 5)

(*iii*) A $\left(-\frac{2}{3}, \frac{1}{2}\right)$ and B $\left(\frac{5}{3}, \frac{3}{2}\right)$.

Sol. (*i*) Let P (x, y) be the mid-point of the line segment joining the points A (5, 7) and B (6, 3)

$$\therefore x = \frac{x_1 + x_2}{2} = \frac{5+6}{2} = \frac{11}{2}$$

$$y = \frac{y_1 + y_2}{2} = \frac{7+3}{2} = \frac{10}{2} = 5$$

∴ Co-ordinates of P are $\left(\frac{11}{2}, 5\right)$

(*ii*) Let P (x, y) be the mid-point of the line segment joining the points A (– 5, – 8) and B (3, 5)

$$\therefore x = \frac{x_1 + x_2}{2} = \frac{-5+3}{2} = \frac{-2}{2} = -1$$

$$y = \frac{y_1 + y_2}{2} = \frac{-8+5}{2} = \frac{-3}{2}$$

∴ Co-ordinates of P are $\left(-1, \frac{-3}{2}\right)$

(*iii*) Let P (x, y) be the mid-point of the line segment joining the points A $\left(-\frac{2}{3}, \frac{1}{2}\right)$, and B $\left(\frac{5}{3}, \frac{3}{2}\right)$

$$\therefore x = \frac{x_1 + x_2}{2} = \frac{\frac{-2}{3} + \frac{5}{3}}{2} = \frac{\frac{3}{3}}{2} = \frac{1}{2}$$

$$y = \frac{y_1 + y_2}{2} = \frac{\frac{1}{2} + \frac{3}{2}}{2} = \frac{\frac{4}{2}}{2} = \frac{2}{2} = 1$$

∴ Co-ordinates of P are $\left(\frac{1}{2}, 1\right)$ **Ans.**

Q. 9. Point A (4, – 1) is reflected as A′ in y-axis. Point B on reflection in x-axis is mapped as B′ (– 2, 5).

(*i*) Write the co-ordinates of A′ and B.

(*ii*) Write the co-ordinates of the middle point of the line segment A′B. **(1993)**

Sol. A′ is the reflection of A (4, – 1) reflected in y-axis.

∴ Co-ordinates of A′ are (– 4, – 1)

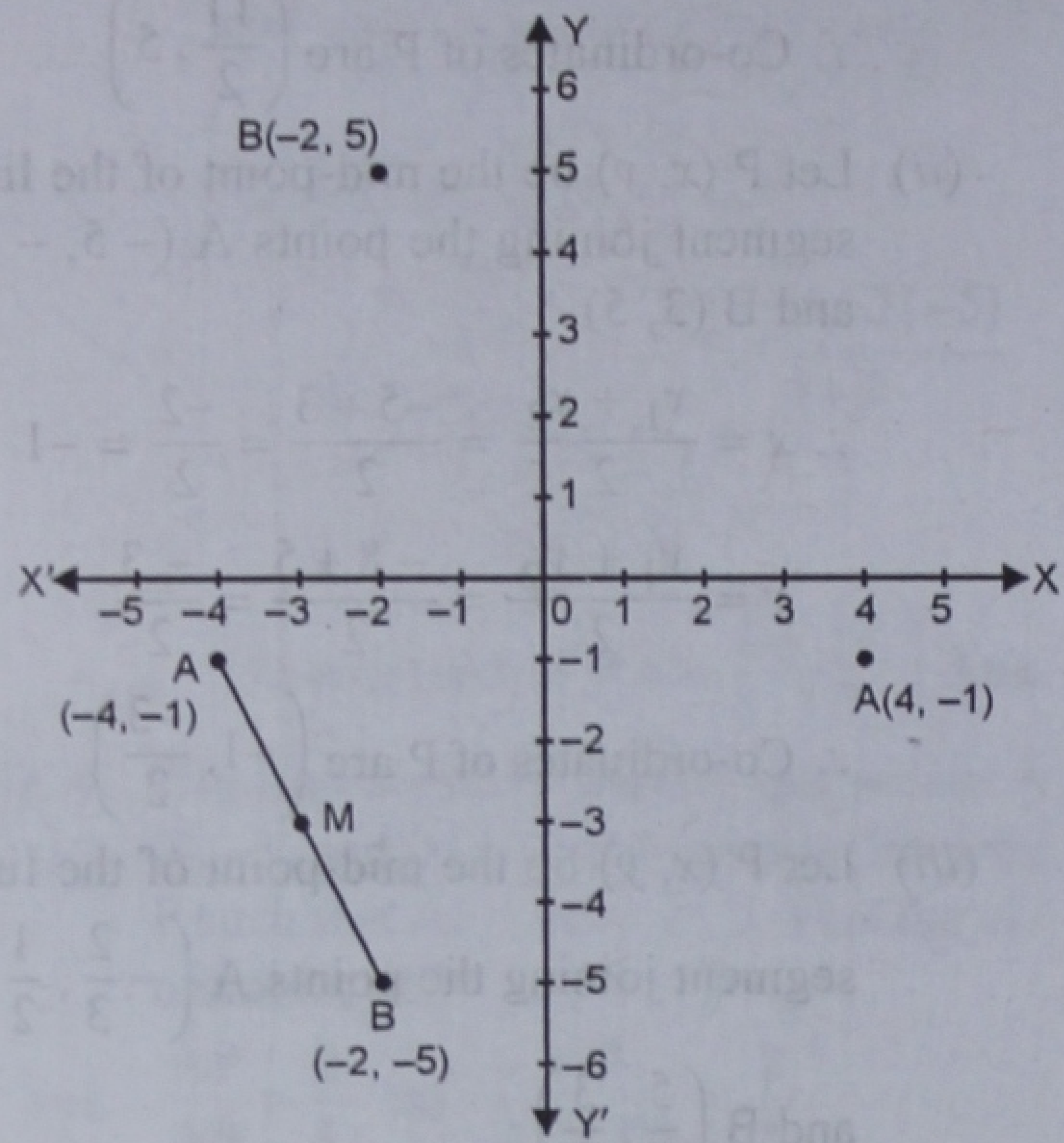
B′ (– 2, 5) is the reflection of B in x-axis.

∴ Co-ordinates of B will be = (– 2, – 5)

Let M is the mid-point of A′B, then coordinates of M will be

$$= \left(\frac{-4-2}{2}, \frac{-1-5}{2}\right) = \left(\frac{-6}{2}, \frac{-6}{2}\right)$$

= (– 3, – 3) **Ans.**



Q. 10. The line segment joining A (– 3, 1) and B (5, – 4) is a diameter of a circle whose centre is C. Find the co-ordinates of the point C. **(1990)**

Sol. ∵ C is the centre of the circle and AB is the diameter.

∴ C is the mid-point of AB.

Let co-ordinates of C (x, y)

$$\therefore \quad x = \frac{-3+5}{2},\ x = \frac{1-4}{2}$$

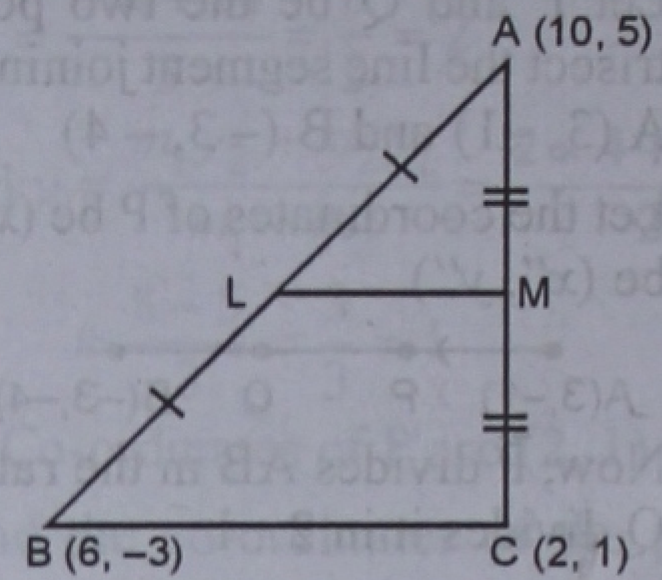
$$\Rightarrow \quad x = \frac{2}{2},\ y = \frac{-3}{2}$$

$$\Rightarrow \quad x = 1,\ y = \frac{-3}{2}$$

∴ Co-ordinates of C are $\left(1, \frac{-3}{2}\right)$ **Ans.**

Q. 11. A (10, 5), B (6, – 3) and C (2, 1) are the vertices of a ΔABC. L is the mid-point of AB and M is the mid-point of AC. Write down the co-ordinates of L and M. Show that $LM = \frac{1}{2}BC$. **(2001)**

Sol. Co-ordinates of A are (10, 5), of B and (6, – 3) and of C are (2, 1)



L is mid point of AB, M is mid-point of AC.

L and M are joined.

∵ L is the mid-point of AB.

∴ Co-ordinates of L will be

$$\left(\frac{10+6}{2}, \frac{5-3}{2}\right) \text{ or } \left(\frac{16}{2}, \frac{2}{2}\right) \text{ or } (8, 1)$$

∵ M is the mid-point of AC

∴ Co-ordinates of M will be

$$\left(\frac{10+2}{2}, \frac{5+1}{2}\right) \text{or} \left(\frac{12}{2}, \frac{6}{2}\right) \text{ or } (6, 3)$$

$$\therefore \text{ Length of } LM = \sqrt{(8-6)^2 + (1-3)^2}$$

$$= \sqrt{(2)^2 + (-2)^2}$$

$$= \sqrt{4+4} = \sqrt{8} = \sqrt{4 \times 2}$$

$$= 2\sqrt{2} \quad \ldots(i)$$

and length in of BC

$$= \sqrt{(6-2)^2 + (-3-1)^2}$$

$$= \sqrt{(4)^2 + (-4)^2}$$

$$= \sqrt{16+16} = \sqrt{16 \times 2} = 4\sqrt{2}$$

from (*i*) and (*ii*), it is clear that

$$LM = \frac{1}{2}BC$$

Hence proved.

Q. 12. (*i*) The co-ordinates of A and B are (– 3, *a*) and (1, *a* + 4). The mid-point of AB is (– 1, 1). Find the value of *a*.

(*ii*) The mid-point of the line segment, joining A $(p, 5)$ and B $(3, q)$ is M $(-1, 4)$. Find the values of p and q.

Sol. (*i*) The points A and B are $(-3, a)$ and $(1, a+4)$.

$\therefore$ Co-ordinates of the mid-point of AB are

$$\left[\frac{-3+1}{2}, \frac{a+a+4}{2}\right] \text{ i.e. } \left[\frac{-2}{2}, \frac{2a+4}{2}\right]$$

i.e. $(-1, a+2)$

But, the mid-point of AB is $(-1, 1)$

$\therefore a+2 = 1 \Rightarrow a = -1$ **Ans.**

(*ii*) Let M $(-1, 4)$ be the mid-point of the line segment A $(p, 5)$ and B $(3, q)$

$\therefore \quad -1 = \frac{p+3}{2}$

$\Rightarrow \quad p+3 = -2$

$\Rightarrow \quad p = -2-3 = -5$

and $4 = \frac{5+q}{2}$

$\Rightarrow \quad 5+q = 8$

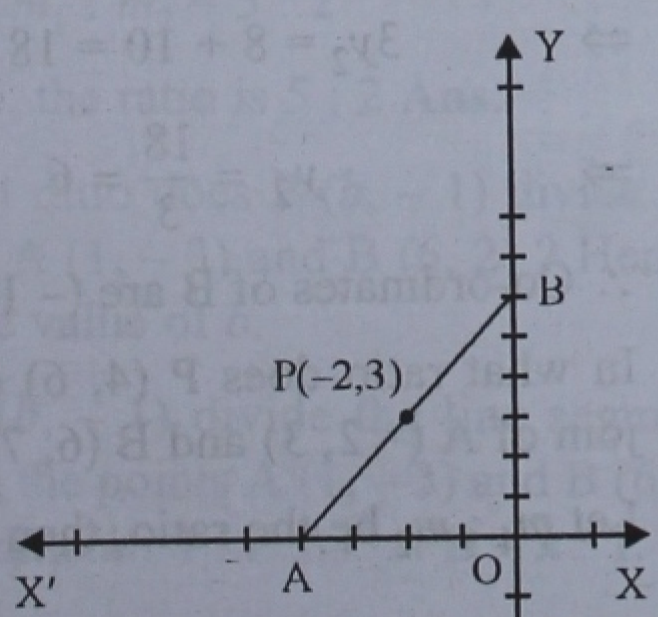
$\Rightarrow \quad q = 8-5 = 3$

$\therefore \quad p = -5, q = 3$ **Ans.**

Q. 13. In the adjoining figure, AB is a line-segment intersecting x–axis at A and y–axis at B. If P$(-2, 3)$ is the mid-point of AB, write down the co-ordinates of A and B.

Sol. A is on x–axis and B is on y–axis.

Let co-ordinates of A be $(x, 0)$ and B $(0, y)$



$\because$ P $(-2, 3)$ is the mid-point of AB

$\therefore -2 = \frac{x+0}{2} \quad \Rightarrow x+0 = -4$

$\Rightarrow x = -4$ and $3 = \frac{0+y}{2}$

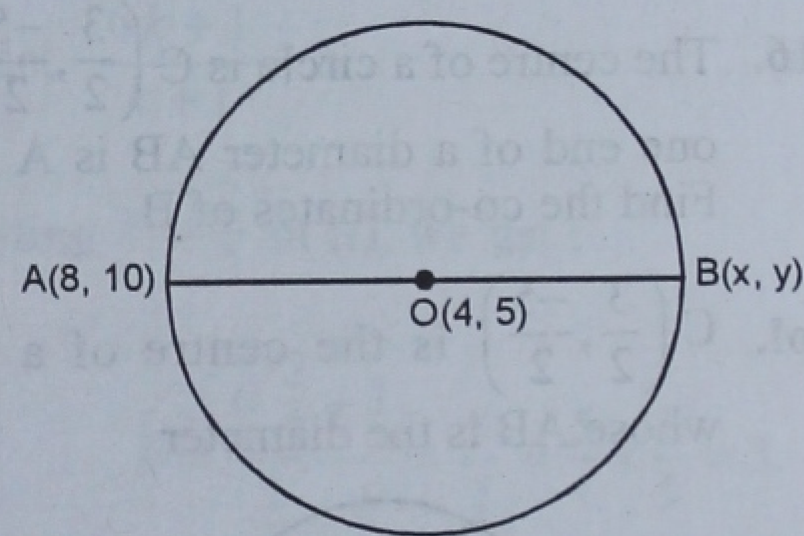
$\Rightarrow 0+y = 6 \quad \Rightarrow y = 6.$

$\therefore$ Co-ordinates of A are $(-4, 0)$ and of B are $(0, 6)$ **Ans.**

Q. 14. The centre O of a circle has the co-ordinates $(4, 5)$ and one point on the circumference is $(8, 10)$. Find the co-ordinates of the other end of the diameter of the circle through this point. **(1998)**

Sol. Co-ordinates of the centre O of a circle are $(4, 5)$

Point A $(8, 10)$ is on the circumference of the circle which is joined to O and produced to meet the circle at B.



$\therefore$ AOB is the diameter of the circle and O is mid-point of AB.

Let co-ordinates of B be (x, y)

$\therefore \quad 4 = \frac{8+x}{2}$ and $5 = \frac{10+y}{2}$

$\Rightarrow 8 = 8+x$ and $10 = 10+y$

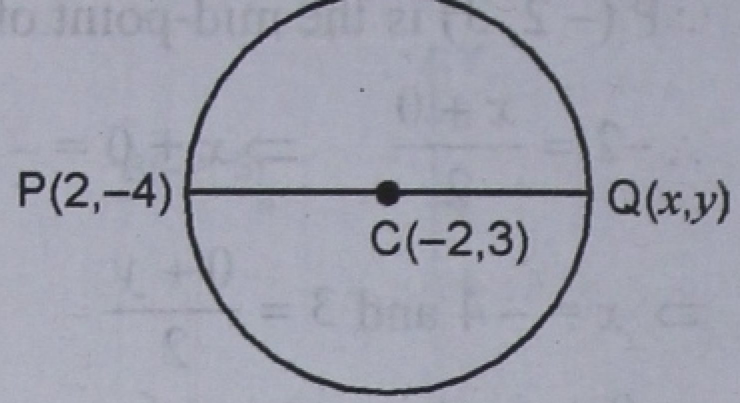
$\Rightarrow x = 8-8 = 0$ and $y = 10-10 = 0$

Hence, co-ordinates of B are $(0, 0)$ **Ans.**

Q. 15. The centre of a circle is C $(-2, 3)$ and one end of a diameter PQ is P $(2, -4)$. Find the co-ordinates of Q.

Sol. Let co-ordinates of Q be (x, y)

$\because$ C $(-2, 3)$ is the centre of the circle whose diameter is PQ.



∴ C is the mid-point of PQ.

$\therefore -2 = \dfrac{2+x}{2}$

$\Rightarrow 2 + x = -4$

$\Rightarrow x = -4 - 2 = -6$

and $3 = \dfrac{-4+y}{2}$

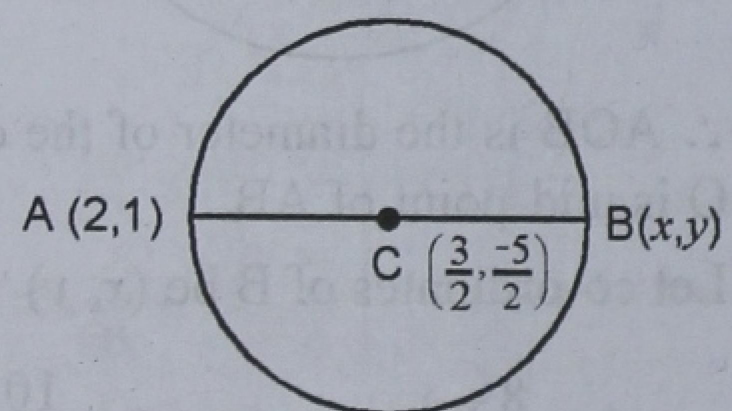
$\Rightarrow -4 + y = 6$

$\Rightarrow y = 6 + 4 = 10$

Hence co-ordinates of Q are (– 6, 10) **Ans.**

Q. 16. The centre of a circle is $C\left(\dfrac{3}{2}, \dfrac{-5}{2}\right)$ and one end of a diameter AB is A (2, 1). Find the co-ordinates of B.

Sol. $C\left(\dfrac{3}{2}, \dfrac{-5}{2}\right)$ is the centre of a circle whose AB is the diameter



Let co-ordinates of B be (x, y)

∵ C is the centre of the line segment joining the points A (2, 1) and B (x, y)

$\therefore \quad \dfrac{3}{2} = \dfrac{2+x}{2}$

$\Rightarrow 4 + 2x = 6$

$\Rightarrow 2x = 6 - 4 = 2$

$\Rightarrow x = \dfrac{2}{2} = 1$

and $\dfrac{-5}{2} = \dfrac{1+y}{2}$

$\Rightarrow 2 + 2y = -10$

$\Rightarrow 2y = -10 - 2 = -12$

$\Rightarrow y = \dfrac{-12}{2} = -6$

∴ Co-ordinates of B are (1, – 6) **Ans.**

Q. 17. The point P (– 4, 1) divides the line segment joining the points A (2, – 2) and B in the ratio 3 : 5. Find the point B.

Sol. Let the co-ordinates of B be(x_2, y_2) and co-ordinates of A are (2, – 2), P (– 4, 1) which divides AB in the ratio 3 : 5

$\therefore \quad x = \dfrac{m_1x_2 + m_2x_1}{m_1 + m_2}$

$\Rightarrow \quad -4 = \dfrac{3 \times x_2 + 5 \times 2}{3+5}$

$\Rightarrow \quad -4 = \dfrac{3x_2 + 10}{8}$

$\Rightarrow 3x_2 + 10 = -32$

$\Rightarrow \quad 3x_2 = -32 - 10 = -42$

$\therefore \quad x_2 = \dfrac{-42}{3} = -14$

and $\quad y = \dfrac{m_1y_2 + m_2y_1}{m_1 + m_2}$

$\Rightarrow \quad 1 = \dfrac{3 \times y_2 + 5 \times (-2)}{3+5}$

$\Rightarrow \quad 1 = \dfrac{3y_2 - 10}{8}$

$\Rightarrow 3y_2 - 10 = 8$

$\Rightarrow \quad 3y_2 = 8 + 10 = 18$

$\Rightarrow \quad y_2 = \dfrac{18}{3} = 6$

∴ Co-ordinates of B are (– 14, 6) **Ans.**

Q. 18. In what ratio does P (4, 6) divide the join of A (– 2, 3) and B (6, 7) ?

Sol. Let m_1 ; m_2 be the ratio, then

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$$

$$\Rightarrow 4 = \frac{m_1 \times 6 + m_2 \times (-2)}{m_1 + m_2}$$

$\Rightarrow 4\ (m_1 + m_2) = 6m_1 - 2m_2$

$\Rightarrow\ 4m_1 + 4m_2 = 6m_1 - 2m_2$

$\Rightarrow\ 4m_1 - 6m_1 = -2m_2 - 4m_2$

$\Rightarrow\ -2m_1 = -6m_2$

$$\Rightarrow \frac{m_1}{m_2} = \frac{-6}{-2} = \frac{3}{1}$$

$\therefore m_1 : m_2 = 3 : 1$

Hence, the ratio is 3 : 1 **Ans.**

Q. 19. In what ratio does P (2, – 5) divide the join of A (– 3, 5) and B (4, – 9) ?

Sol. Let $m_1 : m_2$ is the ratio, then

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$$

$$\Rightarrow 2 = \frac{m_1 \times 4 + m_2(-3)}{m_1 + m_2}$$

$$\Rightarrow 2 = \frac{4m_1 - 3m_2}{m_1 + m_2}$$

$\Rightarrow 4m_1 - 3m_2 = 2m_1 + 2m_2$

$\Rightarrow 4m_1 - 2m_1 = 2m_2 + 3m_2$

$\Rightarrow\ 2m_1 = 5m_2$

$$\Rightarrow \frac{m_1}{m_2} = \frac{5}{2}$$

$\Rightarrow\ m_1 : m_2 = 5 : 2$

Hence, the ratio is 5 : 2 **Ans.**

Q. 20. In what ratio does P (*b*, – 1) divide the join of A (1, – 3) and B (6, 2) ? Hence, find the value of *b*.

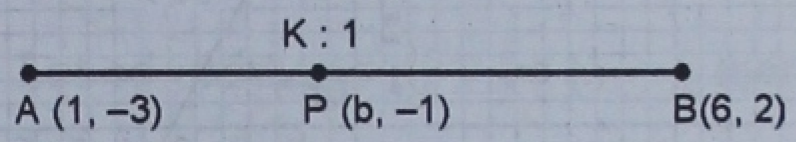
Sol. Let P (*b*, – 1) divide the line segment joining the points A (1, – 3) and B (6, 2) in the ratio *k* : 1 *i.e.* AP : PB = *k* : 1.

$\therefore$ Co-ordinates of P are

$$\left\{\frac{k.6 + 1.1}{k+1}, \frac{k.2 + 1.(-3)}{k+1}\right\}$$

But, P is (*p*, – 1)

$$\Rightarrow \frac{2k-3}{k+1} = -1$$



$\Rightarrow\ 2k - 3 = -k - 1$

$\Rightarrow 3k = 2$

$$\Rightarrow\ k = \frac{2}{3}.$$

$\therefore$ The required ratio is $\frac{2}{3}$; 1 *i.e.* 2 : 3 (internally).

$$\text{Also,}\quad \frac{6k+1}{k+1} = b \qquad \ldots(i)$$

Putting $k = \frac{2}{3}$ in (*i*), we get :

$$b = \frac{6.\frac{2}{3}+1}{\frac{2}{3}+1} = \frac{5}{\frac{5}{3}} = \frac{5}{1} \times \frac{3}{5} = 3.$$

Hence $b = 3$ **Ans.**

Q. 21. The line segment joining A (2, 3) and B (6, –5) is intercepted by the *x*-axis at the point K. Find the ratio in which K divides AB. Also, write the co-ordinates of the point K. (2006)

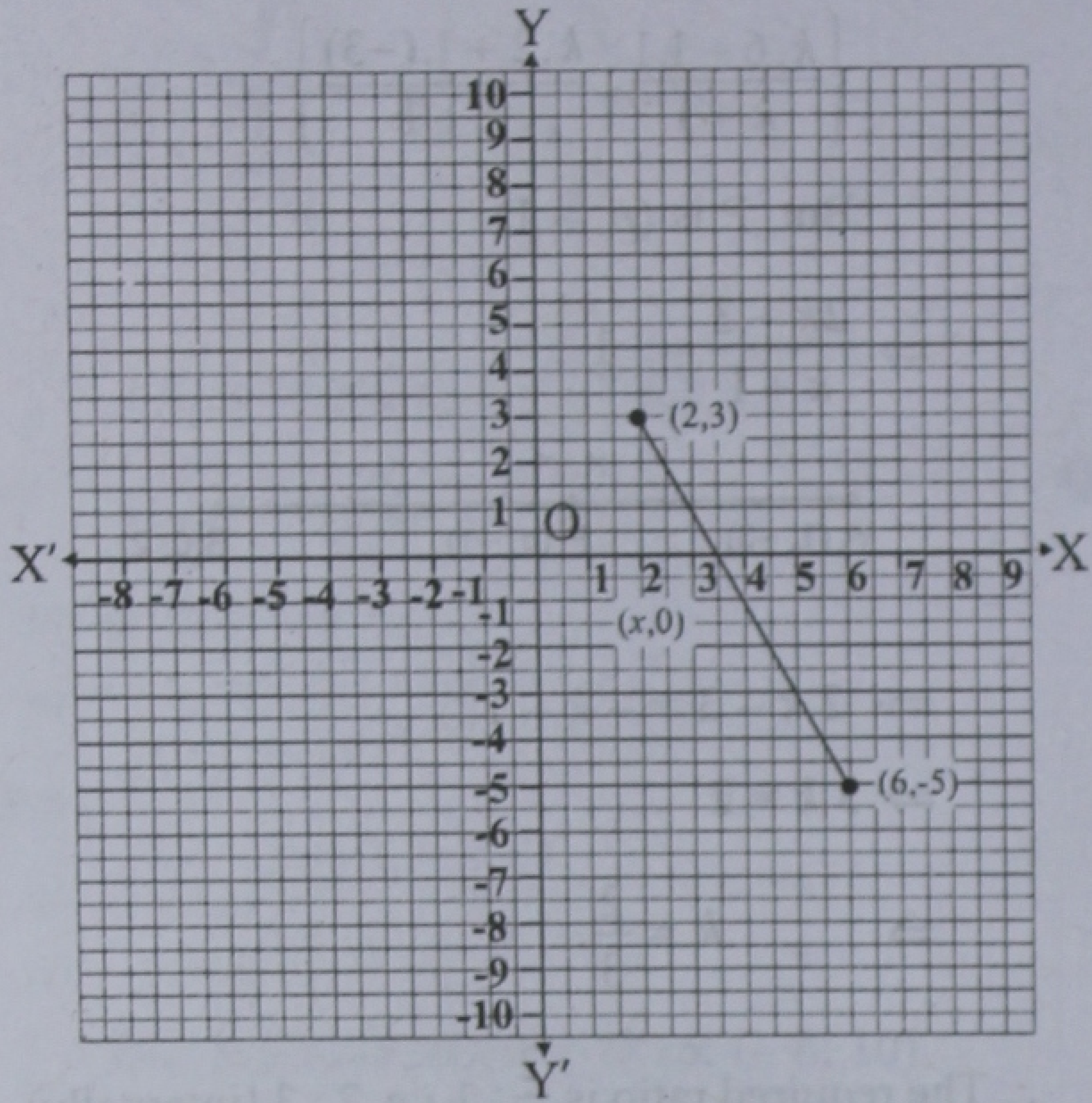
Sol. Let the line segment Intersect the *x*-axis at the point P

$\therefore$ Co-ordinates of P are (*x*, 0)

Let P divide the line segment in the ratio *k* : 1 then

$$\frac{-5k+3}{k+1} = 0 \Rightarrow -5k = -3 \Rightarrow k = \frac{3}{5}$$

Hence, required ratio is 3 : 5

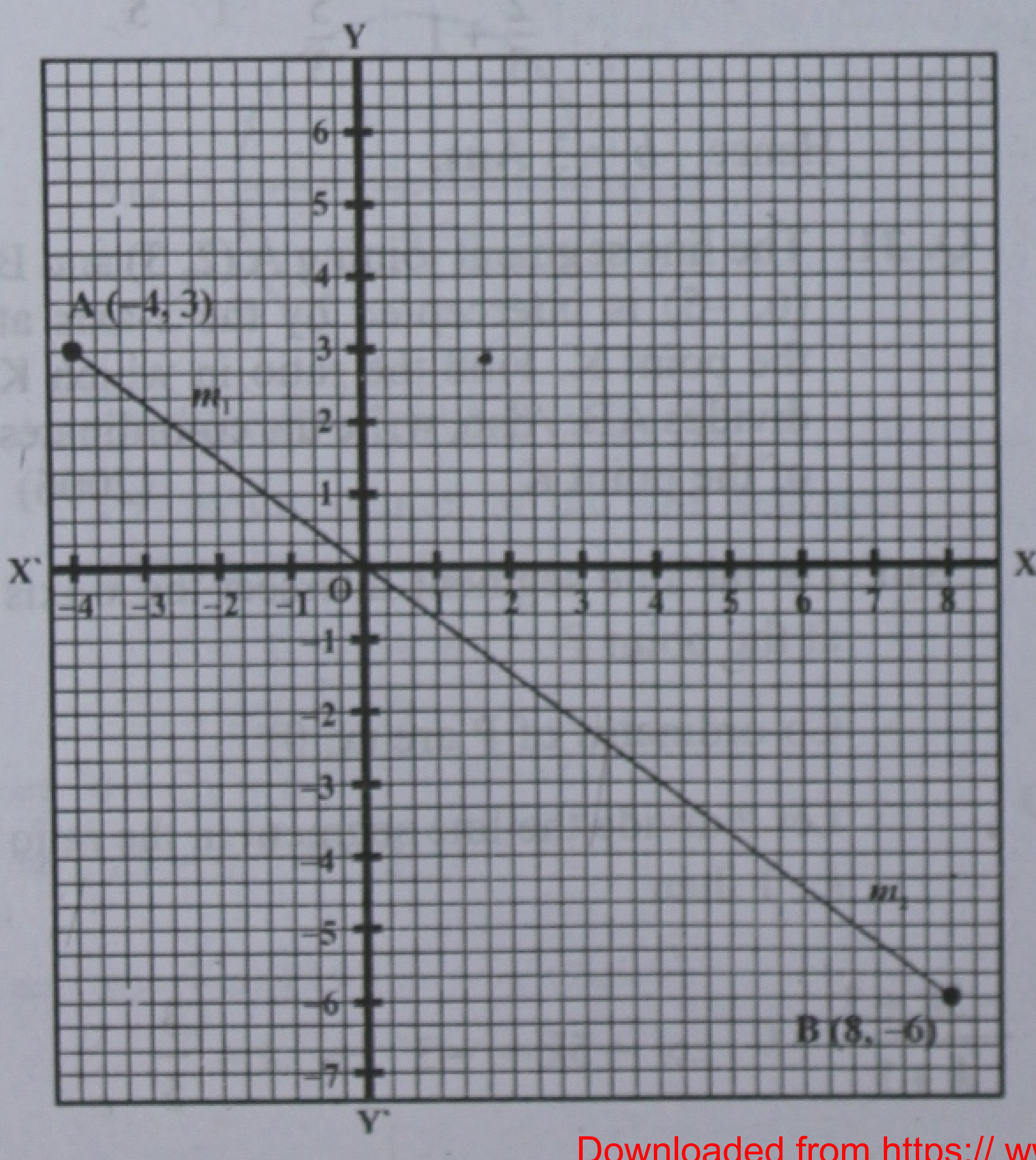


Q. 22. If A = (–4, 3) and B = (8, –6)

(i) find the length of AB

(*ii*) in what ratio is the line joining A and B, divided by the *x*-axis ? **(2008)**

Sol. A = (– 4, 3), B = (8, – 6)



$\therefore$ Length of AB = $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

$= \sqrt{[8-(-4)]^2+(-6-3)^2}$

$= \sqrt{(8+4)^2+(-6-3)^2}$

$= \sqrt{(12)^2+(-9)^2} = \sqrt{144+81} = \sqrt{225}$

= 15

Join AB. It passes through the origin O(0, 0)

Let O divides AB in the ratio $m_1 : m_2$

$\because x = \dfrac{m_1x_2+m_2x_1}{m_1+m_2}$

$\Rightarrow 0 = \dfrac{m_1\times 8+m_2\times(-4)}{m_1+m_2}$

$\Rightarrow 8m_1 - 4m_2 = 0$

$\Rightarrow 8m_1 = 4m_2$

$\Rightarrow \dfrac{m_1}{m_2} = \dfrac{4}{8} = \dfrac{1}{2}$

$\therefore m_1 : m_2 = 1 : 2$

Hence O, divides AB in the ratio 1 : 2 **Ans.**

Q. 23. In what ratio is the segment joining the points A (6, 5) and B (– 3, 2) divided by the *y*–axis ? Find the point at which the *y*–axis cuts AB.

Sol. Let P divides the line segment joining the points A (6, 5) and B (– 3, 2) in the ratio $m_1 : m_2$

$\because$ P lies on *y*–axis

$\therefore$ Its $x = 0$

Let co-ordinates of P be $(0, y)$, then

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$$

$$\Rightarrow \quad 0 = \frac{m_1(-3) + m_2 \times 6}{m_1 + m_2}$$

$$\Rightarrow \quad -3m_1 + 6m_2 = 0$$

$$\Rightarrow \quad 6m_2 = 3m_1$$

$$\Rightarrow \quad \frac{m_1}{m_2} = \frac{6}{3} = \frac{2}{1}$$

$$\Rightarrow \quad m_1 : m_2 = 2 : 1$$

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} = \frac{2 \times 2 + 1 \times 5}{2 + 1}$$

$$= \frac{4+5}{3} = \frac{9}{3} = 3$$

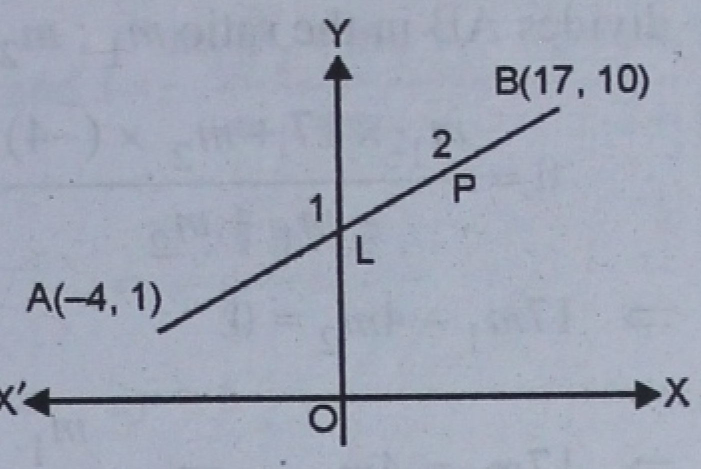
$\therefore$ Co-ordinates of point P are (0, 3) **Ans.**

Q. 24. (*i*) Write down the co-ordinates of the point P that divides the line joining A (– 4, 1) and B (17, 10) in the ratio 1 : 2.

(*ii*) Calculate the distance OP, where O is the origin.

(*iii*) In what ratio does the y-axis divide the line AB ? **(1995)**

Sol. Point P, divides a line segment giving the points A (– 4, 1) and B (17, 10) is the ratio 1 : 2.



(*i*) Let co-ordinates of P be (x, y), then

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$$

$$= \frac{1 \times 17 + 2 \times (-4)}{1+2} = \frac{17-8}{3}$$

$$= \frac{9}{3} = 3$$

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

$$= \frac{1 \times 10 + 2 \times 1}{1+2} = \frac{10+2}{3} = \frac{12}{3} = 4$$

$\therefore$ Co-ordinates of P will be (3, 4).

(*ii*) O (0, 0) is the origin

$$\therefore \quad OP = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(3-0)^2 + (4-0)^2}$$

$$= \sqrt{(3)^2 + (4)^2}$$

$$= \sqrt{9+16} = \sqrt{25} = 5 \text{ units.}$$

(*iii*) Line AB intersects y-axis at L

$\therefore$ abscissa of L is zero

Let, co-ordinates of L be $(0, y)$ and let L divides AB in the ratio $m_1 : m_2$

$$\therefore \quad 0 = \frac{m_1 \times 17 + m_2 \times (-4)}{m_1 + m_2}$$

$$\Rightarrow \quad 17m_1 - 4m_2 = 0$$

$$\Rightarrow \quad 17m_1 = 4m_2 \quad \Rightarrow \quad \frac{m_1}{m_2} = \frac{4}{17}$$

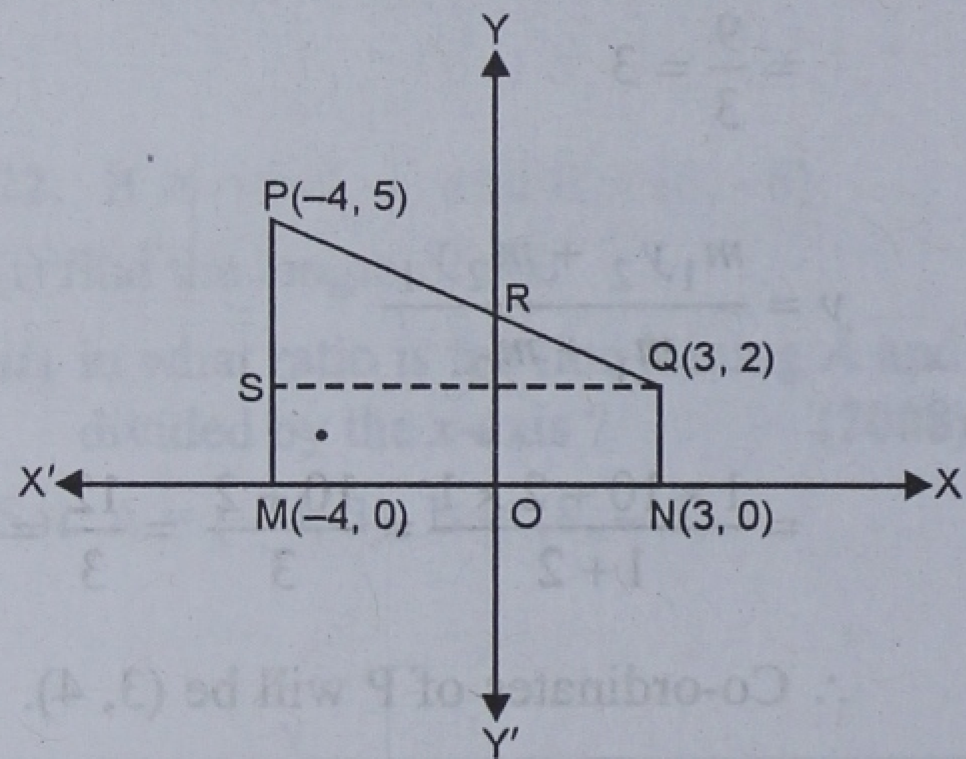
$\Rightarrow$ Ratio = 4 : 17 **Ans.**

Q. 25. The line joining P (– 4, 5) and Q (3, 2) intersects the y-axis at R, PM and QN are perpendiculars from P and Q on the x-axis. Find :

(*i*) The ratio PR : RQ.

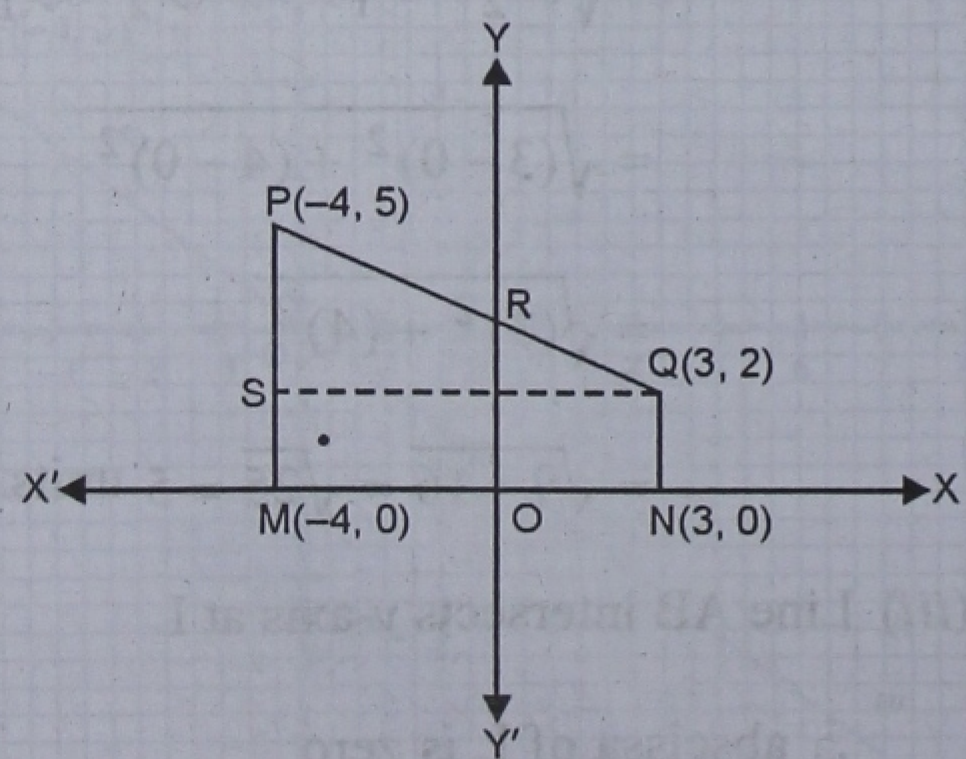
(*ii*) The co-ordinates of R.

(*iii*) The area of the quadrilateral PMNQ.



(2004)

Sol. (*i*) Let R divides the line joining the points P (– 4, 5) and Q (3, 2) in the ratio $k : 1$



$\therefore$ Co-ordinates of R will be

$$\left(\frac{3k - y}{k+1}, \frac{2k+5}{k+1}\right)$$

As R lies on y-axis

$\therefore$ its $x = 0$

$$\therefore \quad \frac{3k-4}{k+1} = 0 \Rightarrow 3k - 4 = 0$$

$$\Rightarrow \quad 3k = 4 \quad \Rightarrow \quad k = \frac{4}{3}$$

$\therefore$ Ratio = 4 : 3

(*ii*) Co-ordinates of R will be

$$\left(\frac{3 \times \frac{4}{3} - 4}{\frac{4}{3} + 1}, \frac{2 \times \frac{4}{3} + 5}{\frac{4}{3} + 1}\right)$$

$$\text{or} \quad \left(\frac{\frac{4-4}{1}}{\frac{4+3}{3}}, \frac{\frac{8+15}{3}}{\frac{4+3}{3}}\right)$$

$$\text{or} \quad \left(0, \frac{23}{3} \times \frac{3}{7}\right) \text{ or } \left(0, \frac{23}{7}\right)$$

(*iii*) Area of trapezium PMNQ

$$= \frac{1}{2}(\text{PM} + \text{QN}) \times \text{MN}$$

$$= \frac{1}{2}(5+2) \times 7 = \frac{1}{2} \times 7 \times 7$$

$= \dfrac{49}{2} = 24{\cdot}5$ sq. units **Ans.**

Q. 26. The line segment joining A $\left(-1, \frac{5}{3}\right)$ and B $(a, 5)$ is intersected by the y–axis at the point P in the ratio 1 : 3. Find

(*i*) the value of a ;

(*ii*) the co-ordinates of P.

Sol. Let, the co-ordinates of P be (x, y)

$\because$ P lies on y–axis

$\therefore$ its $x = 0$,

$$\text{Now, } x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$$

$$\Rightarrow \quad 0 = \frac{1 \times a + 3 \times (-1)}{1+3}$$

$$\Rightarrow \quad 0 = \frac{a-3}{4} = 0$$

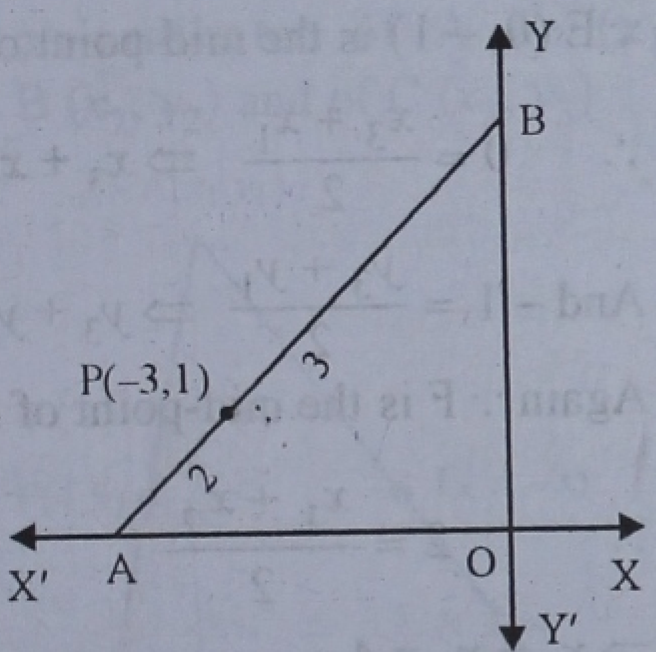
$$\Rightarrow a - 3 = 0 \qquad \Rightarrow a = 3$$

$$\text{And } y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} = \frac{1 \times 5 + 3 \times \frac{5}{3}}{1+3}$$

$$= \frac{5+5}{4} = \frac{10}{4} = \frac{5}{2}$$

$\therefore$ Co-ordinates of P are $\left(0, \frac{5}{2}\right)$ **Ans.**

Q. 27. In the given figure, the line segment AB meets x-axis at A and y-axis at B. The point P (–3, 1) on AB divides it in the ratio 2 : 3. Find the co-ordinates of A and B.



Sol. $\because$ A lies on x-axis

$\therefore$ its $y = 0$

And B lies on y-axis

$\therefore$ its $x = 0$

Now, let co-ordinates of A be $(x, 0)$ and of B be $(0, y)$ and point P (–3, 1) divides AB in the ratio 2 : 3

$$\therefore \ -3 = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$$

$$\Rightarrow -3 = \frac{2 \times 0 + 3 \times x}{2+3} \quad \Rightarrow -3 = \frac{0 + 3x}{5}$$

$$\Rightarrow 3x = -15 \qquad \Rightarrow \quad x = \frac{-15}{3} = -5$$

$$\text{and } 1 = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} = \frac{2 \times y + 3 \times 0}{2+3}$$

$$= \frac{2y + 0}{5} = \frac{2y}{5}$$

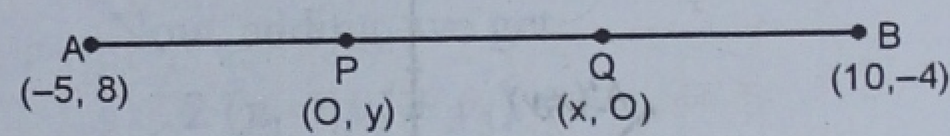
$$\Rightarrow \quad 2y = 5$$

$$\Rightarrow \quad y = \frac{5}{2}$$

$\therefore$ Co-ordinates of A are (–5, 0) and of B are $\left(0, \frac{5}{2}\right)$ **Ans.**

Q. 28. Show that the line segment joining the points A (– 5, 8) and B (10, – 4) is trisected by the co-ordinate axes. Also, find the points of trisection of AB.

Sol.

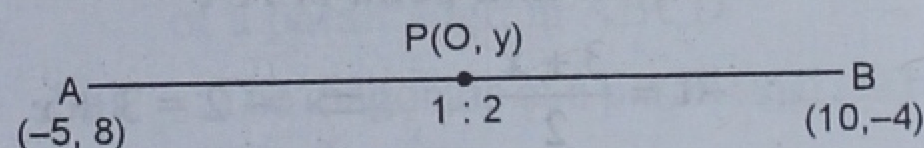


Let the points A (– 5, 8) and B (10, – 4). Let P and Q be the two points on the axis which trisect the line joining the points A and B.

$\because \qquad$ AP = PQ = QB

$\therefore \qquad$ AP : PB = 1 : 2

and AQ : QB = 2 : 1



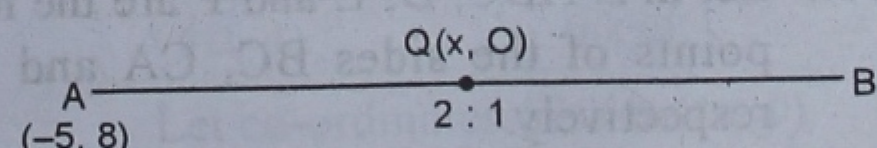
Now, co-ordinates of P will be

$$x = \frac{1 \times 10 + 2 \times (-5)}{1+2} = \frac{10-10}{3} = 0$$

$$y = \frac{1 \times (-4) + 2 \times 8}{1+2}$$

$$= \frac{-4+16}{3} = \frac{12}{3} = 4$$

$\therefore$ Co-ordinates of P are (0, 4)



Co-ordinates of Q will be,

$$x = \frac{2 \times 10 + 1 \times (-5)}{2+1}$$

$$= \frac{20-5}{3} = \frac{15}{3} = 5$$

$$y = \frac{2 \times (-4) + 1 \times 8}{2+1}$$

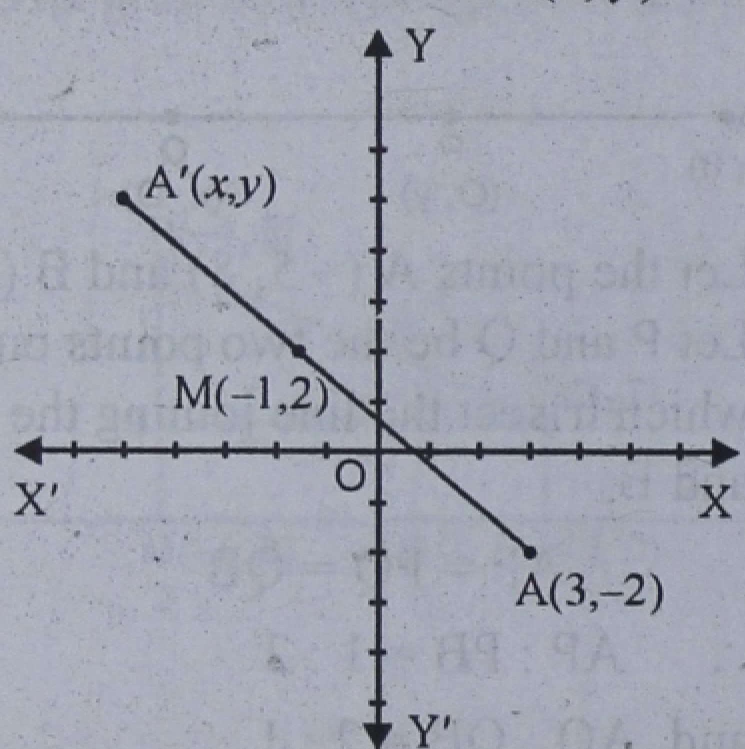
$$= \frac{-8+8}{3} = \frac{0}{3} = 0$$

$\therefore$ Co-ordinates of Q are (5, 0).

Hence proved.

Q. 29. Find the image of the point A (3, – 2) under reflection in the point M (– 1, 2).

Sol. Let A′ be the image of point A (3, – 2) under reflection M (– 1, 2) and let the co-ordinates of A′ be (x, y)



$\because$ M is the mid-point of AA′

$$\therefore -1 = \frac{3+x}{2} \Rightarrow -2 = 3 + x$$

$$\Rightarrow x = -2 - 3 = -5$$

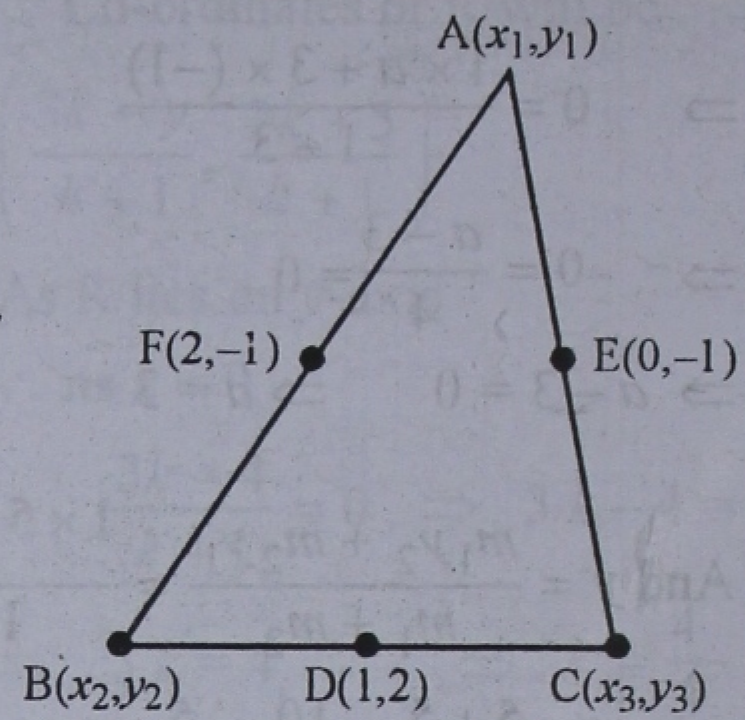
$$\text{And } 2 = \frac{-2+y}{2} \Rightarrow 4 = -2 + y$$

$$\Rightarrow y = 4 + 2 = 6$$

$\therefore$ Co-ordinates of A′ are (– 5, 6) **Ans.**

Q. 30. The co-ordinates of the mid-points of the sides of a triangle are (1, 2), (0, – 1) and (2, – 1) respectively. Find the co-ordinates of the vertices of the triangle.

Sol. Let in Δ ABC, D, E and F are the mid-points of the sides BC, CA and AB respectively.



Let the co-ordinates of vertices, A, B and C are A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3)

$\because$ D (1, 2) is the mid-point of BC.

$$\therefore \quad 1 = \frac{x_2 + x_3}{2}$$

$$\Rightarrow x_2 + x_3 = 2 \text{ and } 2 = \frac{y_2 + y_3}{2}$$

$$\Rightarrow y_2 + y_3 = 4$$

$\because$ E (0, – 1) is the mid-point of CA

$$\therefore \quad 0 = \frac{x_3 + x_1}{2} \Rightarrow x_3 + x_1 = 0$$

$$\text{And } -1 = \frac{y_3 + y_1}{2} \Rightarrow y_3 + y_1 = -2$$

Again $\because$ F is the mid-point of AB.

$$\therefore \quad 2 = \frac{x_1 + x_2}{2}$$

$$\Rightarrow x_1 + x_2 = 4$$

$$\Rightarrow \quad -1 = \frac{y_1 + y_2}{2}$$

$$\Rightarrow y_1 + y_2 = -2$$

Now, $x_1 + x_2 = 4$ ...(*i*)

$x_2 + x_3 = 2$ ...(*ii*)

$x_3 + x_1 = 0$ ...(*iii*)

Adding, we get

$$2(x_1 + x_2 + x_3) = 6$$

$$\Rightarrow x_1 + x_2 + x_3 = 3 \quad \text{...(iv)}$$

Subtracting (*i*), (*ii*) and (*iii*) from (*iv*), we get

$$x_3 = 3 - 4 = -1,\ x_1 = 3 - 2 = 1$$

and $x_2 = 3 - 0 = 3$

Similarly, $y_2 + y_3 = 4$ ...(v)

$y_3 + y_1 = -2$ ...(vi)

$y_1 + y_2 = -2$ ...(vii)

Adding, we get

$2(y_1 + y_2 + y_3) = 0$

$\Rightarrow \quad y_1 + y_2 + y_3 = 0$ ...(viii)

Subtracting (v), (vi) and (vii) from (viii),

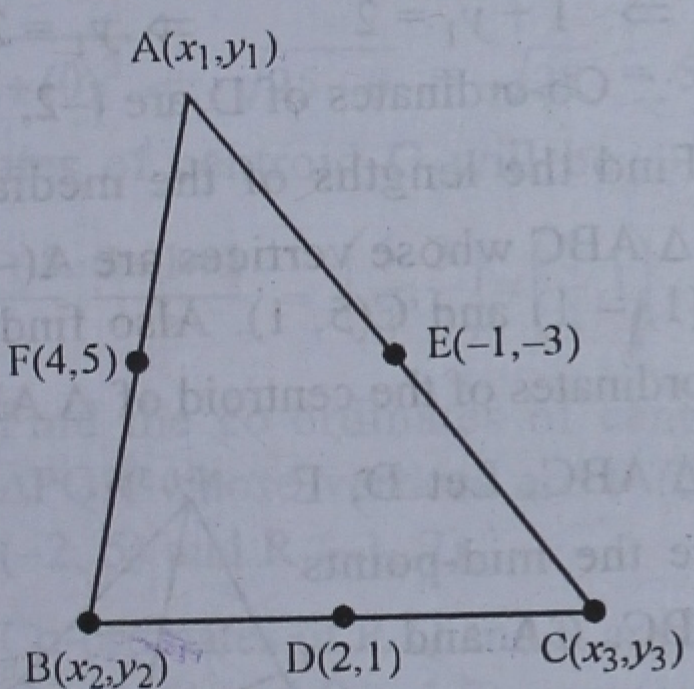
$y_1 = 0 - 4 = -4, y_2 = 0 - (-2) = 2$

$y_3 = 0 - (-2) = 2$

$\therefore$ Co-ordinates of A, B and C are (1, – 4), (3, 2), (– 1, 2) **Ans.**

Q. 31. The mid-points of the sides BC, CA and AB of Δ ABC are D (2, 1), (– 1, – 3) and F (4, 5) respectively. Find the co-ordinates of A, B and C.

Sol. $\because$ D (2, 1), E (– 1, – 3) and F (4, 5) are the mid-points of the sides BC, CA and AB of ΔABC respectively. Let the co-ordinates of A be (x_1, y_1) of B (x_2, y_2) and of C (x_3, y_3)



$\because$ D (2, 1) is the mid-point of BC

$\therefore 2 = \dfrac{x_2 + x_3}{2} \quad \Rightarrow x_2 + x_3 = 4$

$1 = \dfrac{y_2 + y_3}{2} \quad \Rightarrow y_2 + y_3 = 2$

Again, E (– 1, – 3) is the mid-point of CA

$\therefore \quad -1 = \dfrac{x_3 + x_1}{2} \quad \Rightarrow x_3 + x_1 = -2$

And $-3 = \dfrac{y_3 + y_1}{2} \quad \Rightarrow y_3 + y_1 = -6$

and F (4, 5) is the mid-point of AB

$\therefore \quad 4 = \dfrac{x_1 + x_2}{2} \Rightarrow x_1 + x_2 = 8$

And $5 = \dfrac{y_1 + y_2}{2} \Rightarrow y_1 + y_2 = 10$

Now, $x_1 + x_2 = 8$ ...(i)

$x_2 + x_3 = 4$ ... (ii)

$x_3 + x_1 = -2$ ... (iii)

Adding, we get

$2(x_1 + x_2 + x_3) = 10$

$\Rightarrow x_1 + x_2 + x_3 = 5$ ...(iv)

Now, subtracting (i), (ii) and (iii) from (iv), we get

$x_3 = -3, x_1 = 1, x_2 = 7$

And $y_1 + y_2 = 10$ ...(v)

$y_2 + y_3 = 2$ ...(vi)

$y_3 + y_1 = -6$ ...(vii)

Now, adding, we get

$2(y_1 + y_2 + y_3) = 6$

$\Rightarrow \quad y_1 + y_2 + y_3 = 3$ ...(viii)

Subtracting (v), (vi), (vii) from (viii),

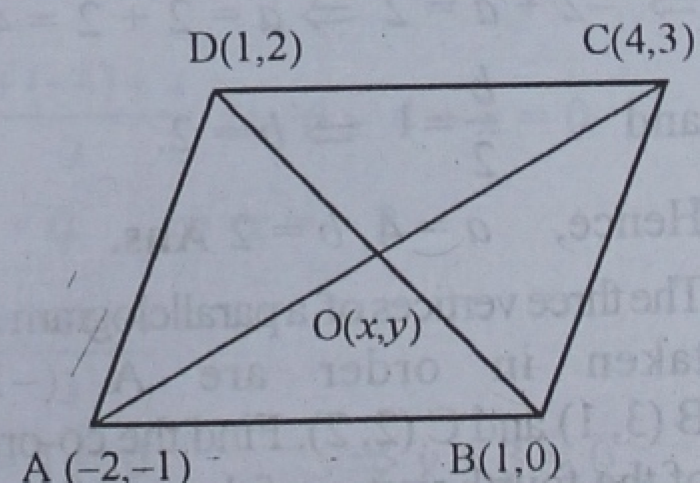
$\therefore \quad y_3 = -7, y_1 = 1, y_2 = 9$

$\therefore$ Co-ordinates of points A, B and C are (1, 1), (7, 9) and (–3, –7) **Ans.**

Q. 32. Prove that the points A (–2, –1), B (1, 0) and C (4, 3) and D(1, 2) are the vertices of a parallelogram ABCD.

Sol. $\because$ The diagonals of a parallelogram bisect each other.

$\therefore$ AC and BD bisect each other at O or O, is the mid-point of AC as well as of BD.



Let co-ordinates of O be (x, y)

(*i*) If O is the mid-point of AC, then

$$x=\frac{-2+4}{2}=\frac{2}{2}=1$$

$$y=\frac{-1+3}{2}=\frac{2}{2}=1$$

∴ Co-ordinates of O will be (1, 1)

(*ii*) If O is the mid-point of BD, then

$$x=\frac{1+1}{2}=\frac{2}{2}=1$$

$$y=\frac{0+2}{2}=\frac{2}{2}=1$$

∴ Co-ordinates of O are (1, 1)

Hence, we can say that ABCD is a parallelogram.

Q. 33. If the points A (–2, –1), B (1, 0), C (*a*, 3) and D (1, *b*) form a parallelogram, find the values of *a* and *b*.

Sol. The vertices of a parallelogram ABCD are A (–2, –1), B (1, 0), C (*a*, 3) and D (1, *b*)

Let, its diagonal AC and BD bisect each other at O *i.e.* O (*x*, *y*) is the mid-point of AC as well as of BD.

When O, is the mid-point of AC, then

$$\therefore\ x=\frac{-2+a}{2} \text{ and } y=\frac{-1+3}{2}=\frac{2}{2}=1$$

Again O, is the mid-point of BD, then

$$x=\frac{1+1}{2}=\frac{2}{2}=1,\ y=\frac{0+b}{2}=\frac{b}{2}$$

∵ O is the mid-point of AC and BD both

$$\therefore\ \frac{-2+a}{2}=1 \text{ and } \frac{b}{2}=1$$

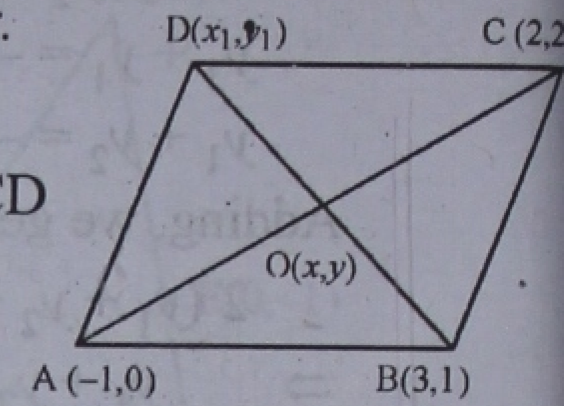
$\Rightarrow -2+a=2 \Rightarrow a=2+2=4$

and $\frac{b}{2}=1 \Rightarrow b=2.$

Hence, $a=4, b=2$ **Ans.**

Q. 34. The three vertices of a parallelogram ABCD, taken in order are A (–1, 0), B (3, 1) and C (2, 2). Find the co-ordinates of the fourth vertex of the parallelogram.

Sol. ∵ The diagonals of a parallelogram bisect each other.



∴ Diagonals AC and BD of the parallelogram ABCD bisect each other at O.

Let co-ordinates of O be (*x*, *y*)

As O is the mid-point of AC.

$$\therefore\ x=\frac{-1+2}{2}=\frac{1}{2} \text{ and } y=\frac{0+2}{2}=1$$

$$\therefore \text{ Co-ordinates of O are } \left(\frac{1}{2},1\right)$$

Again, O is mid-point of BD and let co-ordinates of D be (x_1, y_1).

Then $\frac{1}{2}=\frac{3+x_1}{2} \Rightarrow 3+x_1=1$

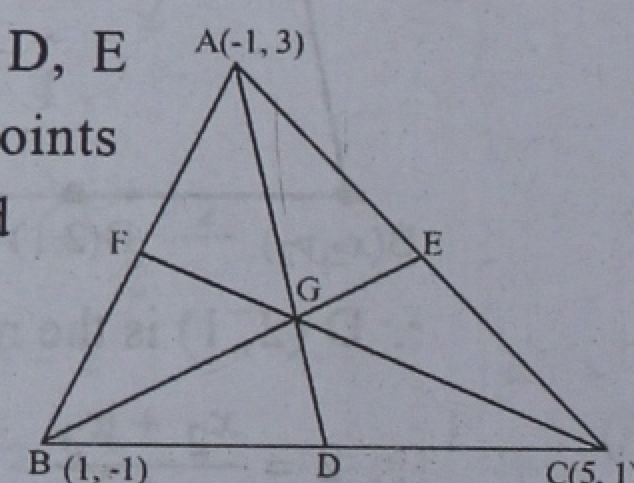
$\Rightarrow x_1=1-3=-2$ and $1=\frac{1+y_1}{2}$

$\Rightarrow 1+y_1=2 \quad \Rightarrow y_1=2-1=1$

∴ Co-ordinates of D are (–2, 1) **Ans.**

Q. 35. Find the lengths of the medians of a Δ ABC whose vertices are A(– 1, 3), B (1, – 1) and C(5, 1). Also find the co-ordinates of the centroid of Δ ABC.

Sol. In Δ ABC, Let D, E and F are the mid-points of sides BC, CA and AB respectively



∴ AD, BE and CF are the medians of Δ ABC which intersect at G.

∴ G is the centroid of the Δ ABC.

Co-ordinates of A, B and C are (– 1, 3), (1, – 1) and (5, 1) respectively

∴ Co-ordinates of D will be

$$\left(\frac{1+5}{2},\frac{-1+1}{2}\right) \text{ or } \left(\frac{6}{2},\frac{0}{2}\right) \text{ or } (3, 0)$$

Co-ordinates of E will be $\left(\frac{5-1}{2}, \frac{1+3}{2}\right)$ or

$\left(\frac{4}{2}, \frac{4}{2}\right)$ or (2, 2)

nd co-ordinates of F will be $\left(\frac{-1+1}{2}, \frac{3-1}{2}\right)$ or

$\left(0, \frac{2}{2}\right)$ or (0, 1)

∴ Length of AD will be = $\sqrt{(3+1)^2+(0+3)^2}$

= $\sqrt{(4)^2+(3)^2}$ = $\sqrt{16+9}=\sqrt{25}=5$ units

Length of BE will be = $\sqrt{(1-2)^2+(-1-2)^2}$

= $\sqrt{(-1)^2+(-3)^2}$ = $\sqrt{1+9}=\sqrt{10}$ units

and length of CF will be = $\sqrt{(5-0)^2+(1-1)^2}$

= $\sqrt{(5)^2+(0)^2}$ = $\sqrt{25+0}$ = $\sqrt{25}$ = 5 units.

Co-ordinates of centroid G will be

= $\frac{-1+1+5}{3}, \frac{3-1+1}{3}$ = $\left(\frac{5}{3}, \frac{3}{3}\right)=\left(\frac{5}{3}, 1\right)$ Ans.

Q.36. Find the co-ordinates of centroid of ΔPQR whose vertices are P (6, 3), Q (–2, 5) and R (–1, 7).

Sol. Co-ordinates of P, Q and R are P (6, 3), Q (–2, 5) and R (–1,7)

Let G be the centroid of the ΔPQR

∴ Co-ordinates of G will be

$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

or $\left(\frac{6-2-1}{3}, \frac{3+5+7}{3}\right)$ or $\left(\frac{3}{3}, \frac{15}{3}\right)$ or (1, 5)

Hence G is (1, 5) **Ans.**

Q.37. Find the co-ordinates of the point of intersection of the medians of the triangle whose vertices are A (–7, 5), B (–1, –3) and C (5, 7).

Sol. ∵ The median of a triangle intersect each other at one point say G.

∴ G is the centroid of the ΔABC.

∴ Co-ordinates of G will be

$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$ or

$\left(\frac{-7-1+5}{3}, \frac{5-3+7}{3}\right)$ or $\left(\frac{-3}{3}, \frac{9}{3}\right)$ or (–1, 3)

Hence, G is (–1, 3) **Ans.**

Q.38. If G (–2, 1) is the centroid of ΔABC, two of whose vertices are A (1, –6) and B (–5, 2), find the third vertex of the triangle.

Sol. Let the co-ordinates of third vertex C be (x, y)

∴ G (–2, 1) is the centroid of the ΔABC in which A is (1, –6) and B is (–5, 2)

$\therefore -2=\frac{1+(-5)+x}{3}=\frac{1-5+x}{3}=\frac{-4+x}{3}$

$\Rightarrow -4+x=-6 \Rightarrow x=-6+4=-2$

and $1=\frac{-6+2+y}{3}=\frac{-4+y}{3}$

$\Rightarrow -4+y=3 \quad \Rightarrow y=3+4=7$

∴ Co-ordinates of vertex C are (–2, 7) **Ans.**

39. A (6, y), B (– 4, 4) and C(x, –1) are the veritces of ΔABC whose centroid is the origin. Calculate the values of x and y.

Sol. ∵ Origin O(0, 0) is the centroid of the ΔABC whose vertices are A (6, y), B(–4, 4) and C (x, –1)

$\therefore 0=\frac{6+(-4)+x}{3} \Rightarrow 6-4+x=0$

$\Rightarrow 2+x=0 \quad \Rightarrow x=-2$

and $0=\frac{y+4-1}{3}$

$\Rightarrow y+4-1=0 \qquad \Rightarrow y+3=0$

$\therefore y=-3 \qquad \therefore x=-2, y=-3$ Ans.